

A Priori Cost of the Energy to Perform Movement on a Predefined Path

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Abstract: The object of the paper is a priori energy cost estimation for the motion work performed by electric drive systems. We tested an original method for finding the efficiency of motion along a pre-specified path. Two mathematical models: the classic point-to-point trajectory planning and the interpolation model for the efficiency of the energy conversion process provide the data for energy cost estimation. First, the interpolation model was built using the electric drive actuator's data specifications. These data are the torque, speed, and efficiency for scattered steady-state operation points. The interpolation model builds the efficiency surface vs. torque and speed, which passes through all specification points of the actuator. If the required trajectory is superposed on the efficiency surface, we can find the incremental values of the efficiency which correspond to trajectories speed and torque. These quantities are necessary and sufficient for energy cost estimation of the motion along the desired trajectory. The a priori calculation of energy costs for electromechanical motion processes is an important means of preventing the energy wastage by adopting the minimum cost trajectory based on execution time, command law and load torque.

Keywords: Electromechanical servo system; trajectories planning; minimum time and minimum RMS torque control; scattered points interpolation; interpolation model; energy efficiency of the motion

1 Introduction

1.1 Literature Review

In any area of activity, it is necessary to perform useful mechanical work by means of electromechanical energy conversion systems. In this paper, we refer to the conversion systems with electric machines powered by static converters and advanced motion control systems. Currently, electromechanical energy conversion systems are widely used for various processes of mechanical motion (robots, electric vehicles, industrial and residential processes etc.). The conventional electromechanical energy conversion systems, realize a wide variety of trajectories of movement at the highest level of quality (accuracy, response time, efficiency). The selection of a particular trajectory becomes a design problem because the same useful work can be obtained with various energy costs and different levels of motion quality.

The interest in the issue of the a priori cost estimation of the electromechanical movement systems has increased due to the rapid progress in the field of the motion planning as discussed in robotics literature and self-driving vehicles. The topics of the motion planning are very extensive and hence only a description of the selected approach is done. See [1]-[5] for surveys on this subject. In the motion planning, the following components must be identified: finding a feasible path, discover the safest maneuvers and determine the optimal trajectory. The actual self-driven vehicles motion planning algorithms originate primarily from the field of mobile robotics. Three categories of path planning methodologies were adopted for autonomous vehicles: graph search based planners, sampler based planners, interpolation curve planners. The last method uses a previous set of points that describes a global road map and on this base generate a new set of data for the trajectory [6]. The interpolating curve planners implement different techniques for path smoothing: lines and circles, clothoid curves, polynomial curves, Bezier curves, spline curves.

The problem of feasible and optimal path planning has been studied extensively over the past few decades [7]. Feasible path planning referees to the problem of determining a path that satisfies given problem constraints.

Optimal path planning refers to the problem of finding a path that optimizes some quality criteria subject to given constraints. Assuming that the cost functional is the arc-length of the path, it is well known that the sufficient family of time-optimal paths for both Dubins', as well as, Reeds-Shepp's car models [8] consist of the concatenation of circular arcs with maximum curvature and straight line segments, all tangentially connected. We consider that shortest obstacle free path is not enough for mitigation of the vehicles effects on the environment. The

functional cost to be optimized must take into account the cost of the energy involved in the motion process.

A first step in the strategy of choosing the minimum cost path is the determination of the a priori cost. A direct method of determining a priori energy cost consists of assessing the losses. The total cost is obtained by summing up the cost of the required useful energy and the estimated losses costs.

Many papers have been written on the time optimal and, respectively, the minimum energy criteria for the motion trajectories generation, in electrical drive systems. There is the detailed references analysis in the domain, e.g. in [9]. A realistic design of the motion path was obtained in [10], [11], by minimizing an indicator that takes into account the tracking time, the energy consumption, the restrictions and system nonlinearities. A survey of the electromechanical control systems for electric vehicles (EV) has been written in [12]. The evaluation of the cost of the different dissipative energy losses especially in dynamic regimes poses a complex problem. The dissipative energy components are depended on the electrical machines material's properties (copper and iron losses), mechanical friction and load nonlinearities and so on. Minimizing a cost function, subject to various constraints, taking into account all the dissipative components of the analytic model, is quite laborious.

In robotic manufacturing systems, much energy is wasted due to an adopted minimum time policy for robot operations. [13]. Energy optimal trajectories for robot applications, is now an enormous research field in and of itself, see e.g. [14]-[17].

