

Identification and Model-based Compensation of Striebeck Friction¹

Lőrinc Márton

Dept. of Electrical Engineering
Sapientia Hungarian University of Transylvania
Trandafirilor 61, 540053 Piata, Romania
martonl@ms.sapientia.ro

Béla Lantos

Dept. of Control Engineering and Information Technology
Budapest University of Technology and Economics
Magyar tudósok krt. 2, H-1117 Budapest, Hungary
lantos@iit.bme.hu

Abstract: The paper deals with the measurement, identification and compensation of low velocity friction in positioning systems. The introduced algorithms are based on a linearized friction model, which can easily be introduced in tracking control algorithms. The developed friction measurement and compensation methods can be implemented in simple industrial controller architectures, such as microcontrollers. Experimental measurements are provided to show the performances of the proposed control algorithm.

Keywords: tracking control, friction modelling and identification, friction compensation

1 Introduction

Many industrial applications require precise positioning of a mechanical system, namely moving an object in a given position in space with a given orientation. Some common applications are material manipulation with industrial robots or cranes, positioning in Hard Disk Drives or optical drives. Other applications

¹ The research results from this paper originally were presented in the [10] and [11] conference papers.

require a controlled motion in space along a predefined path, for example welding, spray painting with robots (trajectory tracking).

These tasks are commonly solved with feedback control. The aim of the control algorithm is to calculate the command signal for electric motors or other type of actuators which drive the mechanical system in order to obtain zero or acceptably small difference between the real and desired position.

Nowadays many types of high resolution position sensors and precise, fast actuators are available at relatively low cost. With these devices very high precision position control tasks became achievable for industrial applications. However in many practical applications it was observed that the high precision position tracking control performances are severely influenced by friction in a negative sense. This is why the search for new friction modelling and identification techniques became a popular research trend.

Friction is a nonlinear phenomenon which is universally present in the motion of bodies in contact. In servo controlled machines friction has an impact in all regimes of operation. In high precision positioning systems it is inevitable to know the value of the friction force to assure good control characteristics and to avoid some undesired effects such as limit cycle and steady state error, tracking lags.

The nonlinear and dynamic behavior of friction is accentuated near zero velocities. Many practical applications require precise motion control in low velocity regime. As examples can be mentioned the space telescopes that should track the motion of a star [1] and positioning applications, when the start point and the end point are near to each other.

Accordingly, in high precision position control systems the friction force should be taken into consideration at the formulation of the control law. To describe the friction phenomena, in [2] nonlinear models were proposed that can explain the nonlinear behavior of friction at low velocities. From theoretical point of view an important result was formulated in [3], in which it was shown that the dynamics of a single input mechanical system with Coulomb friction has a well defined, absolutely continuous solution (Carathéodory solution). To measure the frictional parameters, friction identification and measurement methods were also discussed in many studies. In [4] a time domain identification method is proposed for static friction models which are not necessarily linear in parameters. The method needs no information of acceleration and mass, the only assumption is that the initial and final velocity during the identification must be identical. Neural network based identification methods are also popular to capture the frictional behavior. In [5] Support Vector Machines were proposed for friction modelling and identification. The advantage of this approach is that it can identify nonlinearities from sparse training data. Genetic algorithm based identification for the Stribeck friction parameters was described in [8]. Frequency domain identification methods were also proposed to identify friction. The study [9] presents a frequency domain identification method for simultaneous identification of velocity and position

dependent friction. The compensation of the frictional effects in positioning systems is also discussed in many papers. In the paper [6] a nonlinear observer is developed to compensate the Coulomb friction. In the works of Laura et al. the extended Kalman filter technique is proposed for friction estimation [7].

In this paper a model based compensation method is presented. The rest of the paper is organised as follows: Section 2 is divided in three subsections. The first subsection presents the friction model, which is used in the control algorithm. Afterward, the second subsection presents a parameter identification for the introduced model. The third subsection in Section 2 presents the proposed tracking control algorithm. In Section 3 experimental measurements are given. The final Section sums up the conclusions of this study.

