

Evaluate the Rollover Stability Characteristics of the Liquid Elliptical Tank Truck when Changing Motion Direction

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Abstract: Rollover accidents of vehicle caused a very serious impact on human life and property. The rollover possibility of a tank truck was usually higher than that of a traditional passenger vehicle because the vibrations of the fluid in tank impacted on the movement. It had a tendency to roll at the ratio of 25% and 50% liquid volume in tank when it is affected by a lateral acceleration in steady turning and lane change. This conclusion is obtained from the load transfer ratio (LTR) and the roll angle value of the suspension (RAS). In order to determine these values, the study built a dynamic model of the liquid elliptical tank truck, applied quasi-static, Lagrange and D'Alembert's principle to describe the oscillations of the liquid in tank and the rollover stability state of the vehicle. In addition, it was also determined that the liquid circular tank truck was less stable than the elliptical tank truck with the same specifications and moving conditions.

Keywords: Elliptical tank; Rollover stability; Tank truck; Quasi-static; Load transfer ratio

1 Introduction

Among all types of car accidents, rollover is the most serious type of accident, no matter how the safety system is equipped modern, it can still roll if the driver makes mistakes. According to a report by [1], although the rollover accident rate in the United States in 2022 will decrease by 9 percent compared to 2021 because this time is related to the Covid 19 epidemic, there are still 3623 accidents. Especially, the liquid tank truck are easy to roller than passenger vehicle as to be mentioned in [2], this study also presents the movement of liquids inside different types of tank shapes. In the most common practice is the elliptical tank, the authors [3] used a semi-static model to analyze the movement of a liquid part

inside the tanks, and the Lagrange method to analyze the dynamic model of the vehicle, the result shows that a liquid level of 40-60% in the tank has a tendency to overturn. In study [4], a new approach is presented to investigate the force and moments of motion caused by a liquid part inside an elliptical tank when excited by lateral forces. An equivalent mechanical model used in this paper to approximate the motion of a fluid is the trammel pendulum model. The mechanical model is derived by calculating the trajectory of the center of gravity of the liquid mass in the tank when the vehicle's lateral acceleration varies from 0 to $1g$. The authors determined that the percentage of liquid in the tank was at most 60%, reaching the maximum moving force. Also, based on the Lagrange method to build the vehicle's rollover stability equation from the elliptical tank dynamics model containing a partial liquid and taking into account the nonlinear stiffness of the suspension [5]. The analysis shows that liquid inside a partially filled tank often affects rollover stability of the vehicle, and the threshold for roll stability can be raised by increasing the stiffness of the vehicle's suspension. It is possible to optimize the parameters of the cross sectional shape of the tank, the stiffness of the suspension to improve the stability when turning.

In addition, the liquid circular tank is also interested by some researchers as in [6] the author used semi-static model and D'Alembert's principle for the circular tank truck corresponding to the liquid levels inside the tank. Simulation evaluates the influence of lateral acceleration which acts role as a double lane change, at the liquid level of 0.4, 0.8m at a speed of 100 km/h, the vehicle will roll. A simple method is proposed in [7] to estimate the turn around threshold of the tank truck when it carries partially and fully filled of liquid. The overturning moment caused by fluid motion in a circular, elliptical or oval cross-section is calculated as a function of the lateral acceleration, the roll angle, the fill level of the tank and the shape of the tank. The roll threshold is defined by two different methods. In the first method, the rollover threshold is estimated by using a non-suspension truck dynamics model, but this method gives a large error for the oval tank. Overturning moments are determined from the multi-axle suspension and tires, individual axle loads and lateral displacements are calculated, integrated in the second method. This method gives a reasonable estimate of the constant acceleration limit for axial loads corresponding to a variety of tank fluid levels and for different axial loads when it fills above 40%. The estimated error for occupancy below 40% ranges from 2% to 17%. However, the simulation results show that the proposed simple methods can be effectively used to estimate the roll threshold limit for different fluid levels under different as well as constant axial loads.

Besides, many authors [8] also use the finite element analysis method to analyze the fluid movement in the elliptical tank in the case of braking or study [9] to analyze the effect on the coefficient of friction between the safety fence and the tire to the vehicle's roll stability through the factor of static stability.

