

# Management Schemes Incorporating all Services for Optimum Railway System Energy Utilization Based on Level Set Calculation Method

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*Abstract: The Rail Technical Strategy aims to meet the goals that reduce carbon emissions, increase rail speed, lower rail costs and improve customer satisfaction. The reduction of fuel consumption and costs for rail travel depends on more effective maintenance and operation. In a railway system, mainly diesel energy is needed for traction followed by 30 percent energy consumption. The engine will start a reaction to extract this carbon by running the gas supplying hot and reactive. The heat released by the reaction will be adequate to support the reaction under such conditions, resulting in very high local temperatures usually as a thermal wave. Energy efficiency of rail vehicles, expressed in the reduction of the size of diesel engines and the consistent development of battery and supercapacitor systems to promote regenerative braking are in focus. For the accessibility study of these issues with regard to the implementation of the engine control approach, level set calculations approaches were used in this study.*

*Keywords: optimization; train diesel engine; maximizing efficiency; driving quality*

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## 1 Introduction

Steam locomotives were the dominant source of electricity for railways in worldwide between the 1830s and 1940s [1]. Steam engines were unreliable and expensive to keep and run. The first petrol engine was invented by Dr. N. A Otto in 1876 and by Gottlieb Daimler in 1884. In 1886-1890 James Ackroyd Stuart created a compression-ignition oil engine and Dr. Rudolf introduced an oil-only engine after many years of work. Theoretical tests by Diesel have shown that compression-ignition engines can attain thermal efficiencies of up to 73% compared to 6-10%

for a steam engine and 18-22% for gasoline spark-ignition engines [1]. The term Diesel is used to create the compression-ignition engine. At the time, diesel trains had many benefits relative to steam locomotives described as follows: It is known that rail operations now use energy and fuel very efficiently. Rail emissions are comparatively low and are referred to as 'green transport' compared with other transport methods (e.g. planes and trucks) [2]. The government supports the modal change from road to track; thus, rail transport development will be accelerated and rail capability pressure brought on. The energy shortage and environmental emissions will be intensified by increased capacity and the expansion of rail traction and operations. Most countries around the globe are seeking to find a way to reduce energy demand and emissions of greenhouse gas (GHG). In Britain, the report of the RSSB called 'Meeting Rail's Carbon Ambition' states that rail emissions could be reduced by 38 percent per passenger per kilometer and saves 400,000 tons/day in 5 years. The industry must meet four criteria: energy efficient driving; weight control for new trains; auto shut down of auxiliary loads and regenerative braking considering the carbon and cost saving viewpoint. The reduction of fuel consumption and costs in rail travel would rely on more efficient rolling stock production, infrastructure modernization, and operational optimization. The reduction to GHG emissions would rely both on improving the energy efficiency of diesel trains and on the electrification of railway networks to ensure that railway emissions are extremely low. Increased track capacity should be introduced with the aim of optimizing traffic flow, increasing the efficiency of railways and reducing upgrading time.

Different policies for improving Energy Efficiency (EE) include a number of sustainability goals for the European Union (EU) and other countries in the world. Transportation accounts for a high percentage of the ultimate energy consumption and could have significant improvement opportunities. In last decade, sustainability is a significant issue for railway sector as the Community of European Railway and Infrastructure Companies (CER) and International Union of Railways (UIC) [3]. Following the 1990 consumption baseline, the objectives are 50% reduction in the intensity of energy use (energy per passenger and km, i.e., kWh/p km) in 2030 (0.09 kWh /pkm), and 60% in 2050. Also, a reduction in emissions of 75% in 2050 is predicted from the environmental point of view.

Various countries work in different fields to reduce the increase and domination of road transport as well as increase the market share of so-called eco-friendly means of transportation (e.g. railway and maritime), such as the model of Polish Intermodal Transport or China's One Belt One Road Initiative (OBOR) and the OBOR in particular. Around 13,000 trains have been operated in the last eight years [4]. Railway transportation has several advantages that explain its development in the last decade both in the passenger sector (for instance, the development of 25,000 km of high-speed infrastructures in China [5]), but mainly in the freight sector: Short time (one-third of shipping), no weather influence, high safety, green environmental protection, and its complementarity with shipping transportation

(and the development of new harbor infrastructures around the world [5]). For trains, one significant comment is that in some countries passenger traveling is cheap, but only on high-speed services (for example in Spain). Transport by freight generates a benefit mostly in certain countries (Canada and the US) or is conducted with high capacities (up to 90% in the North Rail Express Freight Service, in Northern Europe), in the German Bahn (DB) (the largest market for freight). For instance, by 2038-2050, the DB Cargo operator is increasing its rail transport volumes in Germany by 70% [3]. It is also understood, however, that conventional passenger and freight services depend heavily on government subsidies (e.g., China Railway Express [5]). The energy management schemes aim at optimum management within the railway system and at incorporating all energy services of the system improve this by properly utilizing of energy (an effective expense for railway operation) [6, 7]. Different organizations and operators aim to integrate the techniques and procedures that could reduce the effect of the transport industry on total energy demand in the medium and long-term.

