

Numerical Analysis of Stabilization Schemes for the Deformed State of Railway Embankment, Using Piles

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Abstract: This article presents a numerical analysis of stabilization schemes for the deformed state of a railway embankment using piles. The study involves the analysis of the concept of reinforcing the railway embankment with horizontal and vertical elements, as well as the main stabilization schemes for the deformed state. The objective is to select a stabilization scheme characterized by a minimal number of reinforcing elements that are sufficient to maximally reduce vertical displacements. For the numerical analysis, six finite element models were developed, which helped obtain the deformation results for the unreinforced embankment and five reinforcement options. Following the numerical analysis of the options with vertical and inclined reinforcement elements, the overall deformed state of the finite element model in the vertical plane was obtained. The results of vertical displacements of the reinforced railway embankment demonstrate that the installation of piles, created based on the jet grouting technology, reduces deformations by 1.50 to 1.88 times.

Keywords: railway embankment; pile; drilling-mixing technology; deformed state; finite element method; numerical analysis

1 Introduction

Long-term operation of any construction object made of soil invariably leads to a complex deformation state. This is particularly evident when considering the operation of a railway embankment between two repairs. If the embankment is constructed without technological violations, the interval between repairs undoubtedly increases. However, if there were violations in the construction, such as, improper soil layer compaction, it is likely that deformations will increase over time. Therefore, the condition of the soil affects the interval between repairs and the maintenance costs of the railway track [1].

Nevertheless, it must be acknowledged that even with an embankment constructed according to all technological requirements, deformations of the ballast and soil will occur due to the dynamic impact of trains. During the operation of the railway track, minor but irreversible deviations of the ballast occur, leading to the propagation of deformations into the upper part of the soil [2].

If the increase in the deformed state of the embankment and the accumulation of irreversible displacements within it is a process that cannot be eliminated, then it should be minimized. From this idea arises the concept of preventive soil reinforcement. Of course, if the embankment is reinforced during its construction, it does not imply that deviations from the technological process are permissible. The purpose of reinforcement (or stabilization) used for the ballast layer is the application of geosynthetic grids or meshes to separate it from the soil [3]. This helps to minimize the penetration of ballast gravel into the soil, thereby reducing deformations. A method for assessing the rate of plastic settlement of the railway track due to prolonged train loads is proposed in [4].

Stabilization of the railway embankment during its construction is also carried out when the construction conditions are uncertain and various external factors are present. Such factors can include deformation of weak foundations under the embankment, disturbance of soil moisture regimes, and increased axle loads. It is undeniable that a pre-reinforced embankment will not deform more intensively than an unreinforced one under increased loads, for example, from 20 to 25 tons. In [5], the dynamic responses of soil layers to the passage of a train with high axle loads are examined. In [6], the issues of bearing capacity and soil stability under increased axle loads and speeds on Polish railways are discussed. The specifics of maintaining and reinforcing the soil with increasing train speeds are analyzed in [7].

The results of measuring the deformation properties of the railway track depending on the condition of its layers are presented in numerous research works. For example, in [8], modern geodetic methods for assessing the geometry of the railway track are substantiated. In [9], stresses in the railway track elements, which are considered indicative of its condition under high-speed movement conditions, were measured. Papers [10] [11] describe methods for assessing the track condition through deformation measurements in the ballast layer supported by the subgrade, which were conducted with the participation of the authors of this study.

A specific case of soil reinforcement is its implementation during reconfiguration. For example, in Ukraine, the prospect of transitioning the railway track gauge from 1520 mm to 1435 mm is currently widely discussed [12]. Solving such a complex task naturally requires time. Therefore, as an intermediate yet effective solution, the reconfiguration of part of the Ukrainian railway to a combined track gauge of 1 435/1 520 mm is proposed. As found in studies of this option, the existence of a combined track gauge creates a non-uniform stress-strain state in

the railway track. This non-uniformity of stresses and deviations necessitates the reinforcement of weak soil, which must be performed on the existing section. Only after such reinforcement can the combined track be laid and the railway line put into operation.