1.2 The Idea of the Paper

The efficiency of the energy transfer in the various physical processes like electrical, mechanical or thermal is univocally characterized by a pair of variables: the effort $e(t)$ and flow $f(t)$.

The inner product of these generalized variables is the instantaneous power. In the electromechanical process, when the electric machine operates as motor, the energy transfer are characterized by the following power variables: the input voltage $u(t)$ and the current $i(t)$ and the output mechanical force $f(t)$ or the corresponding torque $\tau(t)$ and velocity $v(t)$ or the corresponding angular velocity $\omega(t)$. The instantaneous efficiency of the energy transfer is the ratio of the output and input instantaneous power. The cost of the energy transfer in the interval 0 to t_1 is described by relationship:

$$E(t) = \int_0^{t_1} \frac{efdt}{\eta(e, f)}. \quad (1)$$

The output power variables $e(t)$ and $f(t)$ are known because these are desired variables.

The novelty of the paper, is to build the surface of the process efficiency in terms of the output power variables with an aim to estimate the instantaneous values of the efficiency in the interval 0 to t_1 . Thus, we can determine the a priori cost necessary to obtain the desired dynamics of the power variables (effort and flow) by minimizing the wasted energy.

Starting with a relatively small number of experimental data, an interpolation model of the actuator efficiency, as a function of torque and angular velocity, was build up. Motion trajectories are transformed from the explicit form (in the time domain) into the implicit form (in the angular velocity and torque domain). Thus, the implicit motion trajectories were projected on the surface of the efficiency in order to obtain the discrete values of the efficiency, corresponding to the discrete torque and angular velocity values. As a result, the discrete values of the cost and then the total cost can be calculated. The novelty of the developed method is that it can provide the possibility of predetermining the energy costs both in the steady-state regime and in the dynamic regimes, starting from a relatively limited experimental database of the efficiency in the steady-state regimes.

The paper is structured as follows. Section 2 presents the point to point trajectories planning method, under the minimal time or minimal RMS torque control laws. Section 3 describes the strategies applied for building an interpolation model which estimate the efficiency of the energy conversion process. Section 4 introduces the numerical tests and based on these, simulations are performed and analyzed. Finally, section 5, concludes the paper.

2 Problem Formulation

In the following, the electromechanical conversion process, which is the object of the a priori energy costs estimation, is briefly described. Currently, most of the physical processes that are subject to human activities require adequate control by computer means. The purpose of these processes is to achieve useful goals. The desired objectives are obtained by means of power actuators powered by primary energy sources. The actuators are controlled by means of informatics systems according to quality and energy criteria in order to obtain real objectives as close as possible to the desired ones. The most widespread actuators elements in electromechanical systems are the electric machines driven by power static converters. The torque τ obtained through the air gap electromagnetic field of electric machine, and the angular velocity ω are transmitted to the mechanical load. The mechanical load transforms the internal rotational quantities ω and τ in external quantity (Cartesian representation) by means of the kinematics specific

to the application domain. There is a large variety of mechanical loads. For the generalization of the mechanical motion equations, the external quantities were rated to the motor shaft. In the case of an electric vehicle, for example, the mere traveled distance x at velocity $\dot{x}$, and acceleration $\ddot{x}$ are the external quantities. These quantities are rated to the following internal quantities at the motor shaft: the angular position θ rad, angular velocity $\dot{\theta}$ and the acceleration $\ddot{\theta}$. The fundamental kinematic model is based on the motor torque τ , the load torque τ_r , and the dynamic torque. The balance equation on the motor shaft is the following:

$$M_d = \left\{ (\omega, \tau, \theta) \left| \begin{array}{l} \dot{\omega} = K_m \tau - K_m \tau_r \\ \dot{\theta} = \omega \\ \tau_r = f(\omega, \theta) \end{array} \right. \right\} \quad (2)$$

The constant K_m is defined by $K_m = \tau_R / J\omega_R$. If the electric machine operate as motor $\tau_r \geq 0$ else if the electric machine operate as generator (braking mode), $\tau_r \leq 0$.

The mechanic load torque τ_r was considered dependent on position and velocity. All the quantities in equations system (2) are expressed in per-unit [pu] values and τ_R Nm, ω_R rad/s are the rated motor torque and speed, J kgm² is the total moment of inertia. The electromechanical system (2) reaches the desired speed ω and position θ controlling the motor torque τ .

