Development of mathematical optimisation models for predicting the structural properties of rice husk ash (RHA) concrete using Osadebe second degree polynomials

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Abstract

The demand for concrete is increasing in tandem with population growth and urbanization. Cement is an important ingredient in concrete production. Cement is a major contributor to global carbon dioxide emissions during its manufacturing processes. Therefore, sustainable alternatives to normal cement are required for the production of sustainable concrete. Rice husk ash has proven intriguing properties as a sustainable alternative for producing green and eco-friendly concrete. Because the laboratory work needed to assess its properties is both time-consuming and complex, regression models can be effectively used to predict the properties of concrete containing rice husk ash. Using Osadebe's second-degree polynomial equation, a mathematical optimization model for predicting the compressive, tensile, and flexural strengths of RHA concrete was developed in this study. The developed model may be used to compute compressive, tensile, and flexural strengths based on the proportions of four constituents in a given mix. Also, a favourable comparison may be drawn between the model and experimental responses. Furthermore, the statistical analysis summary revealed that the model-predicted values were in good agreement with the experimental values. Finally, the strength values achieved from some of the optimised mixes are adequate for use as structural or load-bearing concrete.

Keywords: concrete, optimisation, compressive strength, flexural strength, Osadebe polynomial, RHA, tensile strength

Kulcsszavak: beton, optimalizálás, nyomószilárdság, hajlítószilárdság, Osadebe-polinom, RHA, szakítószilárdság

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1. Introduction

Global cement usage started to increase in the second half of the twentieth century, resulting in a significant growth in the production of cement [1]. This trend is predicted to continue through the 2020s, with significant growth by 2030 [1-2]. With the massive global increase in greenhouse gas (GHG) emissions, the Antarctic and Arctic polar ice caps have been rapidly melting, extreme weather events have incurred economic damage, and climate change impacts have been intensifying [3]. The cement and concrete industries contribute significantly to GHG emissions [4-5]. Based on infrastructure development, cement production accounts for 5-8% of current global CO_2 emissions [6]. Approximately 850 kg of CO_2 are emitted into the environment for each tonne of clinker produced using present cement production procedures [7]. The global demand for concrete is increasing. Portland cement (PC) is the concrete constituent that contributes the most to GHG emissions. Aside from GHG emissions, both advanced and developing countries have struggled in recent decades with the safe disposal and effective utilisation of industrial and agricultural by-products regarded as merely solid waste [8-13]. Concrete is often regarded as the most resilient and widely used man-made material for infrastructure development on planet Earth [14]. Because of rising urbanisation in developing countries, the global use of concrete is increasing every day. Annual concrete production is expected to be over 30 billion metric tonnes [3, 14-15]. The concrete industry currently requires over 1.5 billion metric tonnes of cement per year [16]. It is expected that cement consumption will increase from 4.2 billion metric tonnes now to 5.2 billion metric tonnes by 2050 [7, 17]. This will put more pressure on the cement industry to generate a significant quantity of cement in order to satisfy the increased demand for infrastructure development. Most cement factories emit a large quantity of waste that causes environmental damage [9].

However, alternative materials for supplementing cement are needed to reduce the negative impact of concrete production. It is important to identify an alternative binder with a lower carbon footprint than cement. The use of supplementary cementitious materials (SCM) or blended cement would be the most viable option for attaining some of the goals of sustainable development. Using recycled or waste materials in concrete might reduce its environmental impact. Using supplemental cementitious materials (SCMs) to minimise PC use while disposing of waste materials from different industries is a viable approach [18-20]. SCMs have been a central focus in the quest to enhance concrete sustainability [21-22]. To mitigate this environmental concern and improve the engineering properties of concrete, industrial SCM such as fly ash, ground granulated blast slag, metakaolin, slag cement, and silica fume (SF) have been used [23-25]. Agricultural wastes such as palm oil fuel ash (POFA), rice husk ash (RHA), olive pomace ash (OOA), sugarcane bagasse ash (SBA), and coconut shell ash can also be used in sustainable concrete as partial replacements for cement [26-28]. If only 30% of total cement usage could be replaced by SCMs, CO_2 emissions from cement production would be minimised [9].

A myriad of studies have been carried out in recent decades to alleviate the negative impacts of using OPC in concrete. *Hussain et al.* [29] investigated whether high-strength fly ash concrete had the same compressive strength as plain concrete; fly ash concrete had higher compressive values than plain concrete. In a study performed by *Garg et al.* [30], various waste materials were used to partially replace sand and cement. Using an adaptive fuzzy logic model, they developed a model to predict the compressive strength of concrete made of fly ash and slag. *De Maeijer et al.* [31] observed that replacing fly ash with concrete might enhance its resistance and chloride migration coefficient, as well as the alkali-silicon reaction; however, carbonization resistance would be reduced. *Liu et al.* [7] examined shrinkage in creep and curing, compression strength, and emissions of carbon dioxide from concrete incorporating fly ash or ground-granulated blast-furnace slag (GGBS). By introducing a parameter to account for the effect of fly ash content, their proposed model successfully predicted the creep strain of concrete. It is widely acknowledged that reducing the quantity of OPC used in concrete could indeed enhance the overall sustainability performance of mix designs, as OPC is the constituent with the highest impact on the environment [32]. It has been established that replacing OPC with SCM is environmentally beneficial in terms of emissions of greenhouse gases [33]. Taking these important factors into account, greater OPC replacement with SCM provides tremendous environmental benefits.

Aside from industrial-based SCMs, RHA, an agro-based SCM, has been found to satisfy the majority of the requirements for durable concrete and to be more effective than other supplementary materials such as fly ash and silica fume [34]. Rice husk (RH) is a primary agricultural waste produced during the milling process from the exterior surface of rice grains. Annual global rice production is estimated to be 748 million metric tonnes (mmt) [35]. RH accounts for 20% of the world's million metric tonnes of rice production [9]. Nigerian rice paddy production was around 8.34 million in 2021, and it has increased at an annual rate of 8.72% in successive years. This implies that rice husk is commercially available and sustainable. Rice husks contain about 30% of the weight of raw rice [36]. Rice husk ash (RHA) is produced by burning dried rice husk at a temperature of around 750°C to make ash and remove volatile organic carbon such as lignin and cellulose [37]. The amorphous RHA contains approximately 75% silica (SiO_2), making it an excellent pozzolan in cementitious materials, where the silicon oxide in the RHA reacts with the $Ca(OH)_2$ from the cement hydration process to form more calcium silicate hydrate (C-S-H), which is responsible for strength development in the cementitious matrix [38].