A large number of methods and options for soil reinforcement can be classified based on geometric parameters, specifically the placement of reinforcement elements within the soil. In general, reinforcement options are divided into those that use horizontal elements (geosynthetic materials [13], additional layers of compacted soil and ballast [11], hot mix asphalt, etc. [14]) and vertical reinforcement elements (rubber granules [15], piles, and micropiles [16] [17], created based on various technologies such as jet grouting, deep soil mixing, bored piles, casing pipes, and dispersed reinforcement [18-23]). There are also combinations of horizontal and vertical elements, such as geosynthetic materials and piles [3] [24].

In this study, the authors considered a soil reinforcement option for a 1 435 mm gauge track based on vertical elements (piles or micropiles created using deep soil mixing technology). The choice of this option is explained by the authors' positive experience, who, in scientific works [18-20], demonstrated that vertical elements most effectively reduce vertical soil deformations.

The choice of deep soil mixing technology, as opposed to, for example, jet grouting, is justified by the simplification of the work process and the lack of need for special technological expenses (for instance, ultra-high pressure for creating soil cement is not required). Additionally, the created pile or micropile has deformation characteristics sufficient to reduce soil deformations by up to 20 % [18]. An important argument in favor of choosing deep soil mixing technology is the relatively low material and financial costs.

Undoubtedly, the first and important parameter is geometric, as the reduction of the vertical component of displacements is directly related to the location of the vertical reinforcement element. However, from the analysis of characteristic scientific works, finding this parameter is not primary, as stabilization schemes include several vertical elements. It can be noted that when using such schemes, it is difficult to isolate the role of a single reinforcement element [3].

Conversely, in some scientific works where single or double piles are studied, the role of each pile is clear and can be assessed [17, 18, 20, 25]. In this context, the search for the geometric placement of a single or double pile to determine its effect on the deformed state is tied to economic considerations.

The result of the analysis is a scientific hypothesis that the insertion of vertical elements into the subgrade has a maximal impact on the vertical component of the deformation state, reducing the overall level of vertical displacements of the railway track elements. Therefore, the objective of this study can be formulated as follows: from the array of existing stabilization schemes for the deformed state of

the railway embankment, select one that is characterized by the minimum number of reinforcement elements, yet is sufficient to maximally reduce vertical deformations. Achieving this goal is possible only through a comprehensive approach, which consists of analyzing existing concepts of the deformation of railway track layers and conducting a numerical analysis of stabilization schemes for the deformed state using the finite element method.

2 Methods and Resources

2.1 Analysis of the Deformation State of the Railway Embankment

To analyze the distribution of vertical deformations within the railway track, three zones with varying intensity can be identified, Fig. 1. The existence of these three zones is subsequently substantiated by the results of numerical analysis.

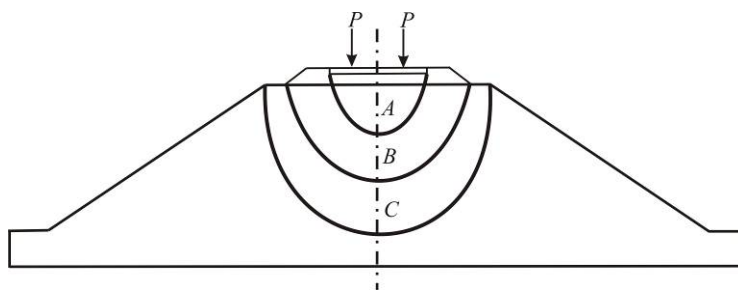


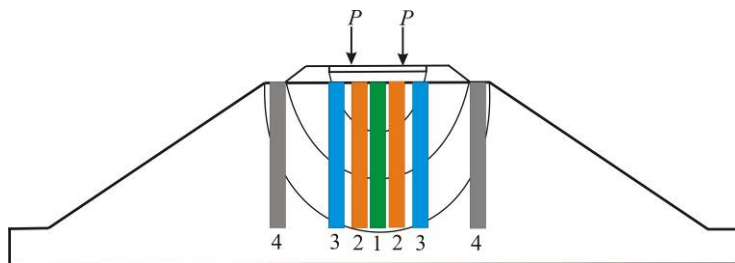
Figure 1

Scheme of dividing the railway track space into three zones of vertical deformations