We consider that the movement is planned as a point-to-point motion. Each motion sequence begins from the conditions known initially and reaches the desired final conditions in a given motion time T_f .

The planning of the trajectory means the design of the variation for the torque τ_{ff} , velocity ω_{ff} and position θ_{ff} according to the required objective with fixed initial and final conditions. These trajectories are imposed in a feed forward (ff) manner to the torque, velocity and position controllers. The feed forward control generates the required torque command to move the mechanical load in accordance to the desired trajectory. The desired trajectory must be such that the required torque and speed are in accordance with the actuating device limitations. The required torque command τ_{ff} is determined at every sampling time $t_k = kT_s$.

The sampling period of the motion trajectory is T_s . The required torque is adopted as piecewise linear:

$$G_{\text{eff}} = \{(\tau_k) | \tau_k = C_1(k) + C_2(k)t_k, \quad k = 1, \dots, N\} \quad (3)$$

The parameters $C_1(k)$ and $C_2(k)$ of torque τ_k are computed at every sampling time step k for the next initial condition:

$$\begin{cases} t = 0 \\ \theta = \theta_i \\ \omega = \omega_i \end{cases} \quad (4)$$

and finally:

$$\begin{cases} t = T_f \\ \theta = \theta_f \\ \omega = \omega_f \end{cases} \quad (5)$$

The set of the C parameters were obtained by solving the mathematical model of equation (6):

$$C = \left\{ (C_1(k), C_2(k)) \left\{ \begin{array}{l} \text{Find } C_1(k) \text{ and } C_2(k) \text{ by solving:} \\ \omega = K_m \int_0^{T_f} (C_1(k) + C_2(k)t - \tau_r(k)) dt \\ \theta = \int_0^{T_f} \omega dt \\ \text{under initial and final conditions} \end{array} \right. \middle| k = 1, \dots, N \right\} \quad (6)$$

The load torque $\tau_r(k)$ depends on the performed applications. For example, the load torque imposed by an Electric Vehicle (EV) may be described by:

$$M_{\tau} = \{(\tau_{rk}) | \tau_{rk} = \tau_{r0} + C_w(k)\omega^2(k), \quad k = 1, \dots, N\} \quad (7)$$

The initial torque is τ_{r0} , while C_w is a global coefficient that takes into account different resistive torque effects [18].

The set of discrete values of the trajectories of the velocity and the position results from the mathematical model (2) after the parameters $C_1(k)$ and $C_2(k)$ was calculated.

$$G_{\omega_{ff}} = \left\{ (\omega_k) \left| \begin{array}{l} \omega(k) = \omega(k-1) + (C_1(k) + C_2(k)t_k - \tau_r(k))T_s K_m \\ \omega(0) = \omega_i \\ k = 1, \dots, N \end{array} \right. \right\} \quad (8)$$

$$G_{\theta_{ff}} = \left\{ (\theta_k) \left| \begin{array}{l} \theta(k) = \theta(k-1) + \omega(k)T_s \\ \theta(0) = \theta_i \\ k = 1, \dots, N \end{array} \right. \right\} \quad (9)$$

A start-stop motion, for example, with the steady-state regime has three sequences: starting, steady-state and braking regimes. If the control law is the minimum time $C_2 = 0$ and the velocity profile is trapezoidal, otherwise, the control law is the minimum RMS torque and the starting and braking velocity profiles are parabolic [19]. The continuity of position, velocity and acceleration (torque) for each sequence of the movement is fulfilled by the fact that final conditions of the past sequence are the initial conditions of the current sequence. The initial and final torque gradient (jerk) is implicitly limited by the torque controller. The requested trajectories were obtained from (2, 7, and 8) in implicit (10) or explicit (11, 12) forms:

$$G = \{(\omega_k, \tau_k, \theta_k) | \theta_k = \theta_{k-1} + T_s \omega(k), k = 1, \dots, N\} \quad (10)$$

$$G_{\omega} = \{(\omega_k, \tau_k) | t_k = kT_s, k = 1, \dots, N\} \quad (11)$$

$$G_{\tau} = \{(\tau_k, t_k) | t_k = kT_s, k = 1, \dots, N\} \quad (12)$$

The a priori energy cost of the trajectory G (11) is completely determined if are known the set of efficiency values $\eta_k (k = 1, \dots, N)$ which corresponds to the discrete values of speed ω_k and torque τ_k .