RHA's suitability for structural concrete production is valued as a green and eco-friendly construction material that seeks to minimise the cement ingredient in the mix [39-40]. The use of agricultural products as a source of energy, preceded by the use of the by-products as a constituent in concrete, may provide a way to contribute to the achievement of sustainable development goals by reducing the environmental impacts of energy production and concrete material manufacturing. If energy-intensive Portland cement is partially replaced with agricultural byproducts, significant energy and cost savings can be achieved [41]. Considering that Portland cement is a porous material with discrete and connected pores of various sizes and shapes, concrete requires a very finely blended material with a fineness comparable to Portland cement for optimum pore size augmentation and decreased permeability [42]. RHA is one of these extremely finely powdered minerals. RHA particles operate as a microfiller in cement paste, enhancing the pore structure and thus aiding in the strength development of the concrete mix [43]. The use of RHA containing an amorphous form of silica can produce durable concrete [39]. Calcium Silicate-Hydrates (CSH) are formed as a result of pozzolanic interactions between RHA's amorphous silica and calcium hydroxyl, filling the voids between the cement grains [39]. As a result, the RHA-concrete microstructure becomes impermeable to deterioration, ensuring greater strength increases than those without RHA [41].

RHA concrete is similar to fly ash or slag concrete, both of which are suitable for high-performance applications [35]. Because of its large specific area, RHA increased the early hydration rate of C3S, resulting in a denser paste [44-45]. *Zareei et al.* [46] examined the effect of RHA as an SCM in concrete prepared by replacing 10% of the cement with micro-silica. They used 5%, 10%, 15%, 20%, and 25% RHA in place of cement. Their findings revealed that up to 15% of cement may be supplemented with RHA to improve strength, and up to 20% RHA can be used in micro-silica-containing concrete to improve durability performance in terms of water absorption, permeability, and chloride ion penetration. *Meddah et al.* [47] reported on the effect of RHA and Al_2O_3 nanoparticles on the mechanical and durability properties of concrete in another study. They used RHA to replace 10% of the cement and Al_2O_3 nanoparticles to replace 1%, 2%, 3%, and 4% of the cement. Their findings revealed that replacing 10% cement with RHA densified the concrete microstructure and enhanced the compressive strength, flexural strength, tensile strength, and durability properties of the concrete in terms of resistance to hydrochloric acid and acid attack. They also discovered that substituting cement with Al_2O_3 densified the concrete microstructure, increasing the concrete's strength and durability. *Ameri et al.* [48] reported that while the early compressive strength of rice husk ash concrete increased significantly, it was constrained by the quantity of RHA. An increased RHA concentration of 15% caused a decrease in compressive strength due to a surplus of unreactive silica. Rice husk ash concrete had compressive strength values that were 9, 12, 13, and 16% higher than the typical OPC mix. Similarly, *Chao-Lung et al.* [49] used RHA as an SCM and reported that rice husk ash concrete produced a compressive strength that was 1.2-1.5 times higher than that of a conventional OPC mix. Meanwhile, *Chindaprasirt et al.* [50] conducted a study to evaluate rice husk ash concrete for sulphate attack resistance and concluded that rice husk ash concrete demonstrated better sulphate attack resistance. Furthermore, *Thomas et al.* [38] have shown that the dense microstructure of RHA can minimise concrete water absorption by 30%. Aside from certain technical advantages, various studies on the environmental effect of RHA have been done. *Gursel et al.* [51], for example, did research on the use of RHA in cement concrete and deemed it effective in minimising the potential for global warming. *Moraes et al.* [52] carried out a comparable study and concluded that the use of RHA in cement mortar contributed to decreasing the adverse effects of cement on the environment. Various studies have also been conducted to investigate the impact of using RHA in soil improvement. *Nguyen et al.* [53], for instance, explored the use of RHA in soil improvement and reported that RHA may be regarded as a suitable stabilising agent for enhancing the geotechnical properties of various types of soil. In their findings, RHA's optimum cement replacement ratio was 7.5%, with the maximum compressive and split tensile strengths.

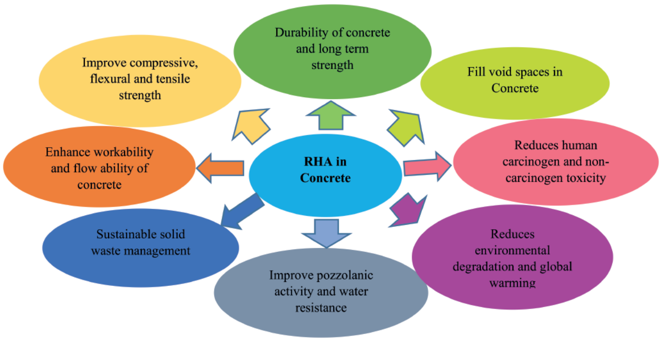

Fig. 1 The multiple structural and environmental benefits of RHA in sustainable concrete
1. ábra Az RHA többszörös szerkezeti és környezeti előnyei a fenntartható betonban

In order to produce concrete with the desired properties, the effect of these supplementary materials on the blended concrete mixture must be identified. Because of the significant complexity of the relationship between the variables and the concrete responses, conventional regression analysis may be insufficient to develop an adequate model. Recently, the Osadebe second decree polynomial has been widely applied in the field of concrete technology for the rapid and precise prediction of concrete properties. However, it has received minimal attention. In light of the aforementioned drawbacks, the aim of this study is to assess the influence of RHA on the properties of concrete, develop and evaluate predictive models for the desired responses, and finally optimise the concrete mixtures for construction purposes using the Osadebe model. In this paper, a mathematical model for optimising the compressive, flexural, and tensile strengths of concrete with 20% RHA as a partial replacement for Portland limestone cement is developed. Concrete with various mix ratios was subjected to compression, flexural, and tensile tests. Then, using Osadebe's second-degree polynomial function and the results of the compression, flexural, and tensile tests, a model for predicting the compressive, flexural, and tensile strengths of RHA concrete was developed. In addition, the developed models were validated using statistical tools.