Although there are many studies on the influence of fluid movement inside the tank on the vehicle's roll stability by different methods. However, when

considering the rollover stability characteristic of the liquid truck, there are still some points that have not been taken into account such as the viscosity of the liquid, the stiffness of the stabilizer bar, the danger volume level of liquid in tank that has not been shown, and finally the comparison between elliptical tank truck (ET) and circular tank truck (CT). Therefore, this study will supplement the above factors.

2 Performance Method

Liquid elliptical tank trucks are widely used for fire trucks or watering public works around the world. They have a cylindrical length and an elliptical cross-section. The main concern of this type of vehicle is the lateral stability because the liquid inside the tank will move when the vehicle changes direction of movement. Therefore, this study will focus on the liquid elliptical tank truck considered in the horizontal plane of the vehicle.

The dynamic model of the liquid elliptical tank truck used an integrated multi-object dynamic model for the analysis of the rollover stability, which is separated into the liquid tank part and the vehicle body part. When the vehicle moves, the liquid in the tank is shared into two parts, including the moving part as a trammel pendulum, the rest will not move below the tank [10,11] as shown in Figure 1.

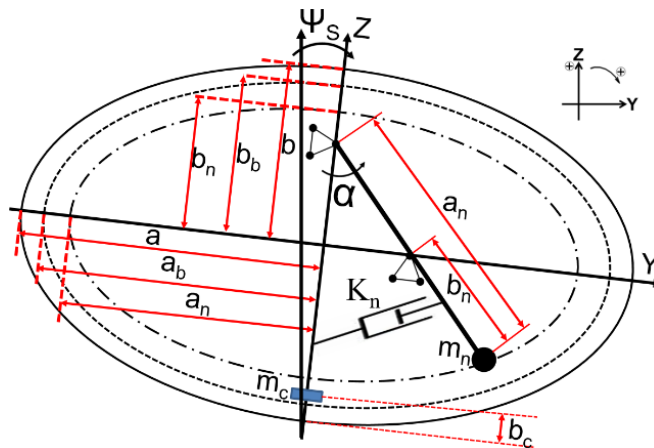


Figure 1

The motion model of the liquid inside the tank

From Figure 1, the study has built an equation describing the movement of liquid inside the tank by quasi-static and Lagrange method as follows:

$$\begin{aligned}
\ddot{\alpha} = & -\left\{ \ddot{\psi}_S (a_n b_n - a_n b \sin \alpha) - \frac{1}{2} \dot{\alpha}^2 \sin(2\alpha) (a_n^2 - b_n^2) \right. \\
& - \ddot{\psi}_S^2 \left[\frac{1}{2} \sin(2\alpha) (a_n^2 - b_n^2) + b_n b \cos \alpha \right] + a_{LT} (a_n \sin \alpha \cos \psi_S + b_n \cos \alpha \sin \psi_S) \\
& \left. - g (a_n \sin \alpha \sin \psi_S - b_n \cos \alpha \cos \psi_S) - K_n \dot{\alpha} \right\} / (a_n^2 \sin^2 \alpha + b_n^2 \cos^2 \alpha)
\end{aligned} \quad (1)$$

With $2a$ and $2b$ are the horizontal and vertical dimension of the tank, $2a_n$ and $2b_n$ are the horizontal and vertical dimension of the pendulum's movement trajectory, $2a_b$ and $2b_b$ are the horizontal and vertical dimension of the liquid bulk movement trajectory, $m_{n,c}$ are the mass of movement liquid and fixed liquid, ψ_S is the angle of inclination of sprung mass, α is the displacement angle a liquid part, g is the acceleration of gravity, K_n is the viscosity coefficient of the liquid, b_c is the height of the fixed liquid center, a_{LT} is lateral acceleration which is generated during change of direction of motion.