## **1.1 Energy Efficiency in Railways**

### **1.1.1 The Use of Regenerative Braking**

Regenerative braking is conceptually simplistic and is a popular solution to enhance energy efficiency, but in fact entails a very complicated problem since there is no certainty that all trains have separate positions in the rail systems at the same time and location (electrically speaking) with different roles in the Rail Power System (i.e., generator vs. load). In traditional railways, this issue is much more difficult since many trains will be in the same track as catenaries. In this case, power cannot be injected into the network because the overhead line voltages are increased or reverse flows are not necessary, so due to the substation configuration, the locomotive resistive braking bank generation is consumed and dissipated [8]. It should be kept in mind that there are two reasons that increase the potential for regenerative braking. Firstly, the reduction in capital cost and the increase of energy density. Energy density and lifetime of storage systems seems now more possible than one decade before (partially driven by the deployment of Electric Vehicles and the Integration of Renewables, RES). Second, regenerative braking capacity can be increased by converters so that the same friction rate (pneumatic braking) does not need to be used in fusion mode during service breakage throughout all practical speed ranges. Pure electric frequency is referred to [9]. This increases the braking efficiency as can be seen in various statistics, but only when energy can be stored or used some way [10, 11, 12, 13, 14]. Table 1 shows the energy losses in diesel train, and Table 2 indicates the comparison of transmission systems in diesel traction.

## 1.2 Level Set Method

### 1.2.1 Basic Level Set Formulation

Interface evolution description is a key component of the flow of gas and liquid. Osher and Sethian's level set method offers an efficiently functional representation of the interface as the level set of a greater dimensional function. It has evolved from a multi-faceted and high-powered interface methodology. The major advantage of this implicit image of a driving interface is the ability to accommodate diesel engine modifications naturally. Following is the level set formulation. The level set method shows the interface ( $\Gamma$ ) as the zero-contour of a higher dimensional function ( $\phi$ ) as level set method defined as the signed distance function to  $\Gamma$ :

$$|\phi(\vec{x}, t)| = |\vec{x} - \vec{x}_r| \quad (1)$$

[20]

$x \rightarrow \Gamma = \text{interface location (closest to } x \rightarrow)$

$\phi$  takes positive and negative values on the sides of interface. Interface curvature  $k$  and interface normal vector  $n \rightarrow$  is computed as:

$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} \text{ and } k = -\nabla \cdot \vec{n} \quad (2)$$

Regarding the velocity field  $u \rightarrow = (u, v, w)$ , the interface evolves based on the level set transport equation is

$$\frac{\partial\phi}{\partial t} + \vec{u} \cdot \nabla\phi = 0 \quad (3)$$

Since the Level Set function has not retained the property of its signed distance function. The re-initialization equation was proposed by Sussman et al. as Eq. (4) to change a level set function  $\phi_0$  into the signed distance function  $\phi$ .

$$\phi_r + \text{sgn}(\phi_0)(|\nabla\phi| - 1) = 0 \quad (4)$$

$\text{sgn} = a$  smoothed-out signum function

$\tau =$  fictitious time controlling the width of band around 0 level set.

Theoretically the signed distance function is preserved without re-initialization in this extension velocity technique. However, the original extension rate is only a first-order velocity approximation close the interface and can lead to unexpected numerical items as local solutions for the flow near the interface. Similar to the extension velocity, Ovsyannikov et al. suggested that the distance function of the signed device is held by inserting a source term direct into the recently established level-set equation. This approach rewrites the level set equation for transport as

$$\frac{\partial\phi}{\partial t} + \vec{u} \cdot \nabla\phi = A(\vec{x}, t)\phi, \quad (5)$$

$A(x \rightarrow, t)$  = an arbitrary regular function not based on  $\phi(x \rightarrow, t)$

Through theoretical analysis, the source term could be expressed as

$$A(\vec{x}, t)\phi = [u_k - (u_k)|_{n=0}] \frac{\partial \phi}{\partial x_k}. \quad (6)$$