2. Osadebe second decree polynomial

According to *Osadebe* [54], concrete is a multivariate unit mass whose strength varies with the volume of the constituent material. Another type of experimental model is his regression equation. He represented the reaction Y as a function of the proportions of the mixture's constituents Z, where the sum of all proportions must *Eq. 1*. It is written as

$$Z_1 + Z_2 + \cdots + Z_q = \sum_{i=1}^{q} Zi = 1 \qquad (1)$$

Where q is the number of mixture components and the proportion of the components in the mixture. Osadebe assumed that the response Y is continuous and differentiable with respect to its predictors and can be expanded in the neighbourhood of a chosen point using Taylor's series.

$$Z(0) = (Z_1^{(0)}, Z_2^{(0)}, \ldots, Z_q^{(0)})^r \qquad (2)$$

$$Y(z) = \sum_{m=0}^{q} F^m (Z)^{(0)} \left(Z_i - Z^{(0)}\right) \qquad (3)$$

Expanding to second order

$$Y(Z) = F(Z^{(0)}) + \sum_{i=1}^{q} \frac{\partial f\left(Z^{(0)}\right)}{\partial Z_i} \left(Z_i - Z^{(0)}\right) +$$
$$\frac{1}{2!} \sum_{i=1}^{q-1} \sum_{i=1}^{q} \frac{\partial^2 f(Z^0)}{\partial Z_i \partial Z_i} (Z_i - Z_i{}^{(0)})(Z_i - Z_i{}^{(0)})$$
$$+ \sum_{i-1}^{q} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i} (Z_i - (o)) \qquad (4)$$

For convenience, the point can be taken as the origin without loss in generality of the formulation and thus;

$$Z_1{}^{(0)} = Z_1{}^{(0)} + Z_2{}^{(0)} + Z_3{}^{(0)} + \ldots, Z_q{}^{(0)} = 0 \qquad (5)$$

$$b_0 = F(0), \quad b_i = \frac{\partial F(0)}{\partial Z_i}, \quad b_{ij} = \frac{\partial^2 F(0)}{2i \partial Z_i \partial j}, \quad b_{ii} = \frac{\partial^2 F(0)}{2i \partial Z_i^2} \qquad (6)$$

Substituting *Eq. 6* into *Eq. 1* gives:

$$Y(Z) = b_0 + \sum_{i-1}^{q} b_i Z_i + \sum_{i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i-1}^{q} b_{ii} Z_i^2 \qquad (7)$$

Multiplying *Eq. 1* by gives the expression:

$$b_0 = b_0 Z_1 + b_0 Z_2 + \cdots \ldots . + b_0 Z_q \qquad (8)$$

Multiplying *Eq. 1* successively by and rearranging, gives respectively:

$$Z_1^2 = Z_1 - Z_1 Z_2 - \cdots \ldots \ldots + Z_1 Z_q$$
$$Z_2^2 = Z_2 - Z_1 Z_2 - \cdots \ldots \ldots - Z_2 Z_q \qquad (9)$$
$$Z_q^2 = Z_1 - Z_1 Z_q - \cdots \ldots \ldots - Z_{(q-1)}$$

Substituting *Eq. 5* and 6 into *Eq. 7* and simplifying yields

$$Y(Z) = \sum_{i-1}^{q} \beta_i Z_i + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i Z_j \qquad (10)$$

$$\beta_i = b_0 + b_i \ldots \ldots + b_{ii} \qquad (11)$$

$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \qquad (12)$$

Eq. 7 is Osadebe's regression model equation. It is defined if the unknown constant coefficients, β_i and β_{ij} are uniquely determined. If the number of constituents, *q*, is 4, and the degree of the polynomial, *m*, is 2. The number of coefficients, *N* is now the same as that for the Scheffe's {4, 2} model given by (15) as:

$$N = C_m^{(q+m-1)} = C_m^{(4+2-1)} = 10 \tag{13}$$

$$\mathrm{N} = \frac{(q+m-1)!}{M!\llbracket q+m-1 \rrbracket - M)!} = \frac{(q+m-1)!}{m!(q-1)!} = \frac{(4+2-1)!}{2!(4-1)!} = \frac{5!}{2!3!} = 10 \tag{14}$$

2.1 Coefficients of Osadebe's Regression Equation

The least number of experimental runs or independent responses necessary to determine the coefficients of the Osadebe's regression coefficients is *N*. Let $y^{(k)}$ be the response at point *k* and the vector corresponding to the set of component proportions (predictors) at point *k* be $Z^{(k)}$.

$$Z^{(k)} = \left(Z_1{}^{(k)}, Z_2{}^{(k)}, \ldots .Zq^{(k)}\right) \tag{15}$$

Substituting the vector of *Eq. 15* into *Eq. 10* gives:

$$Y^{(k)} = \sum_{i=1}^{q} \beta_i Z_i{}^{(k)} + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i{}^{(k)} Z_j{}^{(k)} \tag{16}$$

Where k=1, 2… N

Substituting the predictor vectors at each of the *N* observation points successively into *Eq. 10* gives a set of *N* linear algebraic equations which can be written in matrix form as:

$$z\beta = Y \tag{17}$$

Where β is a vector whose elements are the estimates of the regression coefficients.

2.2 The coefficient of the regression equation

Let the Kth response (compressive strength for the serial number k) be $y^{(k)}$ and the vector of the corresponding set of variables be

$$Z^{(k)} = \left[Z_1{}^{(k)}, Z_2{}^{(k)}, Z_3{}^{(k)}, Z_4{}^{(k)}\right]^T \tag{18}$$

Substitution of the above vector in *Eq. 15* for $k = 1, 2, \ldots, 10$, generates the following system of ten linear algebraic equations in the unknown coefficients β_i and β_{ij}.

$$Y^{(k)} = \sum \beta i Z i^{(k)} + \sum \beta i j Z i^{(k)} Z j^{(k)} \tag{19}$$

1 $1 \le i \le 4$ and $k = 1, 2, 3 \ldots 10$

Let

$$[\mathrm{y}^{(\mathrm{k})}] = \begin{bmatrix} \mathrm{y}(1) \\ \mathrm{y}(2) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}(10) \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{10} \end{bmatrix} \begin{bmatrix} & \mathrm{Z}_1{}^{(1)}, \mathrm{Z}_1{}^{(2)}, \ldots, \mathrm{Z}_1{}^{(10)}, & \\ \mathrm{Z}_2{}^{(1)}, & & \mathrm{Z}_2{}^{(2)}, \ldots, \mathrm{Z}_2{}^{(10)}, \\ \mathrm{Z}_3{}^{(1)} \mathrm{Z}_4{}^{(1)}, & \mathrm{Z}_3{}^{(2)}, \mathrm{Z}_4{}^{(3)} & \ldots, \mathrm{Z}_3{}^{(10)} \mathrm{Z}_4{}^{(10)} \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_{10} \end{bmatrix} = \begin{bmatrix} & \mathrm{Z}_1{}^{(1)}, \mathrm{Z}_1{}^{(2)}, \ldots, \mathrm{Z}_1{}^{(10)}, & \\ \mathrm{Z}_2{}^{(1)}, & & \mathrm{Z}_2{}^{(2)}, \ldots, \mathrm{Z}_2{}^{(10)}, \\ \mathrm{Z}_3{}^{(1)} \mathrm{Z}_4{}^{(1)}, & \mathrm{Z}_3{}^{(2)}, \mathrm{Z}_4{}^{(3)} & \ldots, \mathrm{Z}_3{}^{(10)} \mathrm{Z}_4{}^{(10)} \end{bmatrix} \begin{bmatrix} \mathrm{y}(1) \\ \mathrm{y}(2) \\ \cdot \\ \cdot \\ \cdot \\ \mathrm{y}(10) \end{bmatrix} \tag{21}$$

$$\text{Where, } \mathrm{Z_i}^{\mathrm{k}} = \frac{\text{Weigth of variable i in k}^{\text{th}}\text{experimental run}}{\text{Total weigth of variable i in k}^{\text{th}} \text{ experimental run}}$$