The level of vertical deformations is maximal in zone *A*, decreases in zone *B*, and diminishes to a minimum value in zone *C*. In a real embankment, characterized by a stepwise distribution of deformation characteristics (elastic modulus or deformation modulus E), the shape of these zones is somewhat altered. However, the nature of the changes is not so pronounced as to highlight the specific distribution of deformations under the sleeper (elastic modulus of reinforced concrete $E=4.0 \cdot 10^7$ kPa), under the ballast (elastic modulus $E=10 \cdot 10^4$ kPa), and in the clay loam embankment ($E=35 \cdot 10^3$ kPa). Naturally, from one zone to another, the values of vertical deformations decrease within the system. Nevertheless, the deformation characteristics of the ballast and the soil of the embankment are not sufficient to reduce the vertical pressure, leading to a process of deformation accumulation, as analyzed above.

Figure 2 outlines the principle of introducing vertical reinforcement elements (piles created using deep soil mixing technology) into the soil. This principle aims to minimize the deformation state and stabilize the soil. There is no doubt that vertical reinforcement elements should be placed within the "core" of maximum vertical deformations, such that the pile intersects all three zones (Fig. 2a, Variant 1). Variant 1 is the only option for the immersion of a single pile. In further numerical analysis, aside from Variant 0 (no reinforcement elements), Variant 1 serves as the baseline. Comparing other soil reinforcement options with Variant 1 allows for the determination of their effectiveness in stabilizing the vertical component of the deformed state.

a)



b)

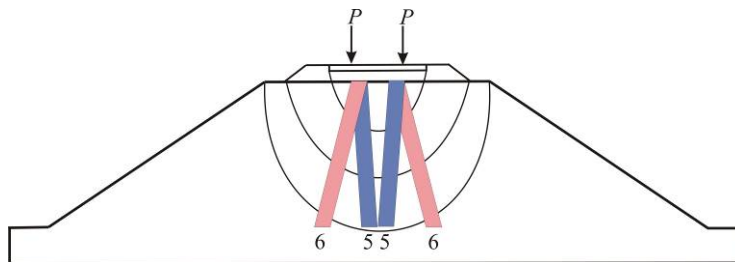


Figure 2

Schemes for stabilizing the deformed state of the railway embankment using vertical (a) and inclined (b) piles created with deep soil mixing technology

Variants 2, 3, and 4 utilize the insertion of double piles symmetrically positioned in the embankment relative to the vertical plane of symmetry. Their placement is not arbitrary. These stabilization schemes are feasible under deep soil mixing technology, specifically the capability of drilling in ballast and embankment without lifting the rail-sleeper grid. Accordingly, there are three drilling options: between the sleepers (Variant 2), near the rail (Variant 3), and at the edge of the embankment (Variant 4).

It should be noted immediately that Variant 4 is the least effective, as proven by the results in [18]. This is logically explained by the analysis in Fig. 1. Since

Variant 4 is outside the "core" of maximum vertical deformations (zone *A* and partially zone *B*), it cannot affect them. Therefore, within the framework of this study, the authors do not consider Variant 4.

Double pile options (Fig. 2a, Variants 2 and 3) have a logical justification from the analysis of the intensity zones of the vertical component of the deformed state (Fig. 1). Their placement within the "core" is symmetrical relative to the vertical plane of symmetry of the railway track, and doubling the piles allows for a reduction in vertical deformations and overall soil stabilization.

