

# Optimal Design of Wind Wheel Column

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*Abstract: The paper deals with the optimal design of a wind wheel column. Have been determined the forces acting on the column. The inner diameter and wall thickness of circular tube are unknown. The minimum of mass was determined in the case of non-linear design constraints. The constraints relate to the maximal stresses and stability of column. A numerical solution to this problem is given by genetic optimization algorithm.*

*Keywords: wind wheel column; optimal design; weld*

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## 1 Introduction

An important structural element of wind turbines is the support column. In the following, we present the optimal design of a wind wheel column with a tubular cross-section (Figure 1). The inner diameter and wall thickness of the pipe are considered unknown. After defining the loads acting on the column, we formulate the objective function and the restrictions, and then solve the optimization problem with a genetic algorithm. We calculate the connecting fillet weld of the column based on the EUROCODE 3 standard.

## 2 Optimal Design of the Column

### 2.1 Mathematical Optimization

The optimum design procedure can be formulated mathematically as follows

$$\begin{aligned} \min f(\mathbf{x}), \quad \mathbf{x} \in R^n; \\ 0 \leq g_j(\mathbf{x}), \quad j=1,2,\dots,m, \end{aligned}$$

where  $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$  the vector of the unknown quantities and  $m$  is the number of constraints.

In our case we define the dimensions of cross-sections: inner diameter  $x(1)$  and wall thickness  $x(2)$  (Figure 1).

## 2.2 Genetic Algorithm

Global optimization procedures include the genetic algorithm (GA), which in certain cases can be used for optimal search more advantageously than traditional search algorithms. The structure and operating principle of the GA differ greatly from the previously applied procedures.

The basis of the method is the natural selection, which can also be observed in nature. This principle is used by the genetic algorithm to solve mathematical problems. Based on the principle, the weaker individual dies, on the other hand, the vigorous one lives on, and the latter can reproduce.

Possible solutions of the search space are called individuals. These have different importance from the point of view of the optimum search. After running the algorithm, the best solution will emerge from these individuals, but there are "elements" among them that are less, but there are also some that do not even come close to the optimal value.

Individuals can be coded in several ways, which basically determines how the algorithm works. The advantage of the bit sequence representation is that it is easy to perform operations with it and requires little storage space. Each bit in the bit sequence can take the value 0 or 1.

When describing the genetic algorithm, the term population refers to the set of individuals that make up the search space. This changes over time as the algorithm creates a new population at each iteration. Populations built up from different individuals over time are called generations.

The selection of the fitness function is one of the most important steps in the optimization with the genetic algorithm. The search time is significantly affected by the fitness function. In practice, this only means a large increase in the case of many variables. In the user interface of the program, the objective function of the problem to be optimized must be specified under the name fitness function.

The creation of the elements of the next generation is preceded by the operation of selection, which basically determines the quality and fitness value of the newly created individuals.

The members of the new population, the new individuals, are created during the genetic operations. Crossover and mutation operations are listed here. Genetic operations, like selection, are extremely important. From the point of view of the search, an important parameter is the relative ratio of mutation and crossover. Basically, the use of crossover means the use of the results of previous iterations, while the search space can be widened with the help of mutation.

Many variations of the crossover operation are known. Its operating principle is to cut the bit sequences representing the parent units at an arbitrary location and join the opposite bit sequences together.

Mutation, like recombination, belongs to genetic operations. The main purpose of its application is to avoid the narrowing of the search space, since in that case it may happen that we get only a local extreme value instead of the global one. During the operation, the algorithm slightly changes the individual. This is usually done by randomly changing a gene on the chromosome that describes the individual. The chance of a change is so small because a significant change could result in the loss of previously earned work.

### 2.3 Formulation of the Stress Constraints

The column is subjected to the following forces:

- the horizontal force acting on the wind wheel,
- the force resulting from the wind load and whirling acting on the column,
- and the compressive force resulting from the self-weight of the wind generator and the column.