The Taylor's expansion of  $A(x \rightarrow, t)$  can be obtained as an arbitrary high-order approximate form as

$$A(\vec{x}, t) = A_0 + A_1 n + A_2 n^2 + O(n^3) \quad (7)$$

This approach simplifies the implementation of the level-set method in contrast with the extension speed method; however, a return to the re-initialization mechanism is needed. It is possible to significantly reduce the number of re-initializations. The hyperbolic tangent function was introduced as a level-set function by Olsson, etc. In the interface bounded field, it is secondhand precision and sufficient for mass conservation. This defines the hyperbolic tangent function

$$\psi(\vec{x}, t) = \frac{1}{2} \left( \tanh h \left( \frac{\phi(\vec{x}, t)}{2\varepsilon} \right) + 1 \right) \quad (8)$$

$\varepsilon$  = a parameter to control the profile thickness.

Table 1  
Energy losses in the diesel train [2]

	Intercity DEMU	Regional DHMU
	Class 221	Class 170
Engine Losses	68%	68%
Engine Idle	5%	5%
Auxiliary Use	5%	5%
Transmission Loss	4%	3%
Running resistance	14%	6%
Inertia	4%	13%

This formulation makes a correlation between the hyperbolic tangent function and level set function.  $\phi = 0$  defined by the location of iso-surface  $\psi = 0.5$  as Eq. (9).

$$\frac{\partial \psi}{\partial \tau} + \nabla \cdot (\psi(1 - \psi)\vec{n}) = \nabla \cdot (\varepsilon(\nabla \psi \cdot \vec{n})\vec{n}). \quad (9)$$

Table 2  
Comparison of transmission systems in diesel traction

	Diesel-mechanic	Diesel-electric	Diesel-hydraulic
Engine efficiency	equal	equal	equal
Transmission efficiency (approximately)	95%	85%	85%
Possibility for optimum engine load	high	high	low

### 1.2.2 Improvement of the Re-Initialization Process

Sussman *et al.* [30] proposed PDE based re-initialization process using the equation  $\phi_\tau = L(\phi_0, \phi) = \text{sgn}(\phi_0)(1 - |\nabla\phi|)$ , so  $\text{sgn}$  is applied to keep  $\phi_0$  unchanged on the interface as Eq. (10)

$$s_\varepsilon(\phi_0) = \frac{\phi_0}{\sqrt{\phi_0^2 + \varepsilon^2}}. \quad (10)$$

$$\partial_2 \int_\Omega H(\phi) = 0 \quad (11)$$

$H$  = a smoothed out approximation of sign function as

$$H_\varepsilon(\phi) = \begin{cases} \frac{1}{2} & \text{if } \phi > \varepsilon \\ -\frac{1}{2} & \text{if } \phi < -\varepsilon \\ \frac{1}{2} \left( \frac{\phi}{\varepsilon} + \frac{1}{\pi} \sin(\pi\phi / \varepsilon) \right) & \text{otherwise} \end{cases} \quad (12)$$

$$\phi_\tau = L(\phi_0, \phi) + Af(\phi) \quad (13)$$

$$A = \frac{-\int_\Omega H(\phi)L(\phi_0, \phi)}{H(\phi)f(\phi)} \quad (14)$$

$$\phi_\tau + \text{sign}(\phi_0)(|\nabla\phi| - 1) = \alpha f \quad (15)$$

$$\frac{\partial\phi}{\partial\tau} + (A_0 - A(t))(-p + k)|\nabla\phi| = 0 \quad (16)$$

The re-initialization equation was modified for McCaslin and Desjardins to take into account considerable spatial variations in the fixed level transport. This local reset is dependent on the local deformation of the flow and numerical diffusion. The re-initialization equation for yields has a spatially and temporally different element

$$\frac{\partial\psi}{\partial\tau} = \nabla \cdot (\alpha(\varepsilon(\nabla\psi \cdot \vec{n}) - \psi(1 - \psi)\vec{n})) \quad (17)$$

The local interfacial value of  $\alpha$  is defined as

$$\alpha(\vec{x}_\Gamma, t) = \max(\lambda_n |u_\Gamma \cdot \vec{n}|, \lambda_s |\vec{n}^T, s_\Gamma \cdot \vec{n}| \varepsilon) \quad (18)$$

$\lambda_n |u| \rightarrow \Gamma \cdot n \rightarrow$  |measures the numerical diffusion