Many researchers have used Osadebe's second-degree polynomial equation to develop prediction models for various engineering applications. Using Osadebe's second-degree polynomial equation, *Onwuka et al.* [55] developed a model for predicting the compressive strength of river sand and termite soil concrete from a given mix ratio of its constituents. The developed model was verified using the student t-test. Their finding shows that the developed model could be used to determine the proportions of the mix that will result in a given or desired compressive strength of a five-component concrete containing a percentage of termite soil. Furthermore, the model's results correspond with the relevant experimental values. Similarly, *Ubi et al.* [56] optimised the flexural and split tensile strength characteristics of polystyrene concrete using Osadebe's model. At a 71% water absorption rate, their optimised results yielded flexural and split tensile strengths of 2.00 N/mm^2 and 4.9 N/mm^2 from a water, cement, sand, and coarse aggregate mix ratio of 0.449, 1, 2.77, and 5.52, respectively. For both flexural and split tensile strengths, most of the model results agreed with their respective laboratory experiments. In a comparative study, *Anyaogu et al.* [57] used both Scheffe's and Osadebe's models to estimate the compressive strength of interlocking tiles produced from recycled plastic bottles. Both Scheffe's and Osadebe's models produced results that were comparable to the experimental results. The developed models were validated using the statistical student's t-test and found to be adequate with 95% confidence.

3. Materials and methods

3.1 Materials

3.1.1 Cement

The cement used in this study was a strength grade of 32.5R Portland Limestone Cement (PLC) produced by UNICEM Cement Company that met the ASTM C150 specifications. The specific gravity of cement was 3.15. *Table 1* details the chemical composition of cement.

3.1.2 Rice Husk Ash (RHA)

In this study, RHA was used as a partial replacement for cement at the optimum volumetric percentage (20%) reported by *Akeke et al.* [40] as the ideal replacement level in concrete. The rice husk was taken from a local rice mill and burned in the laboratory for approximately 3 hours in a muffle furnace at 700 °C. RHA materials were pulverised in a ball mill after cooling to an average particle size using a 75-micron BS sieve before use as SCM. The RHA had a specific gravity of 2.08 as determined by ASTM C188 [58]. The chemical compositions of rice husk ash are shown in *Table 1* and were determined using X-ray fluorescence (XRF) analysis.

Binder	**Chemical composition (%)**								
	SiO_2	**Al_2O_3**	**ZnO**	**CaO**	**Fe_2O_3**	**K_2O**	**MnO**	**MgO**	**Na_2O**
Cement	23.5	5.45	0.12	65.2	3.4	0.4	0.18	1.35	0.3
RHA	84.3	0.18	0.2	0.25	0.09	0.27	0.2	0.03	0.16

Table 1 Chemical composition of cement and RHA
1. táblázat A cement és az RHA kémiai összetétele

3.1.3 Aggregates

The coarse aggregates used in this study were crushed granite with a nominal maximum size of 20 mm, sourced from a local supplier. The coarse aggregates used were graded in accordance with ASTM C 33 [59]. *Table 2* presents the fineness modulus, specific gravity, water absorption, and other physical properties of coarse aggregates. The coarse aggregates used in this work were obtained from a granite quarry in Abakaliki, Ebonyi State, Nigeria. The fine aggregates were river sand sourced locally in Nsukka, Enugu State. The grading distribution of the sand is within acceptable limits. Therefore, the sieve analysis and the curve are within the ASTM C33 [59] acceptable ranges. Sieved sand had a fineness modulus of 2.67. We ensured that there were no deleterious materials contained in the aggregates.

Pro perty	Fineness modulus	Density (kg/m^3)	Specific gravity	Moisture content (%)	Absorp-tion (%)	Poro-sity (%)	Void ratio (%)
Value	2.4	1457	2.67	0.58	1.26	1.26	1.44

Table 2 Physical properties of coarse aggregates
2. táblázat Durva adalékanyagok fizikai tulajdonságai

3.2 Laboratory test programme

Table 3 shows the mix design for the RHA concrete composition, and the compositions were made to obtain grade 20 concrete. The following is a description of the mixture, which includes RHA as an additive material: To get the entire binder material, the appropriate amount of RHA according to the prescribed percentage was added to the cement content and properly mixed. The batching of ingredients was done by volume. Individual ingredients, such as coarse aggregate, fine aggregate, and cement content, were manually mixed dry using a mixing tray for around 2 minutes during the dry mixing process. The mixture was then gradually added to the boiling water, and mixing was continued for 4 to 7 minutes, until a homogeneous mixture was obtained. The simplex design points' mix ratios were determined using pentahedron factor space for a four-component mixture. The ingredients for RHA concrete mixtures were selected based on the required mix proportions with varying water/cement ratios.

The mixed concrete was made in a 150 mm metallic cube mould and automatically vibrated in three layers for the compressive strength test. 500 mm x 100 mm sample beams were fabricated to determine the flexural strength. Three layers of freshly mixed concrete were poured into each mould for the flexural strength test. With a 25 mm steel rod, each layer of concrete was manually compacted 150 times. Furthermore, 100 mm x 200 mm cylindrical concrete specimens were also moulded for split tensile strength. Freshly mixed concrete was poured in each mould in two layers of approximately 100 mm thickness for the split tensile strength. Each layer was manually compacted by tamping the rod 35 times with 25 mm steel rods on each layer. The cast concrete samples were removed from the moulds after 24 hours and stored in a typical curing tank. The strength properties of the concrete samples were examined after 28 days of curing. Each concrete mixture's average was examined. Furthermore, the compressive, tensile, and flexural strengths were evaluated in accordance with BSEN 206 (2001), Part 3, BS 1881-117 (1983), and BS 1881-118 (1993), in that order.

3.3 Mathematical optimization technique

Actual and pseudo-components

The requirement of the simplex that $X_1+X_2+X_3+X_4=1$ makes it impossible to use the normal mix ratios such as 1:3:6, etc. at a given water-cement ratio. Therefore, a transformation of the actual components (normal mix ratios) to meet this condition is necessary. The design matrix as shown in *Table 3* for the X_i experimental points is called a "pseudo-component", and Z_i are the actual experimental components.

$$X=AZ \tag{22}$$

Where A is the inverse of Z matrix and

$$Z=AX^T \tag{23}$$

Where A is the inverse of Z matrix, X^T is the transpose of matrix X.