2 Friction Modelling, Identification and Compensation

2.1 Friction Modelling near Striebeck Velocities

Many models were developed to explain the friction phenomenon. These models are based on experimental results rather than analytical deductions and generally describe the friction force (F_f) in function of velocity (v).

The classical *static + kinetic + viscous friction model* is the most commonly used in engineering. This model has three components: the constant Coulomb friction term ($F_C \text{sign}(v)$), which depends only on the sign of velocity, the viscous component ($F_V v$), which is proportional with the velocity and the static term (F_S), which represents the force necessary to initiate motion from rest and in most of the cases its value is greater than the Coulomb friction: (see Figure 1.)

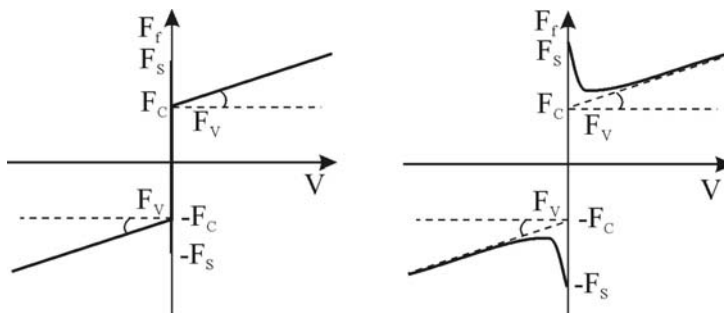


Figure 1

Static+kinetic+ viscous and Striebeck friction

$$F_f = \begin{cases} F_C \text{sign}(v) + F_V v, & v \neq 0 \\ F_S, & v = 0_+ \\ -F_S, & v = 0_- \end{cases} \quad (1)$$

The servo-controlled machines are generally lubricated with oil or grease (hydrodynamic lubrication). Tribological experiments showed that in the case of lubricated contacts the simple static +kinetic + viscous model cannot explain some phenomena in low velocity regime, such as the *Stribeck effect*. This friction phenomenon arises from the use of fluid lubrication and gives rise to decreasing friction with increasing velocities.

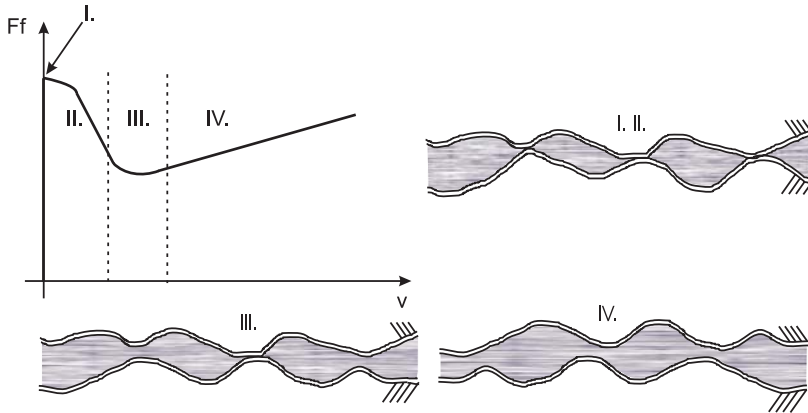


Figure 2
Stribeck Friction Regimes

To describe this low velocity friction phenomenon, four regimes of lubrications can be distinguished (see Figure 2). *Static Friction: (I.)* the junctions deform elastically and there is no excursion until the control force does not reach the level of static friction force. *Boundary Lubrication: (II.)* this is also solid to solid contact, the lubrication film is not yet built. The velocity is not adequate to build a solid film between the surfaces. A sliding of friction force occurs in this domain of low velocities. The friction force decreases with increasing velocity but generally is assumed that friction in boundary lubrication is higher than for fluid lubrication (regimes three and four). *Partial Fluid Lubrication: (III.)* the lubricant is drawn into the contact area through motion, either by sliding or rolling. The greater the viscosity or motion velocity, the thicker the fluid film will be. Until the fluid film is not thicker than the height of asperities in the contact regime, some solid-to-solid contacts will also influence the motion. *Full Fluid Lubrication: (IV.)* When the lubricant film is sufficiently thick, separation is complete and the load is fully supported by fluids. The viscous term dominates the friction phenomenon, the solid-to-solid contact is eliminated and the friction is 'well behaved'. The value of the friction force can be considered as proportional with the velocity.