An important innovation that the authors of this paper examined in detail is the use of inclined elements (Fig. 2b). Such pile options are not often considered. Indeed, reducing the vertical component of the deformed state is more effective with the use of vertical reinforcement elements. However, the nature of deformation distribution, especially in zone *A*, allows for the prediction of a positive impact from inclined elements. This impact is explained by the fact that an inclined reinforcement element more actively intersects zones of intense deformations. From a technological standpoint, there are no problems in implementing deep soil mixing technology from a railway platform to achieve an inclination of the reinforcement element of 10-15° (Variants 5 and 6 have an inclination of 10.6° to the vertical).

Thus, for further numerical analysis of stabilization schemes using the finite element method, the following options are adopted: Variant 0 – the soil has no reinforcement; Variant 1 – the soil is reinforced with a single vertical pile, Fig. 2a; Variant 2 – the soil is reinforced with double vertical piles inside the railway track, Fig. 2a; Variant 3 – the soil is reinforced with double vertical piles outside the railway track, Fig. 2a; Variant 5 – the soil is reinforced with double inclined piles inside the railway track, inclined towards the plane of symmetry, Fig. 2b; Variant 6 – the soil is reinforced with double inclined piles inside the railway track, inclined away from the plane of symmetry, Fig. 2b.

2.2 Numerical Analysis of Stabilization Schemes for the Deformed State Using the Finite Element Method

The numerical analysis of stabilization schemes for the deformed state using the finite element method is implemented through the SCAD for Windows software suite. A finite element model of the railway embankment has been developed, consisting of 87150 nodes and 57526 finite elements. This model represents a 4-meter-high embankment at a 1:1 scale. The length of the reinforcement element matches the height of the embankment. The dimensions of the finite element model are shown in Fig. 3.

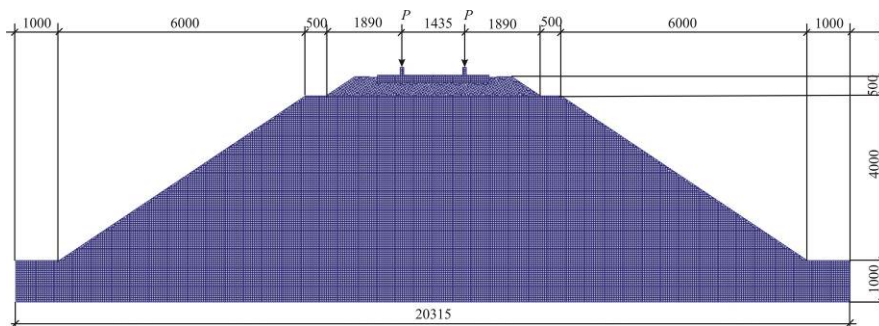


Figure 3

Finite element model of the railway embankment

The problem setup is three-dimensional. Accordingly, the finite elements are volumetric prisms and tetrahedrons with characteristic dimensions of $0.05 \times 0.05 \times 0.05$ m. The mesh is fine, with further refinement in areas representing the rails spaced 1 435 mm apart and the sleepers. The number of finite elements classifies the numerical analysis task as medium-sized.

The model constraints are applied in a way that reduces the boundary effect and its influence on the deformed structure. A soil base with dimensions of 1×1 m is modeled under the embankment, with horizontal deformations restricted on the sides and both horizontal and vertical deformations restricted on the bottom plane.

The deformation characteristics of the model are as follows: rail – modulus of elasticity of steel $E = 2.06 \cdot 10^8$ kPa; sleeper – modulus of elasticity of reinforced concrete $E = 4.0 \cdot 10^7$ kPa; crushed stone ballast – modulus of elasticity $E = 10 \cdot 10^4$ kPa; embankment soil (loam) – $E = 35 \cdot 10^3$ kPa; soil-cement – $E = 37.5 \cdot 10^4$ kPa.