	Mix Ratios				Component's Fraction			
S/N	**Water**	**Binder**	**FA**	**CA**	**Z_1**	**Z_2**	**Z_3**	**Z_4**
1	0.35	1	1	2	0.08	0.229885	0.229885	0.45977
2	0.44	1	1.5	3	0.074	0.16835	0.252525	0.505051
3	0.45	1	2	3	0.07	0.155039	0.310078	0.465116
4	0.5	1	3	6	0.048	0.095238	0.285714	0.571429
5	0.43	1	2	4	0.058	0.13459	0.269179	0.538358
6	0.48	1	2.5	5	0.053	0.111359	0.278396	0.556793
7	0.51	1	4	6	0.044	0.086881	0.347524	0.521286
8	0.33	1	3	5	0.035	0.107181	0.321543	0.535906
9	0.55	1	2	5	0.064	0.116959	0.233918	0.584795
10	0.6	1	2.5	6	0.059	0.09901	0.247525	0.594059

Table 3a Actual (Zi) and pseudo (Xi) components for Osadebe's {4, 2} simplex lattice
3a. táblázat Aktuális (Zi) és pszeudo (Xi) komponensek Osadebe {4, 2} szimplex rácsához

S/N	Water	Binder	FA	CA	Z_1	Z_2	Z_3	Z_4
11	0.55	1	1	2	0.121	0.21978	0.21978	0.43956
12	0.6	1	1.5	3	0.098	0.245902	0.245902	0.491803
13	0.44	1	2	4	0.059	0.134409	0.268817	0.537634
14	0.5	1	2.5	5	0.056	0.277778	0.277778	0.555556
15	0.4	1	3	6	0.038	0.096154	0.288462	0.576923
16	0.43	1	3.5	6.5	0.038	0.306212	0.306212	0.568679
17	0.35	1	4	7	0.028	0.080972	0.323887	0.566802
18	0.51	1	4.5	7.5	0.038	0.333087	0.333087	0.555144
19	0.48	1	4.8	7.6	0.035	0.072046	0.345821	0.54755
20	0.47	1	5	8	0.032	0.345543	0.345543	0.552868

Table 3b Osadebe's {4, 2} simplex lattice for control
3b. táblázat Osadebe {4, 2} szimplex rácsa a vezérléshez

S/N	Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
1	0.080	0.230	0.230	0.460	0.018	0.018	0.037	0.053	0.106	0.106
2	0.074	0.168	0.253	0.505	0.012	0.019	0.037	0.043	0.085	0.128
3	0.070	0.155	0.310	0.465	0.011	0.022	0.032	0.048	0.072	0.144
4	0.048	0.095	0.286	0.571	0.005	0.014	0.027	0.027	0.054	0.163
5	0.058	0.135	0.269	0.538	0.008	0.016	0.031	0.036	0.072	0.145
6	0.053	0.111	0.278	0.557	0.006	0.015	0.030	0.031	0.062	0.155
7	0.044	0.087	0.348	0.521	0.004	0.015	0.023	0.030	0.045	0.181
8	0.035	0.107	0.322	0.536	0.004	0.011	0.019	0.034	0.057	0.172
9	0.064	0.117	0.234	0.585	0.008	0.015	0.038	0.027	0.068	0.137
10	0.059	0.099	0.248	0.594	0.006	0.015	0.035	0.025	0.059	0.147

Table 4 Z-Matrix of Osadebe's mix proportions
4. táblázat Osadebe keverékarányainak Z-mátrixa

Microsoft Excel was used to compute the inverse of the 10x10 matrix in *Table 4* since the process cannot be achieved manually except with some special electronic operations. The result is shown in *Table 5*.

The regression coefficients in *Table 6* are a product of the inverse of the Z-matrix and the laboratory responses using the Microsoft Excel package.

3.4 Derivation of the mathematical models

Mathematical models, whether explicitly or implicitly, are also constrained because they must be tractable; they are ineffective unless they can be solved or modified to give relevant results. Because complex mathematical systems are difficult to analyse, the underlying equations must sometimes

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
50760.36	-11002	36980	-310722	-519334.985	922457.9	-53973.4	29016.3	219307.5	-264470
5413.457	-11729	3794.1	-32991	-53983.0315	96690.91	-5537.67	2977.072	22500.95	-27134.7
114.9767	-341.49	127.58	-442.87	-917.628116	1513.992	-64.768	34.81956	701.784	-725.404
193.9706	-464.45	173.81	-1010.6	-1879.62461	3256.507	-209.515	92.85216	921.0915	-1072.99
-89708.7	194886	-64811	550058	912725.9204	-1626042	94593.74	-50854	-384358	463510.8
-38176.2	82143	-27585	235923	392504.3854	-696327	40692	-22667.5	-164451	197944.7
-60214.3	130887	-44134	366597	615328.7412	-1092388	64094.36	-34041.9	-261166	315037.1
-10594.8	23168	-7558	63587.4	105968.8725	-189131	10804.49	-5571.13	-45396.7	54722.7
-2069.13	4397.9	-1388	12675	19943.07682	-35921.3	2034.306	-1192.57	-7807.35	9328.248
-678.567	1753.6	-653.6	3328.58	6221.183305	-10745.1	639.3391	-284.36	-3465.06	3883.936

Table 5 The inverse Z-matrix of Table 4 using Microsoft Excel
5. táblázat A 4. táblázat inverz Z-mátrixa Microsoft Excel használatával

β	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	β_{10}
RC	-644781	-66895	-1437	-2440	1133237	485161	765461	132728	24108	8511
LR	4.1	3.25	2.9	2.6	3.4	2.5	1.95	3.25	2.2	3.05

RC-regression coefficients; LR-laboratory responses

Table 6 The regression coefficients and the laboratory responses
6. táblázat A regressziós együtthatók és a laboratóriumi válaszok

be linear or easily convertible to linear, such as exponential or log-linear.

$$Z_1 + Z_2 + \cdots + Z_q = \sum_{i=1}^{q} Zi = 1 \quad (24)$$

Where q is the number of mixture components and Z_i the proportion of the components in the mixture. Z_1= Water/Cement Ratio; Z_2= Binder (80% OPC and 20% RHA); Z_3 = Fine Aggregates (FA); Z_4= Coarse Aggregates (CA).