3 Determination of the a Priori Energy Cost Using the Interpolation Model

The efficiency of the energy conversion process through electric machines and power converters is determined by velocity and torque. The measurements by experimental methods of the power quantities (velocity and torque), as well as the corresponding efficiency, allows knowing a limited number of values, usually with a scattered distribution in the admissible range. The interpolation model has

as input the velocity and the torque, the output quantity being the corresponding value to the efficiency. The following set of experimental data in the steady-state operation is considered:

$$\{X, Y, Z\} = \{(\omega_m, \tau_m, \eta_m) | m = 1, \dots, M_{obs}\} \quad (13)$$

Where, M_{obs} are the observation number of points.

The problem formulation of the interpolation model is the following:

Starting from a given data set of irregularly (scattered) distributed points over space $\mathbb{R}^2$ described by:

$$P_m = \{(\omega_m, \tau_m) | m = 1, \dots, M_{obs}\} \quad (14)$$

Considering the scalar values of efficiency $\eta_m, m = 1, \dots, M_{obs}$ associated with each point P_m , satisfying:

$$\eta_m = F(\omega_m, \tau_m) \quad (15)$$

Denoting the unknown function by $F(\omega, \tau)$, look for an interpolating function $\tilde{F}$, such that for every observation points P_m , the interpolating function fulfills the condition:

$$\tilde{F}(\omega_m, \tau_m) = \eta_m \quad (16)$$

We assume that all points P_m , also referred as nodes, or “support points” are distinct and not collinear. Due to the intrinsic inertial nature of involved conversion process, the function F is geometrically represented as a continuous differentiable surface which allows finding the desired function $\tilde{F}$ that passes exactly through each M_{obs} observation points. Consequently, the ill-posed problem is avoided, no regularization being necessary for an adequate approach.

The main objective of the paper is not to approach, in details, the current methods used for approximation of the surfaces. However, it can be that noted that the scattered data interpolation methods may be divided into two important classes:

- *Local methods*, where the interpolated value depends only on the “nearly” points
- *Global methods*, where the value of the interpolate at a point P depends on all data points used

Two methods, which are most widely used in different application fields (in areas such as computer graphics, physical modeling, geographic information systems, medical imaging, and more), have been adopted. These methods are:

- *Delaunay triangulation* and related methods
- *Radial Basis Function* (RBF) networks interpolation

Local methods usually need a triangulation of the set of points (support points).

The interpolated value of a point, other than support points:

$$\{x, z, y\} \notin \{X, Y, Z\} \quad (17)$$

is obtained by local interpolation techniques with the three nearby points. One widely used approach is Delaunay triangulation of data and the Voronoi diagram of a set of points which is the dual of the first approach.

The interpolation technique by the RBF neural networks is based on the theory of learning systems. The function $\tilde{F}(\omega, \tau)$ to be interpolated is, in a pragmatic consideration, represented as a linear combination of the non-linear basis functions weighted by a learning process based on the data points $\{X, Y, Z\}$.

The interpolation error of the energy conversion process efficiency was estimated using two sets of the experimental data: a learning set $\{X, Y, Z\}$ and an assessing set $\{U, V, W\}$ such that $\{U, V, W\} \notin \{X, Y, Z\}$.

Let be the assessing experimental data set as

$$\{U, V, W\} = \{(\omega_n, \tau_n, \eta_n) | n = 1, \dots, N_{obs}\} \quad (18)$$

The assessing points were:

$$P_n = \{(\omega_n, \tau_n) | n = 1, \dots, N_{obs}\} \quad (19)$$

and the interpolation values of efficiency was

$$\tilde{\eta}_n = \tilde{F}(\omega_n, \tau_n) \quad (20)$$

A Root-mean-square error (RMS-error) of interpolation is given by:

$$RMS - error = \sqrt{\left(\sum_{n=1}^{N_{obs}} \frac{(\tilde{\eta}_n - \eta_n)^2}{N_{obs}} \right)} \quad (21)$$

The error of the above method adds experimental data measurement errors.

Several papers have studied the comparative assessment of performances of the RBF networks to approximate nonlinear static characteristics accurately [20]. The

techniques adopted in the paper has been shown that the surface of the efficiency vs. velocity and torque was interpolated (reconstructed) with an error less than 2% by means of a reasonable volume of experimental data. The absolute error is about one order of magnitude larger near the boundary, than it is in interior of the domain. By including points on the boundary the error is much smaller.

The Matlab function used in the paper for local scattered data interpolation was: "griddata", "griddatan" and the "scatteredInterpolant" class in 3D space.