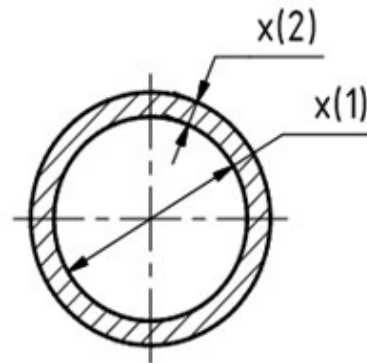


Figure 1

The self-supporting steel column [1] and its cross-section

The force acting on the wind wheel can be determined based on the following relationship [2]

$$F_k = A_k v_a \rho_a (v_1 - v_2) \quad (1)$$

where  $A_k = D_k^2 \pi / 4$  area of the wind wheel,

$D_k$  the diameter of the wind wheel,

$v_a$  the average velocity of the air, entering and exiting ( $v_1$  and  $v_2$ ) the wind wheel,

$\rho_a$  air density.

The output value of the wind speed

$$v_2 = \sqrt{1-\eta} v_1 \quad (2)$$

where

$\eta$  is the efficiency of the wind wheel.

The moment of the force acting on the wind wheel ( $F_k$ )

$$M_{hk} = F_k h, \quad (3)$$

where

$h$  is the height of the column.

The wind pressure can be determined with the following equation [3]

$$p = q_p(z)c, \quad (4)$$

where

$q_p(z)$  the value of dynamic pressure,

$c$  the shape factor, the value of which can be determined based on table 11 of the MSZ 15021 standard.

Force acting on the projection area of a cylindrical column

$$F_w = Dh p, \quad (5)$$

where

$D$  is the outer diameter of the column, ( $D = x(1)+2x(2)$ , Figure 1).

This force is considered to be acting at half the height of the holder, thus the moment at clamping

$$M_{hw} = F_w \frac{h}{2}. \quad (6)$$

Force of the whirl burling [4]

$$F_{rez} = \frac{\pi \rho_l v_{cr}^2 c_y Dh}{2\delta}, \quad (7)$$

where

$v_{cr}$  the critical wind speed can be calculated based on the literature [4],

$\delta$  is the damping factor,

$c_y$  is the transverse form factor.

The bending moment of the  $F_{rez}$  force at the clamped end of column

$$M_{hrez} = F_{rez} \frac{h}{2}. \quad (8)$$

The bending stress at the clamped end

$$\sigma_b = \frac{M_k + M_w + M_{rez}}{W}, \quad (9)$$

where

$W$  is the elastic section modulus of column.

At the lower cross-section of the column the compressive stress from the weight of the nacelle ( $G_g$ ) and the weight of the column ( $G_o$ ) arises

$$\sigma_N = \frac{G_g + G_o}{A_o}, \quad (10)$$

where

$A_o$  is the cross-sectional area of the column.

Unidirectional stresses can be added

$$\sigma_{max} = \sigma_b + \sigma_N. \quad (11)$$

The maximum of stresses ( $\sigma_{max}$ ) must be lower than the admissible stress ( $\sigma_{adm}$ )

$$\sigma_{max} \leq \sigma_{adm} \quad (12)$$

## 2.4 Formulation of the Stability Constraint

Elements subjected to combined bending and axial compression must satisfy the following conditions (EUROCODE 3). Since in our case it is a column with a ring cross section and the bending takes place around an axis, the stability conditions according to the standard will be as follows

$$\frac{\frac{N_{Ed}}{\chi_y N_{Rk}}}{\gamma_{MI}} + k_{yy} \frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\gamma_{LT} \frac{M_{y,Rk}}{\gamma_{MI}}}}{\gamma_{MI}} \leq 1, \quad (13)$$

$$\frac{\frac{N_{Ed}}{\chi_z N_{Rk}}}{\gamma_{MI}} + k_{zy} \frac{\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\gamma_{LT} \frac{M_{y,Rk}}{\gamma_{MI}}}}{\gamma_{MI}} \leq 1, \quad (14)$$

where

$N_{Ed} = G_g + G_o$  is the design value of the compression force,

$M_{y,Ed} = M_{hk} + M_{hw} + M_{hrez}$  the maximum moment about y-y axis along the member,

$\Delta M_{y,Ed}$  is the moment due to the shift of the centroidal axis, in our case  $\Delta M_{y,Ed} = 0$ ,