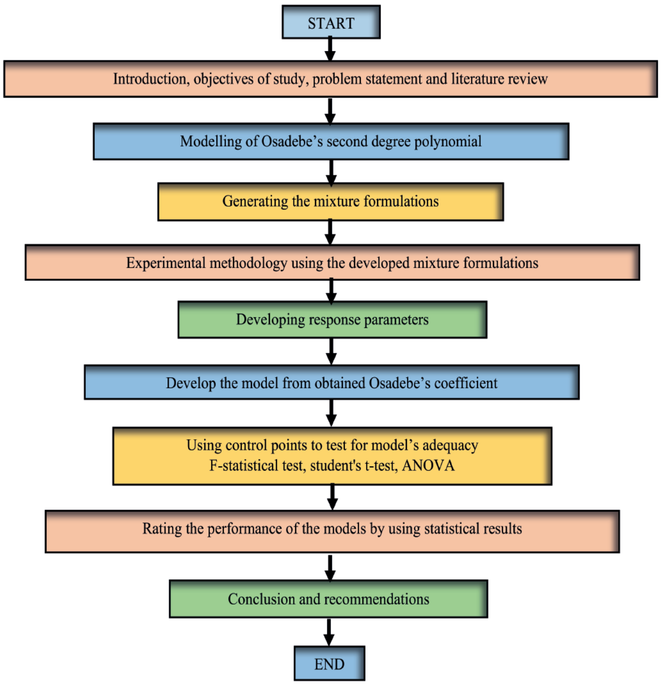

Fig. 2 Adopted methodology flowchart in the study
2. ábra A tanulmányban alkalmazott módszertan folyamatábrája

Osadebe assumed that the response Y is continuous and differentiable with respect to its predictors and can be expanded in the neighbourhood of a chosen point Z_o using Taylor's series.

$$Z(0) = (Z_1^{(0)}, Z_2^{(0)}, \ldots, Z_q^{(0)})^r \quad (25)$$

$$Y(z) = \sum_{m=0}^{q} F^m (Z)^{(0)} \left(Z_i - Z^{(0)}\right) \quad (26)$$

Expanding to second order

$$Y(Z) = F(Z^{(0)}) + \sum_{i=1}^{q} \frac{\partial f(Z^{(0)})}{\partial Z_i} \left(Z_i - Z^{(0)}\right) +$$

$$\frac{1}{2!} \sum_{i=1}^{q-1} \sum_{i=1}^{q} \frac{\partial^2 f(Z^0)}{\partial Z_i \partial Z_i} (Z_i - Z_i{}^{(0)})(Z_i - Z_i{}^{(0)}) +$$

$$\sum_{i-1}^{q} \frac{\partial^2 f(Z^{(0)})}{\partial Z_i} (Z_i - (o)) \quad (27)$$

For convenience, the point Z^0 can be taken as the origin without loss in generality of the formulation and thus;

$$Z_1{}^{(0)} = Z_1{}^{(0)} + Z_2{}^{(0)} + Z_3{}^{(0)} + \ldots, Z_q{}^{(0)} = 0 \quad (28)$$

$$b_0 = F(0), \quad b_i = \frac{\partial F(0)}{\partial Z_i}, \quad b_{ij} = \frac{\partial^2 F(0)}{2i \partial Z_i \partial j}, \quad b_{ii} = \frac{\partial^2 F(0)}{2i \partial Z_i^2} \quad (29)$$

Substituting *Eq. 28* into *Eq. 24* gives:

$$Y(Z) = b_0 + \sum_{i-1}^{q} b_i Z_i + \sum_{i \le j \le q}^{q} b_{ij} Z_i Z_j + \sum_{i-1}^{q} b_{ii} Z_i^2 \quad (30)$$

Multiplying *Eq. 24* by gives the expression:

$$b_0 = b_0 Z_1 + b_0 Z_2 + \cdots \ldots + b_0 Z_q \quad (31)$$

Multiplying *Eq. 24* successively by Z_1, Z_2 ... Z_q and rearranging, gives respectively:

$$Z_1^2 = Z_1 - Z_1 Z_2 - \cdots \ldots \ldots + Z_1 Z_q$$
$$Z_2^2 = Z_2 - Z_1 Z_2 - \cdots \ldots \ldots - Z_2 Z_q \quad (32)$$
$$Z_q^2 = Z_1 - Z_1 Z_q - \cdots \ldots \ldots - Z_{(q-1)}$$

Substituting *Eq. 28* and *30* into *Eq. 31* and simplifying yields

$$Y(Z) = \sum_{i-1}^{q} \beta_i Z_i + \sum_{i \le j \le q}^{q} \beta_{ij} Z_i Z_j \quad (33)$$

$$\beta_i = b_0 + b_i \ldots \ldots + b_{ii} \quad (34)$$

$$\beta_{ij} = b_{ij} - b_{ii} - b_{ij} \quad (35)$$

Osadebe's regression model equation is defined if the unknown constant coefficients, β_i and β_{ij} are uniquely determined. If the number of constituents, q, is 4, and the degree of the polynomial, m, is 2 then the regression equation is given as:

$$Y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_4 Z_4 + \beta_{12} Z_1 Z_2 +$$

$$\beta_{13} Z_1 Z_3 + \beta_{14} Z_1 Z_4 + \beta_{23} Z_2 Z_3 + \beta_{24} Z_2 Z_4 + \beta_{34} Z_3 Z_4 \quad (36)$$

Therefore, *Eq. 36* is the mathematical model based on Osadebe's second degree regression method.

3.5 Criteria for evaluations

In our study, the developed models were validated using three statistical indicators for accuracy, namely the *F-statistical test*, a *student's t-test*, and a *single-factor ANOVA* at a 95% confidence level. The following hypotheses were set in this test:

Null hypothesis: There is no significant difference between the laboratory tests and the model-predicted strength results.

Alternative hypothesis: There is a significant difference between the laboratory test and the model-predicted strength results.

4. Results and discussions

4.1 Osadebe's regression equation for compressive strength of RHA concrete

The solution of *Eq. 36* given the responses in *Table 4* gives the values of the unknown coefficients of the regression equation as follows;

$\boldsymbol{\beta}_1$ = 3265660; $\boldsymbol{\beta}_2$ = 33765.9; $\boldsymbol{\beta}_3$ = 3820.453; $\boldsymbol{\beta}_4$ = 11097.75; $\boldsymbol{\beta}_5$ = 5729225; $\boldsymbol{\beta}_6$ = -2471994; $\boldsymbol{\beta}_7$ = - 3865974; $\boldsymbol{\beta}_8$ = - 661169; $\boldsymbol{\beta}_9$ = 123759; $\boldsymbol{\beta}_{10}$ = - 33934.7.

from *Eq. 36*, the regression equation is given by;

$$Oc = 3265660Z_1 + 33765.9Z_2 + 3820.453Z_3 + 11097.75Z_4 + 5729225Z_1Z_2 - 2471994Z_1Z_3 - 3865974Z_1Z_4 - 661169Z_2Z_3 + 123759Z_2Z_4 - 33934.7Z_3Z_4 \quad (37)$$

Eq. 37 is the mathematical model for the optimisation of the compressive strength of RHA concrete, based on Osadebe's second degree polynomial.