The energy cost of the electromechanical conversion processes is completely determined by the so-called energy quantities: the torque τ , the velocity ω , and the actuator efficiency η . The discrete values of the speed ω_k and torque τ_k are known due to the fact that these are requested quantities. These requested quantities were obtained by the trajectories planning system (11). The efficiency values $\eta_k (k = 1, \dots, N)$ which corresponds to the discrete values of speed ω_k and torque τ_k was obtained from the interpolation model:

$$E = \left\{ (\tilde{\eta}_k) \middle| \tilde{\eta}_k = \tilde{F}(\omega_k, \tau_k), \quad k = 1, \dots, N \right\} \quad (22)$$

The final cost may be expressed in the implicit or respectively explicit forms:

$$W_N = \sum_{k=1}^N \frac{\theta_k \tau_k}{\tilde{\eta}_k} \quad (23)$$

$$W_N = \sum_{k=1}^N \frac{\omega_k \tau_k T_s}{\tilde{\eta}_k} \quad (24)$$

Thus, the discrete values of final cost are:

$$W = \left\{ (w_k) \middle| w_k = \sum_{k=1}^N \frac{\omega_k \tau_k T_s}{\tilde{\eta}_k} \right\} \quad (25)$$

In Figure 1 was represented the flowchart of the main functions of the a priori cost estimation technique of the electromechanic motion process. The learning database in the Figure 1 contains the M_{obs} of the measurements points (13).

These points generate the surface $\tilde{F}(\omega_m, \tau_m) = \eta_m$ (16) by the interpolation model. Once the surface $\tilde{F}$ was built, we can estimate the efficiency $\tilde{\eta}_k$ for any point (τ_k, ω_k) by assessing the interpolation model. The a priori cost estimation for a required motion trajectory (11) was accomplished based on discrete points (τ_k, ω_k) of the G considered trajectory and the corresponding efficiency $\tilde{\eta}_k$.

The same technique can be used for a posteriori cost calculation of the motion. In these case, the measurement quantities we need are the actual value of the velocity ω and the feed forward torque value τ_{ff} . The actual value of the torque τ may be considered very close to the required torque τ_{ff} because the error of the torque controller usually is very small.