$\lambda_s |n| \rightarrow T \cdot S\Gamma \cdot n \rightarrow |\varepsilon$  measures the deformation of local flow kinematics

$$\begin{aligned} & \max(D_{\bar{x}}\phi_{i,j,k} - D_x^+\phi_{i,j,k}, 0)^2 + \max(D_{\bar{y}}\phi_{i,j,k} - D_y^+\phi_{i,j,k}, 0)^2 \\ & + \max(D_{\bar{z}}\phi_{i,j,k} - D_z^+\phi_{i,j,k}, 0)^2 - 1 = 0 \end{aligned} \quad (19)$$

$D_{x,y,z\pm}$  = the first order upwind finite difference notations

The direction and the radius of each particle applied to correct the error to the level set are stored. The radius is:

$$r_p = \begin{cases} r_{max} & \text{if } s_p \phi(\vec{x}_p) > r_{max} \\ s_p \phi(\vec{x}_p) & \text{if } r_{min} \leq s_p \phi(\vec{x}_p) \leq r_{max} \\ r_{min} & \text{if } s_p \phi(\vec{x}_p) < r_{min} \end{cases} \quad (20)$$

Because evaporation rates are normally determined from the jumping conditions in areas just next door to the interfaces, the governing equations for incompressible evaporating gas-liquid fluxes are as "Jump State Form"

$$\nabla \cdot \vec{u} = 0 \quad (21)$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \otimes \vec{u}) = -\nabla p + \nabla \cdot (\mu [\nabla \cdot \vec{u} + \nabla \cdot \vec{u}^t]) + \rho \vec{g} \quad (22)$$

$$\frac{\partial \rho C_p T}{\partial t} + \nabla \cdot (\rho C_p \vec{u} T) = \nabla \cdot (\lambda \nabla T) \quad (23)$$

$$\frac{\partial \rho Y}{\partial t} \nabla \cdot (\rho \vec{u} Y) = \nabla \cdot (\rho D_m \nabla Y) \quad (24)$$

$T$  = the temperature

$\lambda$  = the thermal conductivity

$C_p$  = the specific heat at constant pressure

$Y$  = the mass fraction,

$D_m$  = the mass diffusion coefficient

The vapor pressure on the interface is described to be the saturation pressure  $P_{vap}^\Gamma$ , which and  $Y_{vap}^\Gamma$  can be given by the Clausius-Clapeyron relation as:

$$P_{vap}^\Gamma = p_{atm} \exp\left(-\frac{h_{lg} m_{vap}}{R} \left(\frac{1}{T^\Gamma} - \frac{1}{T_B}\right)\right) \quad (25)$$

$$Y_{vap}^\Gamma = \frac{P_{vap}^\Gamma m_{vap}}{(p_{atm} - P_{vap}^\Gamma) m_g + P_{vap}^\Gamma m_{vap}} \quad (26)$$

$$\dot{\omega} = \frac{\rho_g D_m \nabla Y \cdot \vec{n}|_g^\Gamma}{1 - Y_{vap}^\Gamma} \quad (27)$$

$$T^\Gamma = \frac{h_{lg} m_{vap} T_B}{h_{lg} m_{vap} - R T_B \ln \frac{P_{vap}^\Gamma}{p_{atm}}} \quad (28)$$

$P_{vap}^\Gamma$  is computed as

$$P_{vap}^\Gamma = \frac{-Y p_{atm} m_g}{(m_g - m_{vap}) Y_{vap}^\Gamma - m_{vap}} \quad (29)$$

Table 3  
Annual financial saving and carbon emissions' saving  
(Source: London Midland Internal Documents)

	Savings in kWh/day @23.5kW	Annual Savings in kWh	Annual savings @ 9p/kWh	Annual savings in kg of CO2
SX daytime stabling	4814.945	962989	£86,669.01	469746.0342
SX over night stabling	8174.805	1634961	£147,146.49	797533.9758
SO day and overnight stabling	25144.894	1005795.76	£90,521.62	490627.1717
SU day and overnight stabling	16770.175	670807	£60,372.63	327219.6546
Total Annual Savings	54904.819	4274552.76	£384,709.75	2085126.836
Annual savings minus losses	49414.3371	3847097.484	£346,238.77	1876614.153

### 3 Level Set Simulation

#### 3.1 Cost Function Variations Due to the Unknowns

Assuming that the 1th and 2<sup>nd</sup> order Fréchet derivatives of  $R(p)$  show them as  $R'(p)[\cdot]$  and  $R''(p)[\cdot, \cdot]$ . The first order Fréchet derivative of a function (if it exists) is a bounded and linear operator. The Fréchet derivatives of the second order are also bounded, but bilinear, meaning that the operator operates on two arguments and is linear for each [32]. Table 3 shows the annual financial saving and carbon emissions' saving. Table 4 shows the modification financial saving.