Test for model adequacy using statistical tools

The model's efficiency was evaluated in comparison to the laboratory results of the check points. The predicted values (Y-predicted) for the test control points were calculated by inserting the appropriate Z_1, Z_2, Z_3, and Z_4 values into the modified model equation, *Eq. 37*. These values were then

Symbol	Water	Binder	FA	CA	Model (Y_K)	Laboratory (Y_E)
C1	0.35	1	1	2	26.62	26.58
C2	0.44	1	1.5	3	28.52	28.93
C3	0.45	1	2	3	28.83	27.23
C4	0.5	1	3	6	19.21	18.15
C5	0.43	1	2	4	17.99	17.10
C6	0.48	1	2.5	5	14.66	15.74
C7	0.51	1	4	6	8.75	7.90
C8	0.33	1	3	5	8.58	9.19
C9	0.55	1	2	5	8.04	7.18
C10	0.6	1	2.5	6	7.71	7.70

Table 7 Laboratory response and model response for the compressive strength test based on Osadebe's {4,2} Polynomial
7. táblázat Laboratóriumi válasz és modellválasz az Osadebe-féle {4,2} polinom alapján végzett nyomószilárdsági vizsgálathoz

Varaiable	Mean	Variance	Observations	Df	F	P (F<=f)	F Critical
Model	16.893	75.774	10	9	1.023	0.4867	3.1789
Laboratory	16.57	74.062	10	9			

Table 8 F-test two-sample for one-tail variances for the compressive strength test of RHA concrete
8. táblázat F-próba kétmintás egyvégű varianciákhoz az RHA beton nyomószilárdságvizsgálatához

Pearson Correlation	Hypothesized Mean Difference	t Stat	P(T<=t) one-tail	t Critical one-tail	P(T<=t) two-tail	t Critical two-tail
0.995	0	1.184	0.133	1.833	0.2668	2.2622

Table 9 T-Statistical test for the compressive strength test of RHA concrete
9. táblázat T-statisztikai vizsgálat az RHA beton nyomószilárdsági vizsgálatához

compared to the experimental results (*Y*-laboratory). The compressive strength at the control points, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, and C_{10}, was tested for adequacy using an *F-statistical test*, a *student's t-test*, and an *ANOVA* at a 95% confidence level. The analysis of variance for the *Fisher test* using the check point is as shown in *Table 8* below. Then, the calculated *F* from the table is 1.023, which is less than the critical (tabulated) *F value* of 3.1789, justifying the adequacy of the model equation (*Table 8*). Again, the *p-value* of 0.4867, which is greater than 0.05, further indicates the adequacy of the model. Using *T-statistical* and *ANOVA tests* further affirmed the adequacy of the model. Their statistically calculated values are far less than the critical values (*Tables 9* and *10*).

Groups	Count	Sum	Average	Variance
Model response	10	168.91	16.891	75.7744
Laboratory response	10	165.7	16.57	74.0615

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.5152	1	0.5152	0.0069	0.9348	4.41387
Within Groups	1348.52	18	74.9180			
Total	1349.04	19				

Table 10 ANOVA Single Factor for the compressive strength test of RHA concrete
10. táblázat ANOVA egytényezős vizsgálat az RHA beton nyomószilárdsági vizsgálatához

4.2 The Osadebe regression equation for flexural strength of RHA concrete

The values of the unknown coefficients of the regression equation as follows;

$\beta_1 = 3265660$; $\beta_2 = 33765.9$; $\beta_3 = 3820.453$; $\beta_4 = 11097.75$; $\beta_5 = 5729225$; $\beta_6 = -2471994$; $\beta_7 = -3865974$; $\beta_8 = -661169$; $\beta_9 = 123759$; $\beta_{10} = -33934.7$

from *Eq. 36*, the regression equation is given by;

$$Of = 1268384Z_1 - 131198Z_2 - 3049.74Z_3 - 4962.78Z_4 + 2227102Z_1Z_2 + 952640.1Z_1Z_3 + 1507630Z_1Z_4 + 261500.1Z_2Z_3 - 46803.39Z_2Z_4 - 17491.04Z_3Z_4 \tag{38}$$

Eq. 38 is the mathematical model for the optimisation of the flexural strength of RHA concrete, based on Osadebe's second degree polynomial

Symbol	Water	Binder	FA	CA	Model (Y_K)	Laboratory (Y_E)
C1	0.35	1	1	2	3.5	4
C2	0.44	1	1.5	3	3.25	2.2
C3	0.45	1	2	3	3.1	2.3
C4	0.5	1	3	6	3.75	4.2
C5	0.43	1	2	4	3.4	3.2
C6	0.48	1	2.5	5	3.8	4.3
C7	0.51	1	4	6	3.5	2.5
C8	0.33	1	3	5	4.5	4.8
C9	0.55	1	2	5	3.8	2.45
C10	0.6	1	2.5	6	4.4	3.5

Table 11 Laboratory response and model response for the flexural strength test based on Osadebe's {4,2} Polynomial
11. táblázat Laboratóriumi válasz és modellválasz a hajlítószilárdsági vizsgálatra az Osadebe {4,2} polinom alapján

Statistical test on the adequacy of the model

The predicted values (*Y*-predicted) for the test check points were calculated by inserting the appropriate Z_1, Z_2,

Z_3, and Z_4 values into the modified model equation, *Eq. 38*. These values were then compared to the experimental results (*Y*-laboratory). The flexural strength at the check points, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, and C_{10}, was tested for adequacy. The analysis of variance for the *Fisher test* at the checkpoint of the flexural strength test is presented in *Table 12* below. Then, the calculated *F* from the table is 0.2312, which is less than the critical (tabulated) *F* value of 0.3146, validating the adequacy of the model equation (*Table 12*). Furthermore, the results of *T-statistical* and *ANOVA tests* show a strong relationship between the model and laboratory responses, reaffirming that the model is acceptable. Again, their *p-values,* which are far greater than 0.05, further indicate the adequacy of the model.