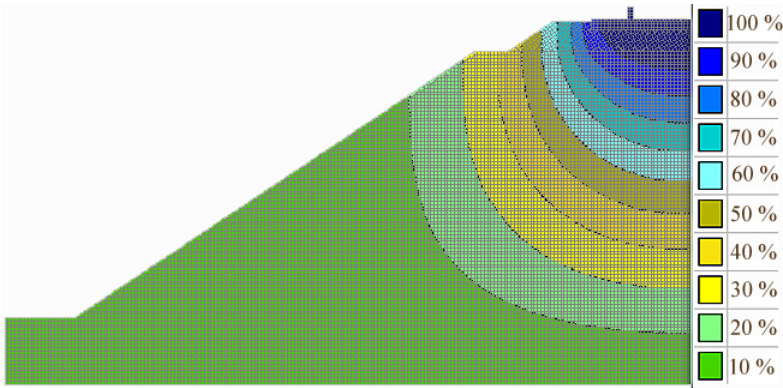
The soil used for constructing the subgrade has strictly regulated normative values for the deformation modulus E , which typically range from 30 to 40 MPa. In the model, a deformation modulus value of 35 MPa was selected. These specified limits of the soil deformation modulus influence the development of vertical deformations, but their variations will not be significant and will not affect the reliability of the pile placement's effectiveness.

A load is applied to the rails, with a value of $P = 98.1$ kN, corresponding to a 20-ton static load applied to the axle. The rails are rigidly connected to the sleepers, which is ensured by the finite elements of the SCAD suite. The part of the rail-sleeper grid present in the model evenly distributes the pressure onto the ballast, which then redistributes the stress onto the embankment.

3 Results and Discussion

Following the numerical analysis of 6 finite element models with vertical and inclined reinforcement elements, the overall deformed state of the finite element model was obtained, with its vertical component presented in Figs. 4-6. Vertical deformations in the figures are presented as percentages. The deformation values for the baseline Variant 1 (Fig. 4a) are set at 100%. Therefore, in Fig. 4b, 5, and 6, the characteristic values indicate the reduction in vertical deformations compared to the unreinforced option.

a)



b)

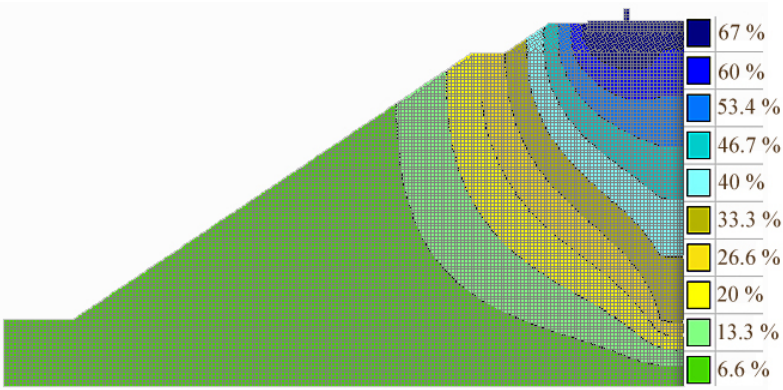


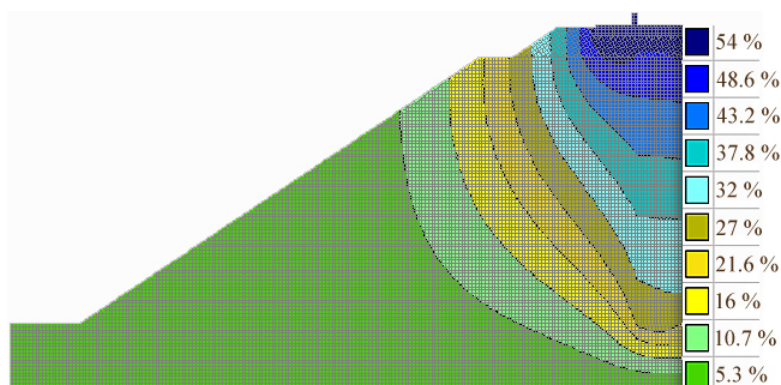
Figure 4

Deformed state of finite element models of the embankment: a) Variant 0; b) Variant 1

For better visualization of isolines and isopoles of the vertical component of the deformed state, only half of the finite element model is shown in the figures.