$$R(P + \varepsilon \delta P) = R(P) + \varepsilon \dot{R}(P)[\delta P] + \frac{\varepsilon^2}{2} R''(P)[\delta P, \delta P] + O(\varepsilon^3) \quad (30)$$

Rewriting  $F(p)$  as

$$F(P) = \frac{1}{2} \langle R(P), R(P) \rangle_{S_u} \quad (31)$$

$$F(P + \varepsilon \delta P) = F(p) + \varepsilon F'(p)[\delta P] + \frac{\varepsilon^2}{2} F''(P)[\delta P, \delta P] + O(\varepsilon^3) \quad (32)$$

$$F'(p)[p_1] = Re \langle \dot{R}(P)[p_1], R(P) \rangle_{S_u} \quad (11)$$

$$F''(P)[p_1, p_2] = Re \langle \dot{R}(P)[p_1], \dot{R}(P)[p_2] \rangle_{S_u} + Re \langle R''(P)[p_1, p_2], R(P) \rangle_{S_u} \quad (33)$$

$$\langle \hat{u}, \dot{R}(P)[\hat{p}] \rangle_{S_u} = \langle \dot{R}(P) * [\hat{u}], \hat{p} \rangle_{S_p}, \quad \forall \hat{u} \in S_u, \forall \hat{p} \in S_p \quad (34)$$

can be written as

$$F'(p)[p_1] = Re \langle \dot{R}(P) *, R(p), p_1 \rangle_{S_p} \quad (35)$$

Regarding that the existence of the 1th and 2<sup>nd</sup> order Fréchet derivatives of  $p$  regarded to  $v$ , showed as  $p'(v)[\cdot]$  and  $p''(v)[\cdot, \cdot]$ , the 1th and 2<sup>nd</sup> order Fréchet derivatives of  $F$  regarded to  $v$  is

$$\hat{F}(v)[v_1] = \hat{F}(v)[p'(v)v_1] \quad (36)$$

$$F''[v_1, v_2] = F''(v)[p'(v)[p_1], p'(v)[p_2]] + \hat{F}(p)[p''(v)[v_1, v_2]] \quad (37)$$

$$v_1, v_2 \in Sv$$

Equations (36) and (37) themselves can be shown in terms of  $R(p)$  and its derivatives using (11) and (12):

for a  $c \in \mathbb{R}$

$$\begin{cases} \phi(X, \mu) > c \quad \forall x \in D \\ \phi(X, \mu) = c \quad \forall x \in \partial D \\ \phi(X, \mu) < c \quad \forall x \in \Omega \setminus D \end{cases} \quad (38)$$

$$P(X, \mu) = p_i(x)H(p_o(X, \mu) - c) + p_o(1 - H(\phi(X, \mu) - c)) \quad (39)$$

$$\frac{\partial p}{\partial \phi} = (p_i - p_o)\delta_{rg}(\phi - c) \quad (40)$$

$$\frac{\partial p}{\partial \mu_j} = \frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \mu_j} = (p_i - p_o)\delta_{rg}(\phi - c) \frac{\partial \phi}{\partial \mu_j} \quad (25)$$

Now using (25) with (15) and (14), the gradient vector for  $F$  is

$$\frac{\partial F}{\partial \mu_j} = \hat{F}(p) \left[ \frac{\partial p}{\partial \mu_j} \right] = \text{Re} \langle R(p), \hat{R}(p) \left[ \frac{\partial p}{\partial \mu_j} \right] \rangle s_u \quad (41)$$

$$= \text{Re} \langle \hat{R}(p) * \left[ R(p), (p_i - p_o)\delta_{rg}(\phi - c) \frac{\partial \phi}{\partial \mu_j} \right] \rangle s_p \quad (42)$$

$$\frac{\partial^2 p}{\partial \mu_j \partial \mu_k} = (p_i - p_o) \left( \delta_{rg}(\phi - c) \frac{\partial^2 \phi}{\partial \mu_j \partial \mu_k} + \delta'_{rg}(\phi - c) \frac{\partial \phi}{\partial \mu_j} \frac{\partial \phi}{\partial \mu_k} \right) \quad (43)$$

where  $\delta'_{rg}(\cdot)$  is the derivative of the regularized Dirac delta function. Based on (16) and (12) we have