$\chi_y = \chi_z$  are the reduction factors due to flexural buckling,

$\chi_{LT}$  is the reduction factor due to lateral torsional buckling, in our case

$$\chi_{LT} = 1,$$

$k_{yy}$  and  $k_{zy}$  are the interaction factors, the interaction factors depend on the method, which is chosen. The interaction factors have been derived from two alternative approaches. Values these factors may be obtained from Annex A (alternative method 1) or from Annex B (alternative method 2) - EUROCODE 3,

$N_{Rk} = f_y A_o$  is the characteristic resistance to normal force of the critical cross-section,

$f_y$  yield strength,

$M_{y,Rk} = f_y W$  characteristic value of resistance to bending moments about y-y axis.

In our case, the cross section is not sensitive to torsion, therefore the recommended value of the interaction factor

$$k_{yy} = C_{my} \left[ I + (\bar{\alpha} - 0, 2) \frac{\frac{N_{Ed}}{\chi_y N_{Rk}}}{\gamma_{MI}} \right]. \quad (15)$$

According to the standard are the values of  $k_{zy} = 0.6k_{yy}$ , and  $C_{my} = 0.6$  from table 5.18. The non-dimensional slenderness ( $\bar{\lambda}$ ) can be calculated as follows

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} \sqrt{\beta_A}, \quad (16)$$

where

$$\lambda = \frac{\beta l}{i}, \quad i = \sqrt{\frac{I}{A}}, \quad \lambda_1 = \pi \sqrt{\frac{E}{f_y}}, \quad (17)$$

where

$\lambda$  slenderness ratio,

$l$  length of column,

$\beta l$  effective column length,

$i$  radius of gyration,

$I$  second moment of area,

$A$  area of cross-section,

$\lambda_1$  slenderness value to determine the relative slenderness,

$E$  modulus of elasticity,

$\beta_A = 1$ , if the cross-section is 1st, 2nd, or 3rd class.

The safety factor  $\gamma_{MI}=1,1$  and  $\chi_{LT}=1$ .

## 2.5 Formulation of the Objective Function

We choose the mass of the column as the objective function (Figure 1)

$$m = h \rho_{ac} \frac{\left[ (x_1 + 2x_2)^2 - x_1^2 \right] \pi}{4}, \quad (18)$$

where

$\rho_{ac}$  steel density,

$x_1$  inner diameter of the pipe,

$x_2$  pipe wall thickness.

## 2.6 Solving the Optimization Problem with Genetic Algorithm

A lot of optimization problem will combine technical and economical requirements against the product or the component, so functional and economical requirements must be equally considered. When specifying the technical and economical approach, product and process optimization is defined. The product optimization can be specified further [5]

*product optimization:*

- topology optimization [6],
- form optimization [7],
- dimension optimization [8],
- material optimization.

Recent years the evolutionary algorithms are used frequently as optimization procedures [9]. Evolutionary algorithms are stochastic search methods, which are based on the principles of the biological evolution. Three optimization directions of the evolutionary algorithms were developed independently from each other: the evolutionary programming, the evolution strategies and the genetic algorithms. All these methods use the variation and selection operations as the basic elements of the evolution process, but they differ in the development of these elements [10]. The usage of these algorithms will increase in the coming years due to the various application possibilities. The calculation of the actual restriction values can be computed in many applications only by numeric methods.

In our case of *dimension optimization* the dimensions of cross-sections  $x(1)$  and  $x(2)$  are computed.

The following data were used for optimization:

$D_k = 16$  m;  $\rho_a = 1,2045$  kg/m<sup>3</sup> at 20 °C;  $v_l = 25$  m/s;  $\eta = 30$  %;  $h = 25$  m;  $q_p(z) = 1,154$  kN/m<sup>2</sup>;  $c = 1$ ;  $v_{kr} = 8,63$  m/s;  $c_y = 0,225$ ;  $\delta = 0,05$ ;  $q_p(z) = 1,15$  kN/m<sup>2</sup>;  $c = 1$ ;  $G_g = 6000$  N; steel qualities:  $f_y = 235$ ; 275 and; 355 MPa;  $n = 2$ ;  $\chi_{LT} = 1$ ;  $\beta_A = 1$ ;  $\beta = 2$ ;  $E = 210$  GPa;  $\rho_{st} = 7800$  kg/m<sup>3</sup>.