The dynamics model of the vehicle without fluids includes the sprung mass (SM) placed on the suspension (dampers, springs, stabilizer bar) and symmetrical about the vertical axis of the vehicle, below which is the unsprung mass (UM) and the tire, all is presented as Figure 2:

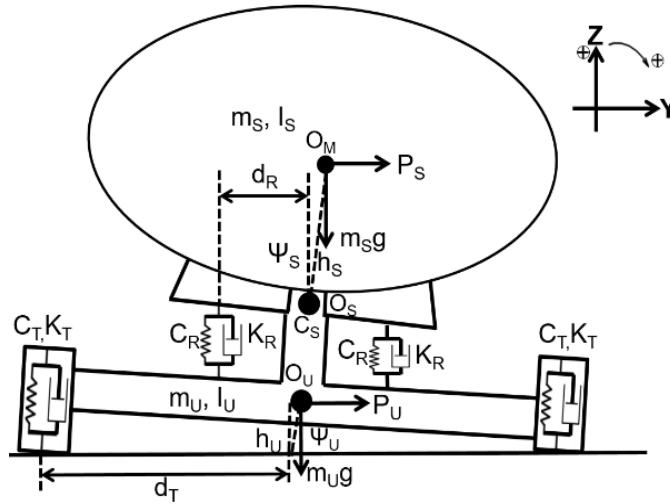


Figure 2

The vehicle dynamic model without liquid

Where $m_{S,U}$ are the SM and UM, $Z_{S,U}$ are the displacement of SM and UM, ψ_U is the angle of inclination of unsprung mass, C_R and K_R are the stiffness and damping of the suspension, C_T and K_T are the cornering and roll stiffness of tyre, $I_{S,U}$ are the moment of inertia of SM and UM, C_S is the stiffness of stabilizer bar, $h_{S,U}$ are the dimension from center of SM to the roll center and center of UM to the ground, h_i is the dimension from center of liquid mass center to roll axis, $d_{R,T}$ are the

dimension from the suspension to the roll center and center of UM to the wheel track, g is the gravity acceleration, $P_{S,U}$ are the centrifugal force of SM and UM.

With the positive direction opposite to the gravity acceleration and clockwise rotation, by applying D'Alembert's principle to the 4-DOF model of Figure 2 and the roll model in [12,13] to establish the system of differential equations of the vehicle motion as Eq. (2):

$$\left\{ \begin{array}{l} (m_s + m_n + m_c)\ddot{Z}_s = -2C_R(Z_s - Z_U) - 2K_R(\dot{Z}_s - \dot{Z}_U) \\ -m_n\dot{\alpha}^2 a_n \cos \alpha - m_n\ddot{\alpha} a_n \sin \alpha \\ I_s\ddot{\psi}_s = -(2d_R^2 C_R + C_s)(\psi_s - \psi_U) - 2d_R^2 K_R(\dot{\psi}_s - \dot{\psi}_U) \\ +m_s a_{LT} h_s \cos \psi_s + m_s g h_s \sin \psi_s \\ +m_n g (h_t \sin \psi_s + a_n \sin \alpha) + m_n g a_{LT} (h_t \cos \psi_s - a_n \cos \alpha) \\ +m_n \dot{\alpha}^2 a_n h_t \sin(\psi_s + \alpha) + m_n \ddot{\alpha} a_n [a_n - h_t \cos(\psi_s + \alpha)] \\ +m_c a_{LT} b_c \cos \psi_s + m_c g b_c \sin \psi_s \\ m_U \ddot{Z}_U = 2C_R(Z_s - Z_U) + 2K_R(\dot{Z}_s - \dot{Z}_U) \\ -C_T(Z_U - d_T \psi_U) - K_T(\dot{Z}_U - d_T \dot{\psi}_U) \\ I_U \ddot{\psi}_U = (2d_R^2 C_R + C_s)(\psi_s - \psi_U) + 2d_R^2 K_R(\dot{\psi}_s - \dot{\psi}_U) \\ +C_T d_T (Z_U - d_T \psi_U) + K_T d_T (\dot{Z}_U - d_T \dot{\psi}_U) \\ +m_U a_{LT} h_U \cos \psi_U + m_U g h_U \sin \psi_U \end{array} \right. \quad (2)$$

The lateral acceleration a_{LT} is generated by centrifugal force [14] in the case of turning or changing lanes, this force will affect the liquid inside the tank, the sprung mass P_s and the unsprung mass P_U , it is expressed by the formula follow:

$$P_i = a_{LT-j} m_i \quad (3)$$

With i corresponds to the liquid, SM and UM object; j are the vehicle movements in case of steady turns (ST) and lane changes (LC) mentioned in the study [15, 16] by Eq. (4,5) below:

$$a_{LT-ST} = \frac{v^2}{R} \quad (4)$$

$$a_{LT-LC} = \left(\frac{2\pi S_{LC}}{t_{LC}^2} \right) \sin \left(\frac{2\pi t_A}{t_{LC}} \right) \quad (5)$$