$$\begin{aligned} \frac{\partial^2 F}{\partial \mu_j \partial \mu_k} &= F''(p) \left[ \frac{\partial p}{\partial \mu_j}, \frac{\partial p}{\partial \mu_k} \right] + F(p) \frac{\partial^2 p}{\partial \mu_j \partial \mu_k} \\ &= \text{Re} \langle \hat{R}(p) \left[ \frac{\partial p}{\partial \mu_j} \right], \hat{R}(p) \left[ \frac{\partial p}{\partial \mu_k} \right] \rangle s_u + \text{Re} \langle R''(P) \left[ \frac{\partial p}{\partial \mu_j}, \frac{\partial p}{\partial \mu_k} \right], R(P) \rangle s_u + \\ &\quad \text{Re} \langle \hat{R}(P) *, [R(P)], \frac{\partial^2 p}{\partial \mu_j \partial \mu_k} \rangle s_p \cdot \end{aligned} \quad (44)$$

$$\frac{\partial^2 F}{\partial \mu_j \partial \mu_k} \simeq \text{Re}$$

$$\left\langle \hat{R}(p) \left[ \frac{\partial p}{\partial \mu_j} \right], \hat{R}(p) \left[ \frac{\partial p}{\partial \mu_k} \right] \right\rangle s_u \quad (45)$$

$$= Re \left\langle \dot{R}(p) \left[ (p_i - p_o) \delta_{rg}(\phi - c) \frac{\partial \phi}{\partial \mu_j} \right], \dot{R}(p) \left[ (p_i - p_o) \delta_{rg}(\phi - c) \frac{\partial \phi}{\partial \mu_k} \right] \right\rangle S_u \quad (32)$$

$$\left[ \tilde{H}_\mu(F) |_{\mu=\mu^{(t)}} + \lambda^{(t)} I \right] (\mu^{(t+1)} - \mu^{(t)}) = -J_\mu(F) |_{\mu=\mu^{(t)}} \quad t \geq 0 \quad (46)$$

$$\frac{\partial \phi}{\partial \beta_j} = \alpha_j \beta_j \frac{\|x - X_j\|^2}{\|\beta_j(x - X_j)\|^\dagger} \psi(\|\beta_j(x - X_j)\|^\dagger) \quad (47)$$

$$\frac{\partial \phi}{\partial x_j^{(k)}} = \alpha_j \beta_j^2 \frac{X_j^{(k)} - x^{(k)}}{\|\beta_j(x - X_j)\|^\dagger} \psi(\|\beta_j(x - X_j)\|^\dagger) \quad (48)$$

Table 4  
Modification financial saving (Source: London Midland internal reports)

		Pessimistic	Realistic	Optimistic
Fuel amount	Total fuel consumption per day (in litres)	117	234	351
	Total fuel consumption per week (in litres)	819	1638	2457
	Total fuel consumption per year (in litres)	42588	85176	127764
Financial saving	Fuel cost in pounds per litre	£ 0.64	£ 0.64	£ 0.64
	Amount of fuel saved per year (in pounds)	£ 27,256.32	£ 54,512.64	£ 81,768.96

### Conclusions

This article presents potential ways to improve the energy efficiency of diesel and hybrid trains without affecting their dynamics. This technologies (on board and off board storage) improve the life cycle of these units and the efficiency of low- and medium-sized traffic energy networks. Several options were assessed to achieve these goals: Save dynamic braking systems fuel resources and cut the size of the diesel engine. For the improvement of the utility of simulations train feature models, Distributed Energy Resources (DER) were combined and aggregated. For this reason, super-capacitors and various battery technologies were selected. The brake energy of the cars and railways can be recovered, and energy efficiency increased. Energy savings by 10-20% and an increase in load factor and peak shavings by nearly 70 percent was recorded. In addition, the stochastic aspects of such occurrences in railways, such as time delays have been analyzed with regard to timing. Delays can have a positive or negative effect in regenerative braking on power efficiencies (and also, their impact on timetable performance). Simulations are accurate for the chosen route in the example, but the program will use the same approach for any other rail line operated by other vehicles. In addition, detailed physical and heat models of batteries will be developed and validated in the future developments of this work with a view to improving integration of energy storage systems in railways, in particular on board solutions, with space, weight and thermal needs to be avoided. The future contribution and assistance of the reporting versatility of the railways modelled on paper will contribute to an efficient energy balance (volatility) in the 2030-2050 Eco energy scenario.

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