The analysis of the deformed state of the unreinforced embankment (Variant 0, Fig. 4a) demonstrates that the distribution of deformations in the rail-sleeper grid, ballast, and embankment corresponds to the three-zone distribution depicted in Fig. 1. The isopoles without discontinuities have a smooth curvilinear shape, indicating the correct choice of the finite element mesh and the sufficient accuracy of the conducted numerical analysis.

a)



b)

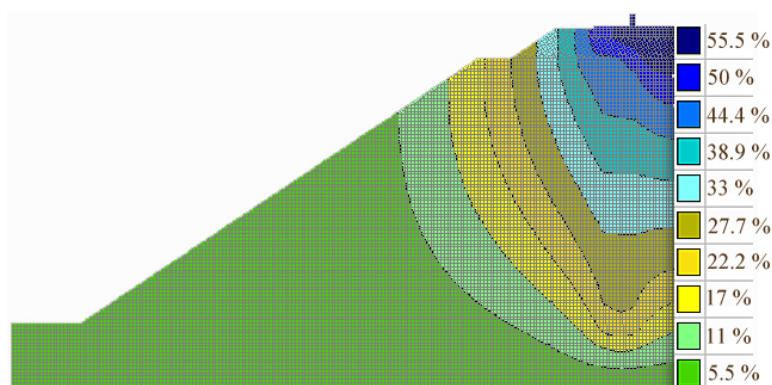
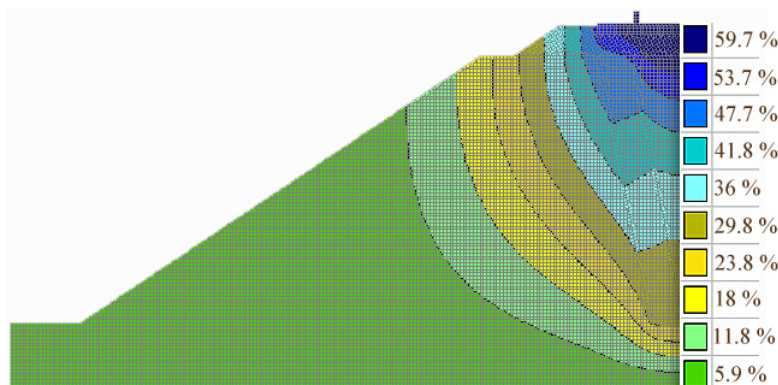


Figure 5

Deformed state of finite element models of the embankment: a) Variant 2; b) Variant 3

An important aspect for further investigation is the qualitative analysis of the change in the shape of isolines and isopoles induced by the introduction of vertical or inclined elements into the homogeneous embankment. Specifically, the alteration in the shape and area of the isopoles, especially in zone A, alongside the results of quantitative analysis, allows us to assert the effectiveness of the stabilization scheme or to identify any negative or insufficient effects, manifested in only a slight reduction in vertical deformations.

a)



b)