Where a_{LT-ST} and a_{LT-LC} correspond to the lateral acceleration in ST and LC, respectively, v is vehicle speed, R is turning radius, S_{LC} is the width of the changing lane, t_{LC} is entire duration of the changing lane, t_A is time to analyze the changing lane.

$$\left\{ \begin{array}{l}
 m = \mu V \Delta \\
 \varepsilon = \frac{a}{b} = \frac{a_b}{b_b} = \frac{a_n}{b_n} \\
 \frac{b_n}{b} = 1.087 + 0.6999\Delta - 0.1407\varepsilon - 0.9291\Delta^2 - 1.178\varepsilon\Delta + 0.05495\varepsilon^2 \\
 \quad - 0.03353\Delta^3 + 0.5404\varepsilon\Delta^2 + 0.1518\varepsilon^2\Delta \\
 \frac{m_n}{m} = 0.7844 - 1.729\Delta + 0.3351\varepsilon + 1.156\Delta^2 + 0.7256\varepsilon\Delta - 0.1254\varepsilon^2 \\
 \quad - 0.3219\Delta^3 - 0.9152\varepsilon\Delta^2 + 0.08043\varepsilon^2\Delta \\
 m_c = m - m_n \\
 b_c = [m(b - b_b) - m_n(b - b_n)] / m_c
 \end{array} \right. \quad (6)$$

In addition, from the results in the study [3, 11], Eq. (6) are used to determine the liquid mass of each part in the tank, the trajectory of motion of the center of gravity of the liquid bulk in the tank and the liquid trammel pendulum depending on the volume ratio of the liquid in tank. With m is mass of the entire liquid in the tank, μ is density of fluid, V is volume of fluid, Δ is volume percentage of liquid, ε is Ratio between horizontal and vertical diameter of tank. The elliptical tank has a horizontal tank diameter of 1.7 m, a vertical tank diameter of 1.5 m, and a length of 2 m. Table 1 displays the liquid tank truck's symbols and specifications to be presented in the equations, but it does not account for the liquid's mass.

Table 1
Meaning and value of symbols

Specifications	Value	Specifications	Value
The sprung mass (SM) m_s	2305 kg	The unsprung mass (UM) m_u	1073 kg
The dimension from center of SM to the roll center h_s	0.61 m	The dimension from center of UM to the ground h_u	0.525 m
The moment of inertia of SM I_s	11174 kgm ²	The moment of inertia of UM I_u	1311.1 kgm ²
Stiffness of the suspension C_R	724704.5 N/m	Damping of the suspension K_R	42629 Ns/m
Cornering stiffness of tire C_T	1015000 N/m	Roll stiffness of tire K_T	25000 Ns/m
Viscosity coefficient of the liquid K_n	0.5	Stiffness of stabilizer bar C_s	35800 N/m
The velocity of vehicle v	30 km/h	The turning radius R	15 m
The dimension from the suspension to the roll center d_R	0.62 m	The dimension from center of UM to the wheel track d_T	1.025 m

3 Theory of Rollover Stability Evaluation

An upcoming rollover accident can be accurately predicted correctly if the load transfer ratio (LTR) value [12,17] in Eq. (7) is calculated when it approaches ± 1 .

$$LTR = \frac{F_{z2} - F_{z1}}{F_{z2} + F_{z1}} \quad (7)$$

When the vehicle turns to the right, the force acting on the left wheel F_{z1} and right wheel F_{z2} will be determined according to Eq. (8). In case the vehicle is rolled, F_{z1} is zero, then only F_{z2} , the value of $LTR=1$.

$$\begin{cases} F_{z1} = 0.5g(m_S + m + m_U) - C_T(Z_U + \psi_U d_T) - K_T(\dot{Z}_U + \dot{\psi}_U d_T) \\ F_{z2} = 0.5g(m_S + m + m_U) - C_T(Z_U - \psi_U d_T) - K_T(\dot{Z}_U - \dot{\psi}_U d_T) \end{cases} \quad (8)$$

However, while trying to focus on the LTR value, it is also important to consider that the roll angle of suspension (RAS) or the difference in roll angle between SM and UM is within the permissible limit of $7 \div 8$ degrees [18, 19].