From these domains results a highly nonlinear behavior of the friction force. Near zero velocities the friction force decreases in function of velocity and at higher velocities the viscous term will be dominant and the friction force increases with velocity. Moreover it also depends on the sign of velocity with an abrupt change when the velocity pass through zero.

For the moment no predictive model of the Striebeck effect is available. Several empirical models were introduced to explain the Striebeck phenomena, such as the Tustin model [2]:

$$F_f = (F_C + (F_S - F_C)e^{-|v|/v_s})\text{sign}(v) + F_V v \quad (2)$$

The model introduced in this paper is based on Tustin friction model and on its development, the following aspects were taken into consideration:

- allows different parameter sets for positive and negative velocity regime
- easily identifiable parameters
- the model clearly separates the high and low velocity regimes
- can easily be implemented and introduced in real time control algorithms

For the simplicity, only the positive velocity domain is considered, but same study can be made for the negative velocities. Assume that the mechanical system moves in $0 \dots v_{\max}$ velocity domain.

Consider a linear approximation for the exponential curve represented by two lines: d_{1+} which cross through the $(0, F_f(0))$ point and it is tangent to curve and d_{2+} which passes through the $(v_{\max}, F_f(v_{\max}))$ point and tangential to curve. (see Figure 3.) These two lines meet each other at the v_{sw} velocity. In the domain $0 \dots v_{sw}$ the d_{1+} can be used for the linearization of the curve and d_{2+} is used in the domain $v_{sw} \dots v_{\max}$. The maximum approximation error occurs at the velocity v_{sw} for both linearizations.

If the positive part of the friction model (2) is considered ($v > 0$), the obtained equations for the d_{1+} and d_{2+} , using Taylor expansion, are:

$$d_{1+} : F_{L1f+}(v) = F_S + \left. \frac{\partial F_f(v)}{\partial v} \right|_{v=0} v = F_S + (F_V - (F_S - F_C)/v_s)v \quad (3)$$

$$\begin{aligned} d_{2+} : F_{L2f+}(v) &= F_f(v_{\max}) + \left. \frac{\partial F_f(v)}{\partial v} \right|_{v=v_{\max}} (v - v_{\max}) \\ &= F_f(v_{\max}) + (F_V - (F_S - F_C)/v_s)e^{-v_{\max}/v_s} (v - v_{\max}) \end{aligned} \quad (4)$$

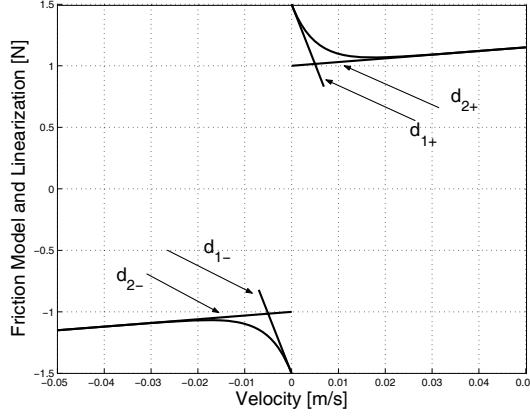


Figure 3

Linearization of Stribeck friction

Thus the linearization of the exponential friction model with bounded error can be described by two lines in the $0 \dots v_{\max}$ velocity domain:

$$d_{1+} : F_{L1f+}(v) = a_{1+} + b_{1+}v, \text{ if } 0 \leq v \leq v_{sw+} \quad (5)$$

$$d_{2+} : F_{L2f+}(v) = a_{2+} + b_{2+}v, \text{ if } v_{sw+} \leq v \leq v_{\max} \quad (6)$$

with:

$$v_{sw+} = \frac{a_{1+} - a_{2+}}{b_{2+} - b_{1+}} = \frac{-F_S + F_f(v_{\max}) + (F_V - (F_S - F_C)/v_s)e^{-v_{\max}/v_s}}{(F_V - (F_S - F_C)/v_s)(-1 + e^{-v_{\max}/v_s})} \quad (7)$$