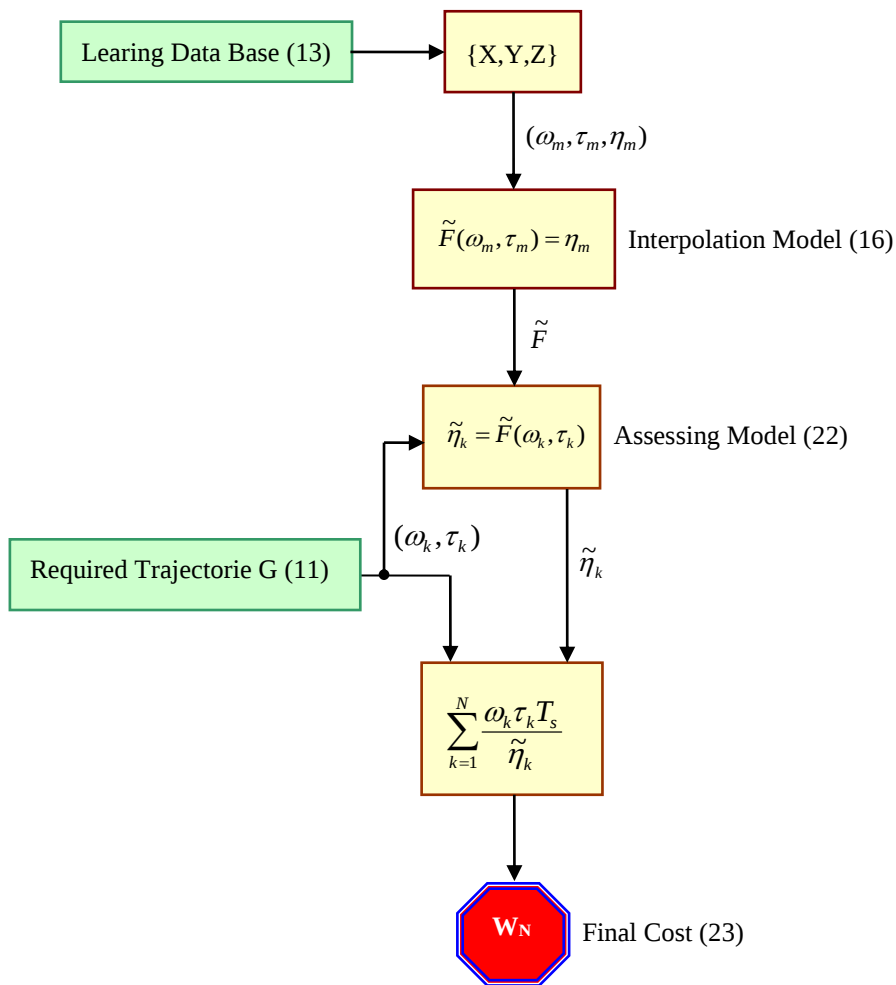


Figure 1

The flowchart of the a priori cost estimation technique of the electromechanic motion process

The test of the above cost estimation method was achieved with an electrical drive system with the permanent magnet synchronous motor. The actuator is a general-purpose power electronic converter. The power electronic converter is provided with control loops for the electromagnetic and mechanical quantities. The main electromagnetic controlling quantity is the motor torque. The mechanical controlling quantities are the angular velocities ω , the angular position θ and eventually angular acceleration $\dot{\omega}$ and jerk $\ddot{\omega}$.

The tests were based on experimental data obtained with the device UQM PowerPhase 220 HD (UQM Technologies, Inc., www.uqm.com).

In Figure 2 are represented some experimental data of the device efficiency vs. mechanical load torque at constant speeds.

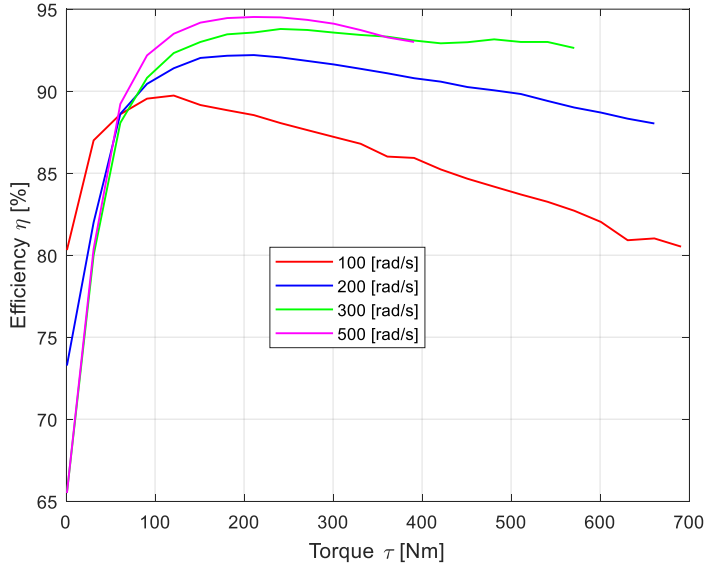


Figure 2
Some experimental data regarding efficiencies vs. torques
at constant velocity of the UQMHD220 device

The coordinates of the steady-state operating points are represented by $(\tau_m, \omega_m), m = 1, \dots, M_{obs}$ with restrictions: $\tau_m \leq \tau_{max}$ and $\omega_m \leq \omega_{max}$. In the paper, the set of the M_{obs} experimental points (τ_m, ω_m) and the corresponding values η_m has been passed to the Matlab function *scatteredInterpolant*, and it has returned the surface $\tilde{F}(\omega, \tau) = \eta$. This surface passes through the sample values at the point locations. We can evaluate this surface at any query point $(\tau_i, \omega_i), i \neq m$, to produce an interpolated value:

$$\tilde{\eta}_i = \tilde{F}(\omega_i, \tau_i) \quad (26)$$

Interpolation technique used in the paper was *Delaunay interpolation of points*. The set of points obtained by sampling the implicit trajectory of the motion $g(\omega, \tau) = 0$ is G_{off} and G_{ff} (3)-(8).

The set E of the efficiency values corresponding to the set G_{off} and G_{ff} are obtained by the interpolation model (22).

The cost W_N of the movement in tracking the trajectory $g(\omega, \tau) = 0$ is computed according to relationships (23)-(24) and depends on the geometry of the trajectory $g(\omega, \tau) = 0$ lying on the surface $\tilde{\eta}_i = \tilde{F}(\omega_i, \tau_i)$. For the same useful work, the same displacement θ_f can be achieved with different control laws and different final execution time T_f , at different energy costs.