4 Valuation Rollover Stability of the Elliptical Tank Truck

4.1 Analysis in the Time Domain

In this part, some analysis of rollover stability of the elliptical tank truck in time domain are shown for two different circumstances: steady turn and a lane change. In an emergency, to avoid an impediment, changing lane is the maneuver of vehicle which it is always used.

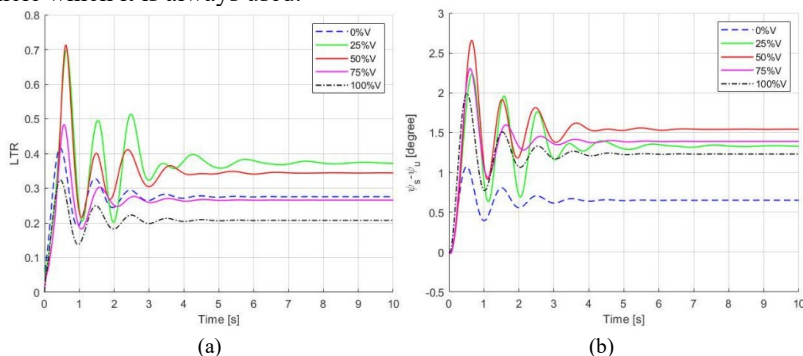


Figure 3
The LTR (a) and RAS (b) value in steady turning

In case of steady turn, the lateral acceleration a_{LT-ST} is a constant.

With Figure 3a, at the liquid volume scale of 25, 50 percent in the tank, the LTR is the highest, its value is greater than 0.7, then it decreases over time and stabilizes. This shows that the force acting on the right wheel F_{z2} is greater than the left wheel F_{z1} because at this time the entire liquid inside the tank vibrates to the right, creating a resonance force that increases the force F_{z2} . The LTR value at these two levels is nearly equal. After a period of time, the LTR value at the 25%V liquid level still oscillates strongly, while at 50%V it has gradually decreased. At a liquid level of 75% of the elliptical tank volume, the liquid vibrates less in the above two cases because the amount of liquid accounts for 3/4 of the tank volume, so the impact force is smaller.

However, at 50%V and 75%V in the tank, the RAS value is the highest in Figure 3b. This case shows that when the liquid vibrates inside the tank, the larger the LTR value is, the more higher the wheel trends lift and is not contact with the road surface leading to the roll angle of the unsuspended mass is bigger, so the roll angle of suspension will be small like the case of 25%V and 75%V. At 50%V, it shows that the liquid mass causes large vibrations to leads the roll angle of the suspended mass is large, so RAS value is large. They are near to 3, but this value is still within the acceptable range. Final, this case also needs attention to the volume levels at 25%, 50%V.

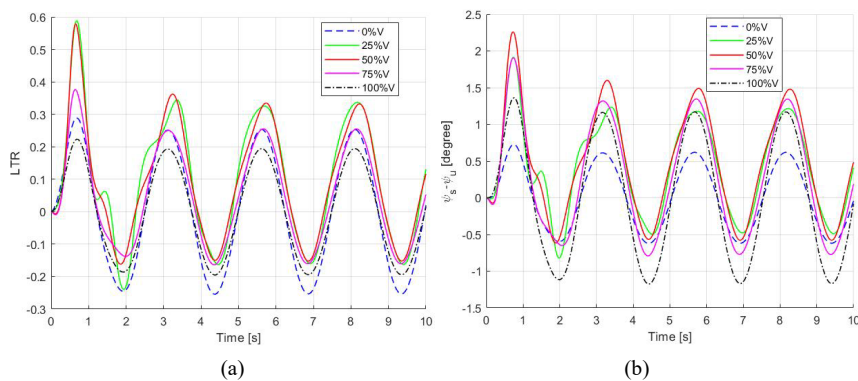


Figure 4
The LTR (a) and RAS (b) value in lane change

In the case when the vehicle changes lane, it is similar to the above case. At the liquid volume of 25, 50 percent in tank, the LTR value is close to 0.6, which is the biggest, it presented in Figure 4a. For the RAS value simulated in Figure 4b, which is still within the safe zone, at a liquid volume of 50, 75 percent, this is the highest.