Same study can be made for negative velocities. Based on linearization, the friction can be modelled as follows:

$$F_f(v) = \begin{cases} a_{1+} + b_{1+}v, & \text{if } 0 \leq v \leq v_{sw+} \\ a_{2+} + b_{2+}v, & \text{if } v_{sw+} \leq v \leq v_{\max} \\ a_{1-} + b_{1-}v, & \text{if } v_{sw-} \leq v \leq 0 \\ a_{2-} + b_{2-}v, & \text{if } v_{\max} \leq v \leq v_{sw-} \end{cases} \quad (8)$$

It can be seen that the model is linearly parameterized and it can be implemented with low computational cost.

2.2 Friction Measurement and Parameter Identification

2.2.1 Friction Measurement

For the friction force measurements it is assumed that the load is driven by a servo motor and the torque developed by the motor is proportional with the command signal. The friction force can appear inside the motor, in the gearbox between the load and the motor and at the load side. The friction to identify is the sum of all these friction forces. As it was presented in the previous subsection, the relationship between the friction behavior F_f and the velocity v is a mapping $F_f = F_f(v)$. The identification task in this is to obtain the parameters of the model (8) from a finite number (N) available measurements (v_i, F_{fmi}) , $i=1..N$, where $F_{fmi} = F_{fi} + d_i$. The term d_i is the measurement error on the i 'th measurement data.

The method is presented for positive velocity regime.

The dynamics of the positioning system reads as:

$$\begin{aligned}\dot{x} &= v \\ m\dot{v} &= u - F_f(v)\end{aligned}\tag{9}$$

with m mass of the load and u is the control input force, x denotes the position.

It can be seen that if the velocity is kept constant, the friction force is proportional with control signal u , $F_f(v) = u$. Hence if the positioning system is stabilized to different angular velocities v_i , the value of the friction force will be proportional with the command signal.

The method needs high precision velocity control. It is known that the linear PI control algorithm assures only poor transient performances for velocity tracking but guarantees precise final tracking accuracy, if the reference velocity is kept constant. It suggests that for parameter identification it is enough to use standard PI algorithm for velocity control: $u = K_P((v_{ref} - v) + 1/T_i \int (v_{ref} - v)dt)$. The well tuned PI controller guarantees precise velocity control.

The measurement algorithm can be summarized as follows: (see Figure 4.)

- Stabilize the velocity to v_{refi}
- Wait a time period $T1$ to get rid of transients.
- After the transients, calculate the average of the speed (v) and the control signal (u) over a time period $T2$ to get rid of measurement noise.
- Save the measurement data (v_i, u_i) .
- Repeat the sequence for the next velocity v_{refi+1}

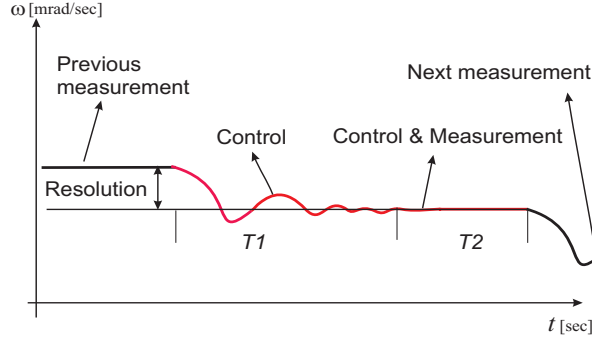


Figure 4
Velocity control for friction measurement

Note that during the data collection the closed loop velocity control algorithm remains active.