We solved the formulated optimization problem with a genetic algorithm. Table 1 shows the optimal inner diameters and wall thicknesses of the pipe column, as well as the mass minimums for different steel qualities. Figure 2 illustrates the change in column mass as a function of steel quality.

Table 1  
The results of the optimization

Steel qualities	S235		S275		S355	
Wind velocity [m/s]	8	25	8	25	8	25
Inner diameter [mm]	585,6	897,2	506,8	796,8	398,7	641,1
Wall thickness [mm]	8,9	14,2	8,8	13,8	9,0	14,5

Outer diameter [mm]	603,4	<b>925,6</b>	524,4	824,4	416,7	670,1
Admissible stress [MPa]	117,5	117,5	137,5	137,5	177,5	177,5
Generated stress [MPa]	117,5	117,5	137,5	137,5	177,5	177,5
Mass [kg]	<b>3115,27</b>	<b>7647,84</b>	<b>2674,75</b>	<b>6611,75</b>	<b>2169,37</b>	<b>5609,55</b>

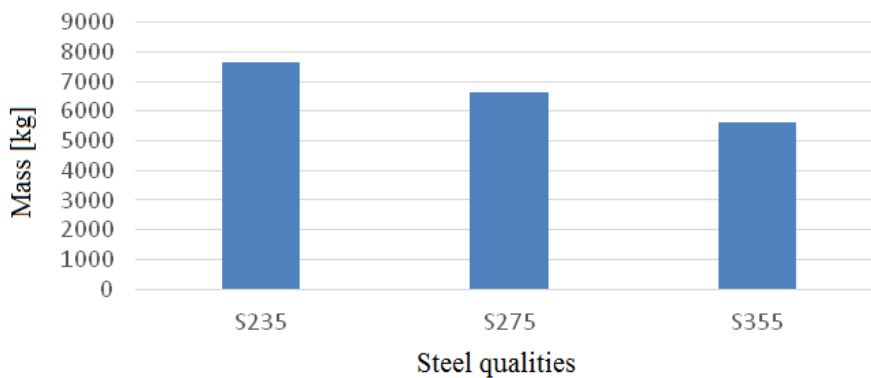


Figure 2

The change in column mass depending on the steel quality

### 3 Design Resistance of Fillet Welds

In this method, the forces transmitted by a unit length of weld are resolved into components parallel and transverse to the longitudinal axis of the weld and normal and transverse to the plane of its throat. The design throat area should be taken as

$$A_w = \sum a l_{eff}, \quad (19)$$

where

$a$  throat thickness of a fillet weld,

$l_{eff}$  the effective length of a fillet weld.

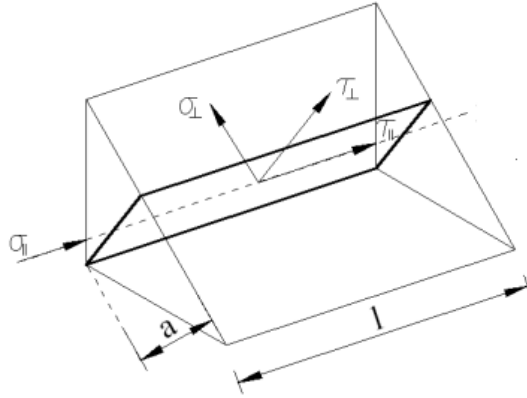


Figure 3

Stresses on the throat section of a fillet weld (EUROCODE 3)

The location of the design throat area should be assumed to be concentrated in the root. A uniform distribution of stress is assumed on the throat section of the weld, leading to the normal stresses and shear stresses shown in Figure 3, as follows:

$\sigma_{\perp}$  is the normal stress perpendicular to the throat,

$\sigma_{\parallel}$  is the normal stress parallel to the axis of the weld,

$\tau_{\perp}$  is the shear stress (in the plane of the throat) perpendicular to the axis of the weld,

$\tau_{\parallel}$  is the shear stress (in the plane of the throat) parallel to the axis of the weld.