Simulation results in two cases show that at level 25, 50 percent of liquid volume, the liquid elliptical tank truck tends to roll when the LTR value is proximate to 1.

The travel of suspension value still conforms to the allowable standards.

In addition to assessing the stability of the vehicle so that it can be applied in practice. The study also compares the rollover stability characteristic between the elliptical tank truck and the circular tank truck with the same specifications of the study [20] which is showed in Figures 5, 6.

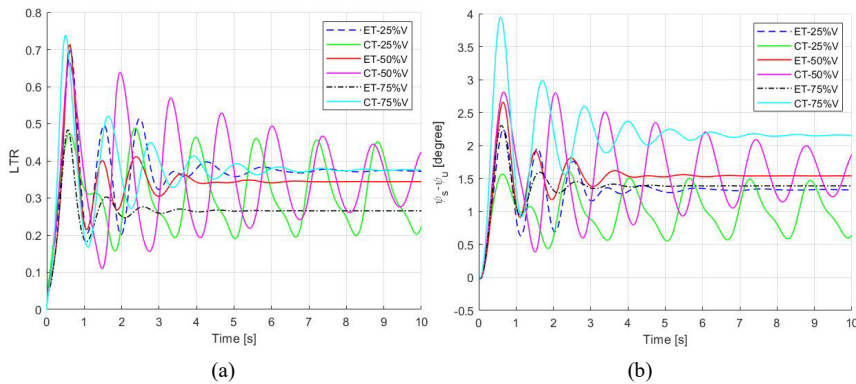


Figure 5

Compare the LTR (a) and RAS (b) value of ET and CT in steady turning

Based on the stability analysis data of both vehicles at the volume levels at which the risk of overturning is simulated in Figure 5, it is shown that in the first oscillation of the liquid in the tank, the LTR value of ET and CT are approximately equal (Figure 5a), but corresponding to each volume scale, the value of CT is higher than that of ET and this value of ET gradually decreases to steady state. In addition, in terms of RAS value, CT is still higher than ET and over time, the values of ET are more stable (Figure 5b). This shows that for circular tank trucks, the amount of liquid in the tank fluctuates in a simple pendulum profile with a wider range than for elliptical tanks because only the upper part of the liquid in the elliptical tank oscillates with a wider range, which is smaller so the resonant impact force will be smaller.

Same as the above case, for the maneuver vehicle is a lane change, the comparison results are presented in Figure 6. In this case, at the first oscillation, the LTR value of ET was higher than that of CT but less than 0.6, then decreased and smaller than CT in subsequent oscillations (Figure 6a). The RAS value is still within the allowable range, but the value of ET is lower than that of CT in the same volume ratio (Figure 6b).

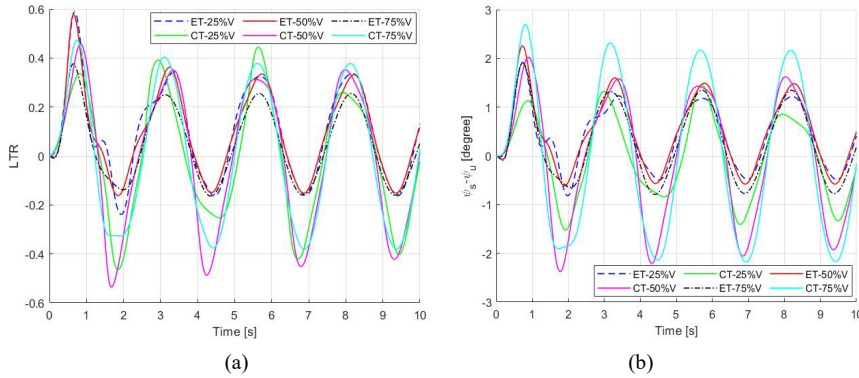


Figure 6

Compare the LTR (a) and RAS (b) value of ET and CT in lane change

Therefore, in both case of maneuver vehicle, ET ensures more rollover stability standard than the CT for different liquid volume ratios in the tank when both vehicles have the same volume of the fluid tank, specifications and structure vehicle.