For precise velocity stabilization, precise velocity measurements are needed. Denote the encoder resolution with N_E . This value is generally given in Pulses Per Turn (*PPT*). The velocity can be measured with the encoder by counting the encoder pulse over a period of time T (*pulse counting method*). The velocity measurement accuracy is: $\Delta v_C = 2\pi / N_E T [\text{rad} / \text{sec}]$. However at low velocities higher accuracy can be obtained using the *pulse timing method*: count the number of pulses of a high frequency external timer over a single encoder pulse. In this case the accuracy of this measurement method is $\Delta v_T = N v^2 / 2\pi f [\text{rad} / \text{sec}]$ where f is the frequency of the timer. For the better accuracy at low speed regime the pulse timing method and at the high speed regime the pulse counting method should be used. The optimal switching between the methods is at the speed $v_{TC} = 2\pi / N_E \sqrt{f / T} [\text{rad} / \text{sec}]$, where $\Delta v_T = \Delta v_C$. By combining these two methods the velocity can be measured using standard industrial encoders ($N_E \leq 5000 \text{ PPT}$) with high accuracy even at low speed regime.

2.2.2 The Parameters of the Lines

The first line (d_{1+}), given by (5), characterizes the friction phenomena at low velocities, where the friction force has a downward behavior in function of velocity. At high speeds the friction increases almost linearly with the velocity, the second line (d_{2+}), given by (6), should be fitted on this part of the Striebeck curve. Hence, let us consider two subgroup of measurement data: the first N_1 measurements at the decreasing part of the curve, and the final N_2 measurements where the friction force increases with velocity.

The parameters of d_{1+} and d_{2+} can be determined as a solution of the following optimization problems:

$$\min_{a_{1+}, b_{1+}} \sum_{i=1}^{N_1} (F_{fi}(v_i) - (a_{1+}v_i + b_{1+}))^2 \quad \min_{a_{2+}, b_{2+}} \sum_{i=N_2}^N (F_{fi}(v_i) - (a_{2+}v_i + b_{2+}))^2 \quad (10)$$

Applying standard optimization techniques such as the the *Least Squares (LS)* method, the friction parameters can easily be calculated.

2.3 Position Tracking Combined with Friction Compensation

The tracking control problem for the system (9) can be formulated as follows: given a reference position trajectory $x_d = x_d(t)$, a twice differentiable function of time, determine the command signal u which guarantees, that $x(t) - x_d(t) \rightarrow 0$, as $t \rightarrow \infty$.

In order to solve the tracking problem, define the following tracking error metric, which combines the time dependent position and velocity errors: ($e_x = x - x_d, e_v = v - v_d$):

$$S = e_v + \lambda e_x \quad (11)$$

where $\lambda > 1$ is a constant parameter.

Based on the plant model (9), the tracking error dynamics reads as:

$$\dot{S} = (u - F_f(v)) / m - \ddot{x}_d + \lambda e_v \quad (12)$$

Consider the control law as:

$$u = -m(-\ddot{x}_d + \lambda e_v + K_S S) + \hat{F}_f(v), \quad K_S > 0 \quad (13)$$

Note that if we have $\hat{F}_f(v) - F_f(v) \approx 0$ yields $\dot{S} + K_S S = 0$, the combined position and velocity error converges to zero.

Hence the tracking problem can be solved, if the control algorithm contains a precise friction compensator term, which can ‘cancel’ the effect of frictional force on the system dynamics. $\hat{F}_f(v)$ can be modelled according to the relation (8) where the model parameters can be obtained as it was described in subsection 2.2.

3 Experimental Results

3.1 Experimental Setup

The experimental setup consists of a permanent magnet 24V DC servo motor with 38.2 [mNm/A] torque constant. The motor drives a metal disc with known inertia ($J = 0.015 \text{ kgm}^2$) through a 1:66 gear reduction ($N=66$). Friction is introduced via a metal surface, which is held against the disc (see Figure 5). The contact between the disc and the metal surface is lubricated with grease. The reaction torque generated by the friction component related to the motor side also can be written as a sum of three terms $F_f = F_{fR} + F_{fG} + F_{fL}/N$, where F_{fR} denotes the friction component inside the motor, F_{fG} denotes the friction component inside the gearhead, F_{fL} is the friction component at the load side.