Varaiable	Mean	Variance	Obser-vations	Df	F	P (F<=f)	F Critical
Model	3.7	0.2094	10	9	0.2312	0.0200	0.3146
Laboratory	3.345	0.9058	10	9			

Table 12 F-test two-sample for one-tail variances for the flexural strength test of RHA concrete
12. táblázat Kétmintás F-próba egyvégű varianciákhoz az RHA beton hajlítószilárdsági vizsgálatához

Pearson Correlation	Hypo-thesized Mean Difference	t Stat	P (T<=t) one-tail	t Critical one-tail	P (T<=t) two-tail	t Critical two-tail
0.6492	0	1.5141	0.0821	1.8331	0.1643	2.2622

Table 13 T-Statistical test for the flexural strength test of RHA concrete
13. táblázat T-Statisztikai vizsgálat RHA beton hajlítószilárdsági vizsgálatához

Groups	Count	Sum	Average	Variance
Model response	10	37	3.7	0.2094
Laboratory response	10	33.45	3.345	0.9058

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.6301	1	0.6301	1.1300	0.3018	4.41387
Within Groups	10.0373	18	0.5576			
Total	10.6674	19				

Table 14 ANOVA Single Factor for the tensile strength test of RHA concrete
14. táblázat ANOVA egytényezős vizsgálat az RHA beton szakítószilárdsági vizsgálatához

4.3 The Osadebe regression equation for tensile strength of RHA concrete

The unknown coefficients of the regression equation as follows;

$\beta1 = -644781.1$ $\beta2 =-66894.62$ $\beta3 = -1436.54$ $\beta4 = -2440.29$ $\beta12 = 1133237$ $\beta13 = 485161.5$ $\beta14 = -765461$ $\beta23 = 132727.7$ $\beta24 = -24108.3$ $\beta34 =-8510.76$

Applying equation 3.50, the regression equation is given by;

$$Ot = -644781.1Z_1 - 66894.62Z_2 - 1436.54Z_3 - 2440.29Z_4 + 1133237Z_1Z_2 + 485161.5Z_1Z_3 - 765461Z_1Z_4 + 132727.7Z_2Z_3 - 24108.3Z_2Z_4 - 8510.76Z_3Z_4 \quad (39)$$

Eq. 39 is the mathematical model for the optimisation of the tensile strength of RHA concrete, based on Osadebe's second degree polynomial.

Symbol	Water	Binder	FA	CA	Model (Y_K)	Laboratory (Y_E)
C1	0.35	1	1	2	3.5	4.2
C2	0.44	1	1.5	3	3.5	3.2
C3	0.45	1	2	3	3	2.5
C4	0.5	1	3	6	3.45	4
C5	0.43	1	2	4	2.95	3.3
C6	0.48	1	2.5	5	3.2	4.4
C7	0.51	1	4	6	3.7	3
C8	0.33	1	3	5	4.3	4.5
C9	0.55	1	2	5	4.5	2.6
C10	0.6	1	2.5	6	3.1	4

Table 15 Laboratory response and model response for the tensile strength test based on Osadebe's {4,2} Polynomial
15. táblázat Laboratóriumi válasz és modellválasz a szakítószilárdsági vizsgálatra az Osadebe {4,2} polinom alapján

Statistical test on the adequacy of the model

For tensile strength, the predicted values (*Y*-predicted) for the test check points were calculated by inserting the appropriate Z_1, Z_2, Z_3, and Z_4 values into the modified model equation, *Eq. 39*. These values were then compared to the experimental results (*Y*-laboratory). At the check points, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9, and C_{10}, were tested for adequacy. The analysis of variance for the Fisher test at the checkpoint of the flexural strength test is presented in *Table 16* below. Then, the calculated *F* from the table is 0.3021, which is less than the critical (tabulated) *F value* of 0.5146, ascertaining the adequacy of the model equation (*Table 16*). However, the *p-value* of 0.1597, which is far greater than 0.05, further indicates the adequacy of the model. Moreover, the results of *T-Statistical* and *ANOVA tests* also prove that the model is adequate and can be used effectively for predicting the tensile strength of RHA concrete.

Varaiable	Mean	Variance	Obser-vations	Df	F	P (F<=f)	F Critical
Model	3.52	0.2757	10	9	0.3021	0.1597	0.5146
Laboratory	3.57	0.549	10	9			

Table 16 F-test two-sample for one-tail variances for the tensile strength test of RHA concrete
16. táblázat F-próba kétmintás egyvégű varianciákhoz az RHA beton szakítószilárdságvizsgálatához

Pearson Correla-tion	Hypo-thesized Mean Difference	t Stat	P (T<=t) one-tail	t Critical one-tail	P (T<=t) two-tail	t Critical two-tail
-0.0283	0	-0.1718	0.4337	1.8331	0.8674	2.2622

Table 17 T-Statistical test for the tensile strength test of RHA concrete
17. táblázat T-statisztikai vizsgálat az RHA beton szakítószilárdságának vizsgálatához

Groups	Count	Sum	Average	Variance
Model response	10	35.2	3.52	0.2757
Laboratory response	10	35.7	3.57	0.549

Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	0.0125	1	0.0125	0.0303	0.8637	4.41387
Within Groups	7.422	18	0.4123			
Total	7.4345	19				

Table 18 ANOVA Single Factor for the tensile strength test of RHA concrete
18. táblázat ANOVA egytényezős vizsgálat az RHA beton szakítószilárdsági vizsgálatához

4.4 Comparison with other research

Onwuka et al. [55] modelled and determined the compressive strength of river sand-termite soil concrete using Osadebe's second-degree polynomial equation. The developed model was validated using the statistical tool *Student's t-test* at a 5% level of significance. The model was confirmed to be adequate, implying that there was no statistically significant difference between model and experimental responses. Furthermore, *Osadebe et al.* [60] used Osadebe's regression theory to develop a model for optimising the compressive strength of sand-laterite blocks. Their model could predict the mix proportion that would result in a desired strength as well as the strength of a sand-laterite block from a desired mix proportion. For model validation, they applied the student's t-test and the *Fisher test* statistics. The statistical analysis showed that their developed model was adequate. *Anyaogu et al.* [21] used Osadebe's method to develop an equation for predicting the compressive strength of interlocking tiles made from recycled plastic bottles. The response function values compared well to the experimental results. The response functions were examined using the statistical *student's t-test*, which was confirmed to be adequate at a 95% confidence level. The intended compressive strength of interlocking tiles created from a mixture of sand, granite dust, and recycled plastic bottles may be predicted from known mix proportions using the response function defined in their study. Furthermore, models based on Osadebe's polynomial, developed by *Ubi et al.* [56], were shown to properly predict the flexural and split tensile strengths of polystyrene concrete.

5. Conclusion

The significant influence of the combined effect of cement and RHA on the compressive, flexural, and tensile strengths of concrete was determined by developing mathematical models based on Osadebe's second decree polynomial. Water content (W), binder content (80% cement and 20% RHA), fine aggregate content (FA), and coarse aggregate content (CA) were used as independent variables to predict the 28-day compressive, flexural, and splitting tensile strengths of RHA-based concrete. With a level of significance of 0.05, the adequacy of the optimisation was compared to the experimental results. The statistical analysis summary showed that the model-predicted responses were in good agreement with the experimental results. The models may easily evaluate any desired compressive, flexural, and tensile strengths of hardened concrete given the mix proportions. The strength values attained in some of the optimised mixes, however, are satisfactory for use as structural or load-bearing concrete.

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