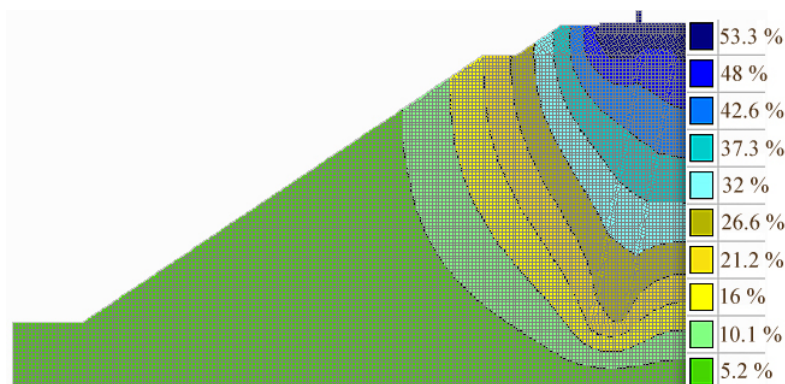


Figure 6

Deformed state of finite element models of the embankment: a) Variant 5; b) Variant 6

For instance, reinforcement with a single pile (Variant 1, Fig. 4b) demonstrates a clear increase in the area of zone *A*, while its values significantly decrease within it. A similar outcome is observed for the case of double piles, both variants of which (especially Variant 2) increase the area of zone *A*, thus reducing the intensity of vertical deformations within it. Even greater changes in the area and shape of the isopoles, as well as their values, are observed in the case of double inclined piles (Fig. 6).

The results of the numerical analysis confirm the hypothesis of the most effective stabilization of the deformed state, attributed to the immersion of the reinforcement element and the increased value of its deformation properties. The modulus of elasticity of soil-cement is 10-11 times greater than the modulus of deformation of the clay embankment, allowing the reinforcement element to bear a larger portion of the pressure from the ballast and the rail-sleeper grid.

The deformations of the unreinforced embankment (Variant 0, Fig. 4a) necessitate engineering intervention. The presence of piles in the ground "spreads out" the zone of maximum vertical deformations, thereby reducing their values by only 1.5 times for a single pile (Variant 1, Fig. 4b).

Introduction of double vertical elements also reduces the value of vertical displacements by 1.85 times (Variant 2, Fig. 5a) and 1.80 times (Variant 3, Fig. 5b). It should be noted that the comparison of the two variants with double piles confirms the hypothesis that the reinforcement element should be placed between the rails to achieve a greater stabilization effect. The choice between the single pile option (Variant 1) and the double pile option (Variant 2) is possible only based on the requirements of technical and economic justification of the corresponding application of materials and works.

The immersion of double inclined elements proves the effectiveness of schemes for stabilizing the deformed state, as the values of vertical deformations decrease by 1.68 times (Variant 5, Fig. 6a) and 1.88 times (Variant 6, Fig. 6b). Variant 6 with inclined piles directed from the middle between the rails towards the slope of the embankment (see Fig. 1) is the best stabilization scheme, based on the obtained effect of reducing vertical deformations among all options. Unlike Variant 2 (Fig. 5a), the angled submerged pile not only changes the area and shape of the isopolles but also distorts the isolines (Fig. 6b), resulting in fractures. This indicates that the most deformed zones change their outline. However, the choice between Variant 2 and Variant 5 also requires technical and economic justification, as carrying out work at an angle requires more expenses than the vertical variant of the drilling and mixing technology.

Conclusions

The variant of the stabilization scheme for vertical displacements is more effective when reinforcement elements (piles) are located within zones of maximum deformation. Due to the ratio of the elasticity moduli of the soil and soil-cement, piles created based on the drilling and mixing technology enlarge the area of maximum deformation, thereby reducing the intensity of the vertical component values of the deformed state.

Introducing a vertical or inclined reinforcement element into the soil significantly alters both the qualitative distribution pattern of vertical deformations and their quantitative values. The reduction in vertical deformations of the reinforced embankment indicates that the immersion of piles created based on the drilling and mixing technology reduces the deformed state by 1.50 - 1.88 times.

Unlike vertical elements, the application of inclined reinforcement elements, although effectively stabilizing the deformed state of the embankment, requires technical and economic justification for their application, as it involves higher implementation costs.

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