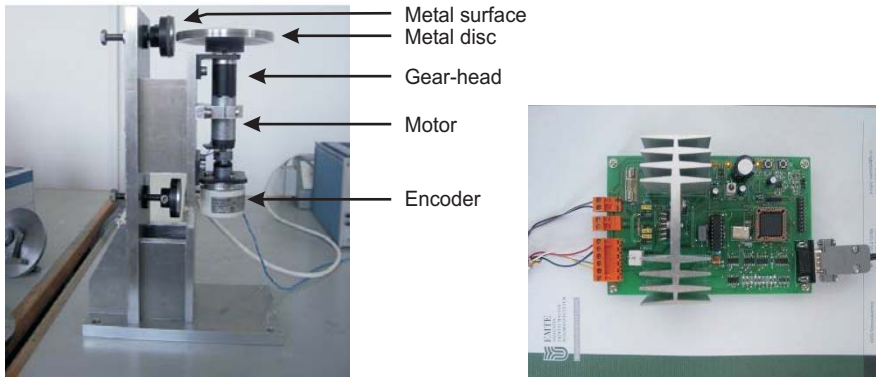


Figure 5

The experimental setup and the control circuit

The friction measurement and control algorithm are implemented on a PIC18 type microcontroller with 40 MHz clock frequency. The used C compiler for the implementation of the control algorithms allows floating point representation. The microcontroller is connected to an IBM-PC computer through RS232 serial port. The PC is used only for data monitoring and off-line data processing.

The DC servo motor is driven by a H-bridge amplifier. The armature current is controlled by a high speed, analog current controller. The microcontroller is interfaced to the current servo amplifier through a 11 bit DAC. The command signal calculated by the control algorithm running on the controller represents the reference for the current controller. Hence the positioning system is controlled by a cascade control architecture.

The angular position and velocity of the mechanical system are measured using a 5000 PPT two channel rotational encoder. The encoder is interfaced through a signal conditioner circuit to microcontroller which also determines the direction of rotation. The impulses of the encoder are counted using the embedded 16 bit timers of the controller. The pulse counting method uses the *Timer 0* block of the controller which has external clock input. The counting period is set to 5 msec. The pulse timing method is implemented using the *Capture* block of the controller, which generates an interrupt when positive signal edge appears on its external input. The high frequency timer, necessary for the measurement in pulse timing mode is derived from the microcontroller clock frequency. The switching between the two methods are implemented in the velocity and position measurement software module.

3.2 Measurement and Identification Results

To obtain the low velocity friction characteristics, the friction force was measured in $0 \dots 0.5$ [rad/sec] velocity domain (at the load side). The speed resolution was chosen 5 [mrad/sec]. Accordingly, totally $N=100$ measurements data were collected. A PI type control algorithm stabilizes the motor speed for each reference speed with $K_P=15$ proportional gain and $T_i=0.24$ [sec] integral time constant. The algorithm was implemented with 5 [msec] sampling period. When the reference speed value is changed, for $T1=50$ sampling period no data was collected in order to get rid of transients, and after that for $T2=16$ sampling period the average of the velocity values and control signal values were calculated to obtain one measurement point.

The measurements clearly capture the increasing and decreasing part of the Striebeck curve. (See Figure 6.) On the first 15 measurements at low velocities a line was fitted using LS method, which optimized the cost function (10), to obtain the parameters a_{1+} and b_{1+} . On the last 50 measurements another line was fitted to obtain the a_{2+} and b_{2+} parameters, which characterize the high velocity regime. The v_{sw+} parameter was determined from the relation (7).