The informatics system (Figure 1) offers the possibility of choosing the minimum cost trajectory depending on the required mechanical load, the desired execution time and the control law. Consider an EV under the conditions of the UQM220HD device specifications. The mechanical rated load torque is $\tau_R = 300Nm$ and the angular rated velocity is $\omega_R = 314rad/s$.

The electromechanical coupling system converts the motor rotational motion in the EV translational motion with position x . We consider a requested longitudinal distance traveled $x_f = 200m$ which correspond to the motor shaft angle $\theta_f = 3000rad$. The moment of inertia corresponding to the total kinetic energy (rotational and translational) is $J = 1Kgm^2$. A possible load torque for EV has the following expression:

$$\tau_r = \tau_{r0} + C_w \omega^2 \quad (27)$$

The initial mechanical load torque was adopted at value $\tau_{r0} = 0.8[pu]$ and the global load coefficient $C_w = 0.3$.

The different motion trajectories were considered for cost estimation tests, keeping the same traveled distance. Hereinafter, we present some experimental results under the following conditions:

- Case 1 when the control law is the minimum time
- Case 2 when control law is the minimum RMS torque

Final time and maximum speed are set to the required values. Acceleration is limited by default via the torque controller.

Figure 3 illustrates the explicit form of the torque and speed trajectories generated under cases 1 and 2.

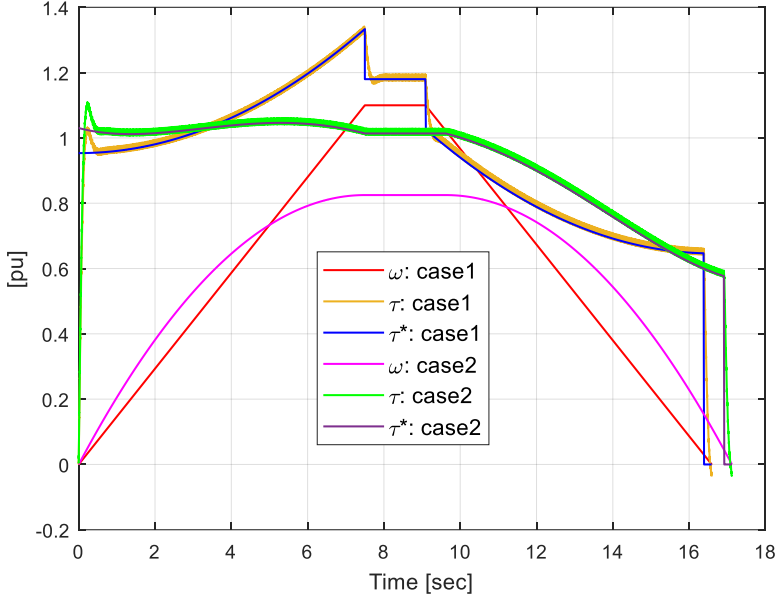


Figure 3
Torque and velocity vs. time in case 1 (minimum time) and
2 (minimum copper energy losses) for UQM device

Both trajectories have the same final required time $T_f = 15s$ but, the execution time was higher because of the speed limit $\omega_{\max} = 1.1[pu]$. In case 1 the control law was the minimum time and resulted $T_f = 16.57s$ and torque $\tau_{RMS} = 1.0189[pu]$. In case 2 the control law was the minimum RMS torque and resulted $T_f = 17.08s$ and torque $\tau_{RMS} = 0.9893[pu]$. In both cases the tracking error of the actual electromagnetic torque vs. the requested one is negligible.

Figure 4 shows how torque vs. speed implicit trajectories is performed on the energy conversion efficiency map. The same movement on two different paths leads to different costs.

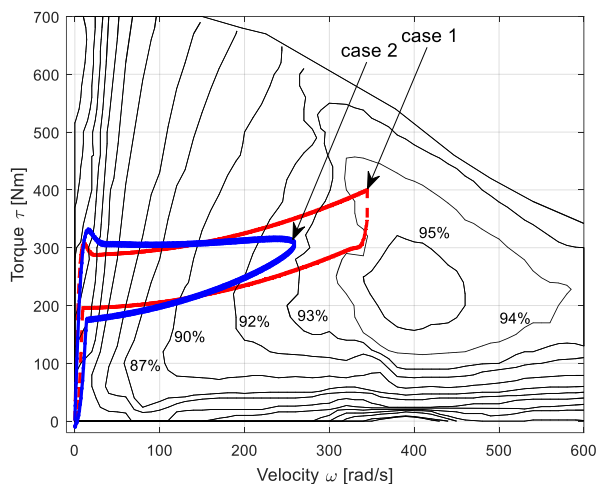


Figure 4

The implicit trajectories of the torques vs. velocities in cases 1 (minimum time) and 2 (minimal copper loss) projected on the efficiency map

Figure 5 shows the energy conversion efficiency based on the distance x travelled. It can be seen that during start-up and braking, the efficiency of the movement is reduced.