The normal stress  $\sigma_{\parallel}$  parallel to the axis is not considered when verifying the design resistance of the weld.

The design resistance of the fillet weld will be sufficient if the following are both satisfied:

1.)

$$\sqrt{(\sigma_{\perp})^2 + 3(\tau_{\parallel}^2 + \tau_{\perp}^2)} \leq \frac{f_u}{\beta_w \gamma_{M2}}, \quad (20)$$

2.)

$$\sigma_{\perp} \leq \frac{f_u}{\gamma_{M2}}, \quad (21)$$

where

$f_u$  is the nominal ultimate tensile strength of the weaker part joined,

$\beta_w$  is the appropriate correlation factor,

$\gamma_{M2}$  is the partial safety factor (resistance of welds).

Welds between parts with different material strength grades should be designed using the properties of the material with the lower strength grade.

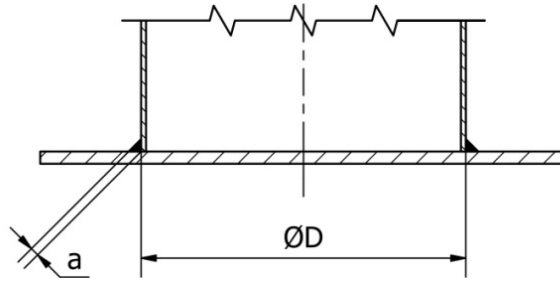


Figure 4

The fillet weld of the column

The following data were used for calculation of fillet weld:

the size of fillet weld is  $a = 10$  mm; the outer diameter of column is  $D_o = D + 2a = 925,6$  mm +  $2 \cdot 10$  mm =  $945,6$  mm;  $A_k = 200,96$  m<sup>2</sup>.

Calculation of elastic section modulus

$$W = \frac{[(D + 2a)^4 - D^4]\pi}{32(D + 2a)} = \frac{[(945,6\text{mm})^4 - (925,6\text{mm})^4]\pi}{32(945,6\text{mm})} = 6,7995 \cdot 10^6 \text{mm}^3$$

The force acting on the wind wheel is according to equation (1)  $F_k = 22,66$  kN.

The moment of this force according to equation (3) is  $M_k = 551,5$  kNm. Force acting on the projection area of a cylindrical column according to equation (5) is  $F_w = 26,7$  kN. The moment of this force according to equation (6) is  $M_w = 333,79$  kNm. The force of the whirl burling according to equation (7) is  $F_{rez} = 14,67$  kN. The moment of this force according to equation (8) is  $M_{rez} = 183,38$  kNm.

Stresses due to bending can be determined according to equation (9)

$$\sigma_b = \frac{5,5150 \cdot 10^8 \text{Nmm} + 3,3379 \cdot 10^8 \text{Nmm} + 1,8932 \cdot 10^8 \text{Nmm}}{6,7995 \cdot 10^6 \text{mm}^3} = 157,2 \text{MPa} .$$

In the fillet weld the stress components can be determined as follows

$$\sigma_{\perp} = \tau_{\perp} = \sigma \cos 45^\circ = 157,2 \frac{\text{N}}{\text{mm}^2} \cos 45^\circ = 111,2 \text{MPa} ,$$

and

$$\tau_{II} = 0.$$

Substituting the values in equations (20) and (21) gives that

$$\sqrt{(111,2 \text{ MPa})^2 + 3(111,2 \text{ MPa})^2} \leq \frac{360 \text{ MPa}}{0,8 \cdot 1,25},$$

222,4 MPa < 360 MPa, the first condition is met.

$$111,2 \text{ MPa} \leq \frac{360 \text{ MPa}}{1,25}, \text{ the second condition is met.}$$

## Conclusions

We developed the optimal design of the support column of a 20 kW wind wheel with a tubular section, taking into account the requirements of the EUROCODE 3 steel structure standard. We chose the mass of the construction as the objective function. In the restrictions, in addition to the stresses from pressure and bending, we also took into account the interaction of the two stresses. In the case of different steel qualities, we determined the minimum mass of the column, as well as its optimal inner diameter and wall thickness. The fillet weld of the column was adjusted according to the EUROCODE 3 standard.

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