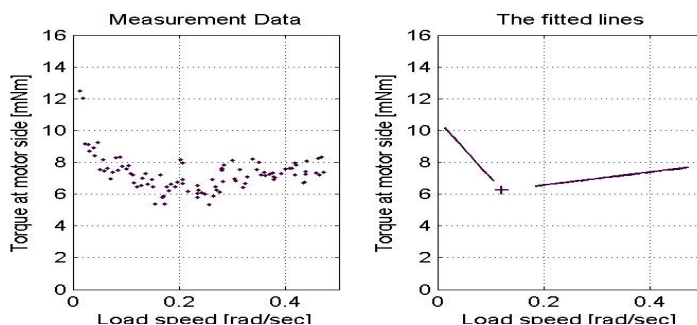


Figure 6

Friction measurement results and the fitted lines

Note that the friction was determined at the load side and the velocity is the velocity of the load. It can be seen (Figure 6) that the obtained model fits well the measurement data.

For the positive velocity regime, the following parameter values obtained during the identification are: $a_{1+} = 11.6 \text{ [mNm]}$, $b_{1+} = -61.2 \text{ [mNmsec/rad]}$, $a_{2+} = 5.7 \text{ [mNm]}$, $b_{2+} = 4 \text{ [mNmsec/rad]}$, $v_{sw+} = 0.085 \text{ [rad/sec]}$

3.3 Compensation Results

In order to test the proposed friction compensation method, the control law (13) was implemented on the microcontroller, with the following parameters: $K_S=I$, $\lambda=10$. Because the motion is rotational, the mass m is replaced by the inertia of system (J). The desired track contains has acceleration, constant speed, and deceleration regimes in both positive and negative velocity regimes.

Two experiments were carried out. In the first experiment, the friction compensator term ($\hat{F}_f(v)$) was neglected from the control law. In the second experiment, the friction compensator term was introduced in the control law according to the model (8). The experimental results are presented in Figures 7 and 8. For the numerical evaluation, the following error sum is considered.

$$E_S = \frac{1}{N} \sum_{i=1}^N |s_i|, \text{ where } N \text{ represents the number of measurements } (N=1000).$$

Without friction compensation, we obtained $E_S=0.3558$. With friction compensation we obtained $E_S=0.0656$. Accordingly, the proposed friction compensation method clearly outperforms the classical linear control algorithms, in which the friction is not taken into consideration.

Conclusions

A friction identification and compensation method was proposed for mechatronic systems, which operates at low velocity regimes near Striebeck velocities. The introduced control algorithm is developed for position tracking tasks and it is based on a linearized friction model, which can easily be implemented on simple industrial controller and its parameters can be identified with standard LS techniques. Experimental measurements shows the proposed tracking control algorithm, modified with a friction compensator term guarantees more precise trajectory tracking than classical control algorithms.