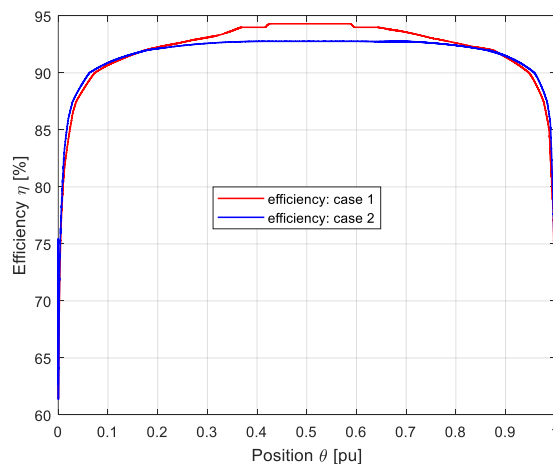


Figure 5

Efficiencies vs. traveled position in cases 1 (minimum time) and 2 (minimum copper energy losses) for UQM device

Figure 6 shows the variation of the a priori cost as a function of the execution time needed to complete a requested distance for an automotive load torque. Two control laws were applied: minimum time and minimum RMS torque control.

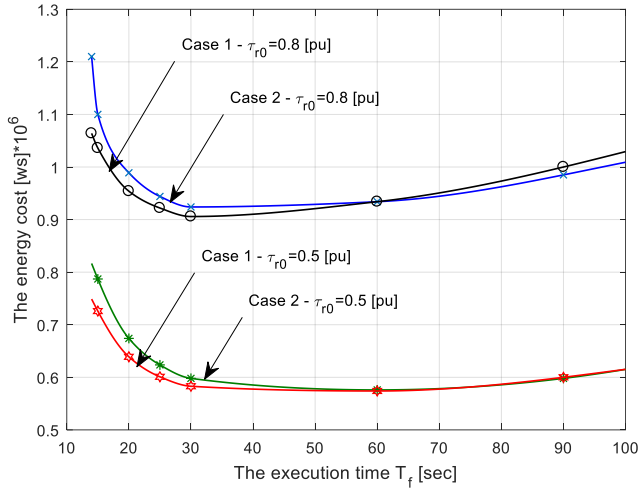


Figure 6

Variation of the cost of movement over a 200 m distance depending on the final execution time and the control law

The influence of the execution time on the energy cost is predominant, in relation to the control law. This outcome is consistent with [19]. For an EV with the initial data specified above, the 200m distance is done at a minimum cost if the travel time is $T_f = 30s$ as shown in Figure 5. For example, if the travel time is $T_f = 20s$, the energy cost increases with 7.5% for minimum time command law and with 6.5% for the minimum RMS torque control law.

Similar analysis can be performed for various motion processes, the method is generalized and not dedicated, since the motion variables are rated to the internal shaft of the motor.

Conclusions

The paper discusses a method of estimating the energy cost for electromechanical motion system. The tested solution is based on a set of experimental data in the steady-state regime for the efficiency, speed and torque of a servo motor with permanent magnet powered by a static converter with the feed forward control system. The experimental data are interpolated to all the allowable range by building the surface for efficiency in terms of speed and torque. Thus, we can evaluate the efficiencies values for each sample of the trajectory, and calculate the

cost and overall efficiency of the movement. It was confirmed experimentally that the same useful work can be performed at different levels of cost and efficiency, which proved the existence of an optimal operation. The advantage of the method developed in the paper is that it eliminates the uncertainties related to the analytical evaluation of the losses of the energy conversion system. Future research will aim to predetermine the optimal trajectories of the movement in both motor and generator operation modes of the electric machine. The presented method can be extended relatively easily for on-line applications, where the load torque needs to be estimated based on the total moment of inertia, motor torque and speed. The a priori calculation of energy costs for electromechanical energy conversion processes provides additional information to the end user, with the aim of avoiding energy waste.

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