4.2 Analysis in the Frequency Domain

Analyze the Amplitude Frequency Characteristic (AFC) of system in operating situations in order to assess the system's operability and limit risks during operation [18, 19]. Therefore, when analyzing the rollover stability characteristics of tank trucks in the frequency domain to determine the dangerous frequency area of lateral acceleration corresponding to different volume ratios in the lane change by changing the steering angle frequency.

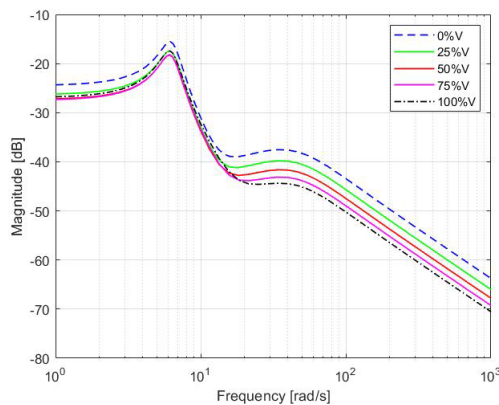


Figure 7

The AFC of the elliptical tank truck in the lane change

At levels of liquid volume ratio in the tank, the response frequency is almost equal, the difference is very little as shown in Figure 7. This result shows that when at high frequency, the vehicle changes lanes as in the simulation, the vehicle's speed is large, so the liquid in the elliptical tank will oscillate back and forth in a short time, it does not cause a great impact on the vehicle, so the AFC values are almost the same. When the vehicle is no load and full load, the frequency value is the highest and are 6.19 and 6.18 rad/s respectively. This value shows that the higher the frequency is, the more longer the time when the vehicle is damaged or unstable. The lower the frequency, the faster the tendency to roll.

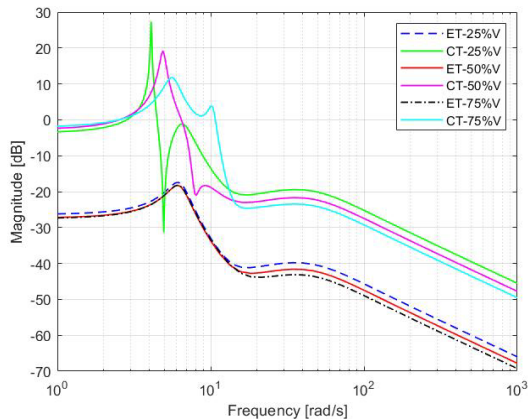


Figure 8

Compare AFC between the circle tank truck and the elliptical tank truck in the lane change

Figure 8 shows the results of comparing the AFC between ET and CT at dangerous volume ratios to clearly show that the use of an elliptical tank has a higher response frequency range so its stable performance is also higher during a lane change. Because at this time the liquid in CT will vibrate with a larger range, the oscillation time will be longer than ET, so the impact force will be larger, it tends to cause danger quickly, this leads to the value AFC of CT is smaller than ET.

Table 2
The response frequency of ET and CT in the lane change

Liquid volume ratio (%)	The response frequency [rad/s]	
	ET	CT
25	6.14	4.09
50	6.123	4.88
75	6.118	5.54

Indeed, the AFC value of CT compared to ET mentioned in Table 2 is well defined. When considering the roll stability characteristics, the CT tends to lose

stabilizes faster than ET when changing lanes. Especially the liquid volume level in the tank at 25 and 50% of the CT.

Conclusions

This research proposed an integrated multi-object dynamic model including a non-linear fluid movement model in tank taking into account the liquid viscosity and a linear single unit heavy vehicle roll model with the stabilizer bar. Applying Lagrange and Quasi-static method, D'Alembert's principle to this model to analyze the influence of liquid volume scale in tank to rollover stability characteristics of the elliptical tank truck.

Simulation results show that the elliptical tank truck running at no-load and full-load ensures stability characteristic during steady turning and a lane change when analyzing in the frequency and time domains. If this truck compares with the liquid circular tank truck with the same specifications and construction, it is still more stable in the same movement condition.

In the process of driving the vehicle, it is important to note that the vehicle must be fully loaded and must use up all the liquid in the tank before being transported to the parking station. If not used up, it is necessary to maintain the mode of operation as suggested in the study.

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