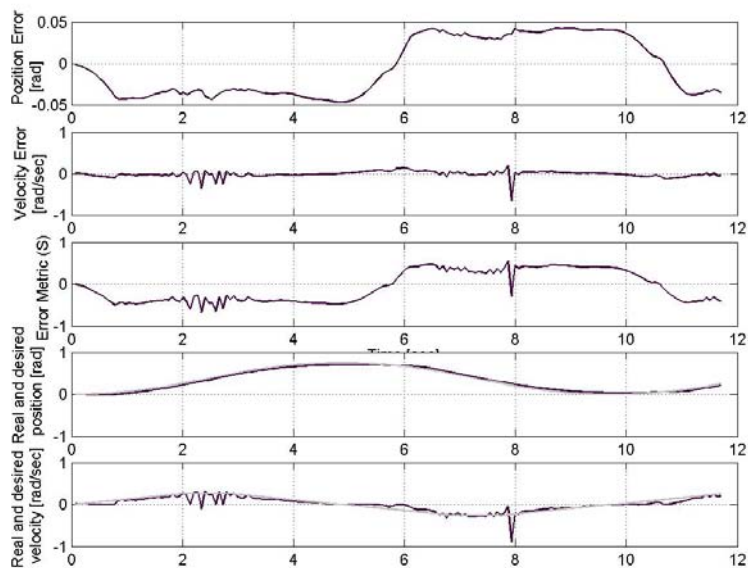


Figure 7
Tracking without friction compensation

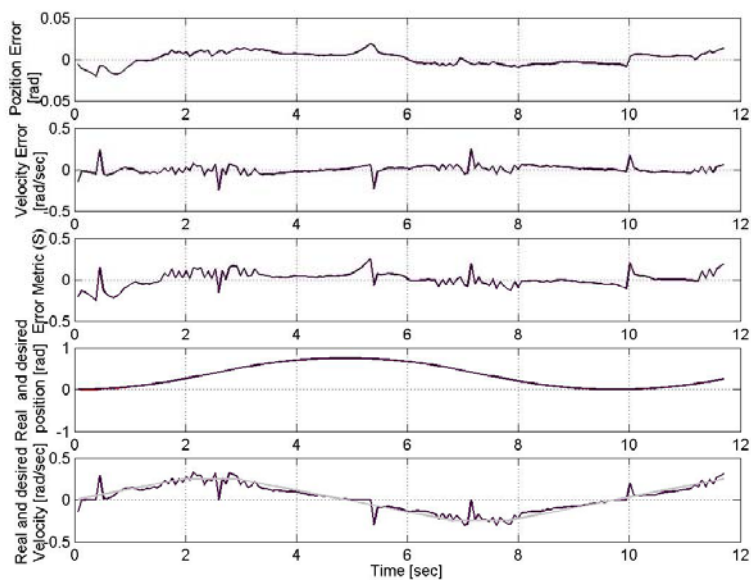


Figure 8
Tracking with friction compensation

Acknowledgement

The research was supported by the Hungarian National Research Program under grant No. OTKA T 042634. The first author research was also supported by Institute for Research Programmes from Sapientia Hungarian University of Transylvania.

References

- [1] Claudio H. Rivetta, Charles Briegel, Paul Czarapata: Motion Control Design of the SDSS 2.5-m Telescope. in Proc. of SPIE, August 2000, pp. 212-221
- [2] Brian Armstrong-Helouvry: Stick Slip and Control in Low-Speed Motion, IEEE Trans. on Automatic Control, October 1990, 38(10): 1483-1496
- [3] Seung-Jean Kim, In-Joong Ha: On the Existence of Caratheodory Solutions in Mechanical Systems with Friction, IEEE Trans. on Automatic Control, November 1999, 44(11): 2086-2089
- [4] Seung-Jean Kim, Sung-Yeol Kim, In-Joong Ha: An Efficient Identification Method for Friction in Single-DOF Motion Control Systems, IEEE Trans. on Control Systems Technology, July 2004, 12(4): 555-563
- [5] D. Bi, F. Li, S. K. Tso, G. L. Wang: Friction Modeling and Compensation for Haptic Display-based on Support Vector Machine, IEEE Trans. on Industrial Electronics, April 2004, 51(2): 491-500
- [6] Bernard Friedland: On Adaptive Friction Compensation. IEEE Trans. on Automatic Control, October 1992, 37(10):1609-1612
- [7] Ashok Ramasubramanian, Laura Ray: Friction Cancellation in Flexible Systems using Extended Kalman-Bucy Filtering. In Proc. of the American Control Conference, Denver, Colorado, June 2003
- [8] M. Verge: Friction Identification with Genetic Algorithms, In Proc. IFAC World Congress, Prague, July 2005
- [9] Milos R. Popovic, Andrew A. Goldenberg: Modeling of Friction Using Spectral Analysis, IEEE Trans. on Robotics and Automation, 14 (1): 114-123, February, 1998
- [10] Márton L, Béla L.: Tracking Control of Mechatronic Systems Based on Precise Friction Compensation, In Proc. 3rd Romanian-Hungarian Joint Symposium on Applied Computational Intelligence, Timisoara, Romania, May 2006
- [11] Márton L., Kutasi N.: Practical Identification Method for Stribeck Friction, In Proc 6th International Symposium of Hungarian Researchers in Computational Intelligence, Budapest, Hungary, November 2005