# Assessing the Limits of Simple Quantum Hardware

**Mihály Szabó, Szabina Fodor**

Corvinus University of Budapest, Fővám tér 13-15, 1093 Budapest, Hungary
mihaly.szabo@stud.uni-corvinus.hu; szabina.fodor@uni-corvinus.hu

*Abstract: Although quantum computing is advancing rapidly, ensuring the accuracy and reliability of the computations available to quantum computer researchers is still a challenge today. This paper reviews the main directions of quantum error correction (QEC) techniques that can potentially be applied in superconducting qubit computers. We present the Mermin-Peres magic square game, where the quantum strategy has theoretical advantages over classical and can provide a benchmark solution for testing the accuracy of quantum computation. In the experimental part of our work, we run the algorithm of the Mermin-Peres magic square game on IBMQ quantum computers accessible via the cloud. Our experimental results failed to demonstrate a proven quantum advantage of the game on the noisy intermediate-scale quantum (NISQ) hardware tested, confirming the inevitable need to implement error correction techniques on physical qubits. By aligning with the Digital and Cognitive Corporate Reality (DCR) theme, this research seeks to inform business strategies incorporating emerging quantum technologies.*

*Keywords: quantum error correction; Mermin-Peres magic square game; Qiskit; IBMQ*

# 1 Introduction

Digital & Cognitive Corporate Reality (DCR) is a comprehensive field that combines digital ecosystems, corporate management, and artificial and natural cognitive capacities. Quantum computing, an emerging technology, could significantly impact corporate ecosystems, especially in areas like data security, optimization, and decision making [1].

Quantum computers have developed rapidly in recent years and have become widely available to the research community in the cloud environment. Although they are freely available and easily accessible to the interested public, their hardware performance and reliability are limited. A common issue with all new innovative technologies is the possibility and method of their use. The potential impact of quantum computing on business management and economics is significant. As the technology matures, companies that strategically integrate

quantum computing into their operations may gain advantages in efficiency, innovation and competitiveness. However, the DCR investigates new technology adaptations, and this new tool is slightly different from the usual ones [1]. From a practical perspective, testing the limits of currently available quantum hardware and its apparent performance is an important issue [2].

The world of quantum computing is different from classical (digital) computing. Superposition, interference, and entanglement – the three essential elements of quantum physics – are employed [3], [4], [5].

- The quantum phenomenon known as superposition describes the ability of a quantum system to exist in many different states at once.
- We can bias quantum systems toward the desired state using quantum interference. The goal is to establish an interference pattern in which the pathways that lead to incorrect responses cancel each other out and interfere destructively while the paths that lead to the correct answer reinforce one another.
- Quantum particles exhibit a robust connection known as entanglement. Even when separated by enormous distances, entangled particles maintain flawless correlation.

The algorithms that quantum computing makes conceivable give it immense capability. These algorithms differ from their classical equivalents as they can solve specific mathematical calculations with reduced complexity. For example, exponential is the best algorithm for factorization problems in classical computation [3]. A proper quantum algorithm (such as Shor's algorithm [6]) solves a factorization problem with polynomial complexity [3].

Noise and errors are among the most significant problems we must cope within quantum computing today. Two states that digital computers must be able to distinguish between are 0 and 1. The circuits must be able to differentiate between low and high voltage. It is always 1 when the voltage is high and 0 when the voltage is low. Because of this discretization, errors must be significant to be seen, but we may still put procedures in place to find and fix them. Since quantum computers maintain a continuous quantum state, they require far more precision than digital computers. The foundation of quantum algorithms is the precise manipulation of parameters that change continually. Errors in quantum computing can be incredibly tiny and undetectable, yet they still have the potential to accumulate and destroy a calculation. The noise from the surroundings around the quantum bit can easily break this delicate quantum state. For instance, noise from control electronics, heat, or impurities in the material of the quantum computer itself might result in significant computation mistakes that would be challenging to fix. However, we need fault-tolerant devices to live up to the promises made by quantum computers. However, millions of quantum bits are required for such systems. Since most of these complex algorithms are noise-sensitive, this cost is necessary for error correction. Current quantum computers typically consist of up to 100 quantum bits.

However, IBM released the first 1000 quantum chips in December 2023 [7]. Even these relatively small quantum devices (around 50 quantum bits) can do things that are impossible for a classical computer. We will soon enter an era when quantum computers, while not fault-tolerant, can do things that classical computers cannot. This era is described by the term "Noisy Intermediate-Scale Quantum" – NISQ [8]. It is noisy because we do not have enough qubits for error correction and "Intermediate-Scale". After all, the number of quantum bits is too small to compute sophisticated quantum algorithms, but it may be large enough to demonstrate unique quantum properties in practice [3], [4]. A specific collection of algorithms, instruments, and tactics is needed for today's NISQ devices. Therefore, this research seeks to answer the question of the usability of the currently publicly available quantum computers and whether the quantum superiority of quantum computers, due to their different computational models, is reflected in the algorithms they run. For our study, we use the Mermin-Peres magic square game algorithm as a benchmark, which has been theoretically shown to have a winning probability of 8/9 in the classical computational model and a winning probability of 100% on quantum computers. We test its results by running it on various available quantum computers. The research also focuses on a quick overview of the quantum computing development environment, namely Qiskit.

The reliability of quantum computations is critical for corporate decision making. Research on quantum error correction (QEC) aligns with DCR by addressing the accuracy and reliability of quantum technologies, which are essential for integrating quantum computing into corporate strategies [1].

The paper aims to test new quantum hardware in near-business environments and decision problems, which seems possible on a small scale. We strive to benchmark current setups by the Mermin-Peres magic square game where it is theoretically possible to reach quantum advance to the digital one. Our further objective is to extend the dialogue in the field of DCR because new technologies should not be exclusively digital to transform business strategies and thinking.

Theoretically, quantum computers can solve problems several orders of magnitude faster than computers nowadays. The business strategist should not negate the potential benefits of new technologies, which is why monitoring the current state of quantum computation seems necessary. Our work can be the first step, which is to use the DCR framework and try to provide broader aspects of available possibilities and further development of quantum technology.

Our research differs from similar ones in purpose and details as well. Dikme et al. [9] created a similar test on similar IBMQ computers using the Mermin-Peres magic square game. However, their paper focused on the best results rather than overall reliability. Xu et al. [10] could provide a slight quantum advance in the same game but used a different hardware architecture. Furthermore, we use this game for benchmarking purposes to compare different IBMQ quantum computers. Following the results, we argue for the necessity of hardware-level QEC, and we take an overview of the most significant correction techniques.

The research addresses critical challenges in quantum computing, particularly the limitations of noise and errors inherent in current quantum hardware. These factors significantly impact the reliability and accuracy of quantum computations, especially on Noisy Intermediate-Scale Quantum (NISQ) devices, which lack robust error correction capabilities. The study aims to test whether contemporary quantum computers can reliably execute algorithms that theoretically demonstrate a quantum advantage, using the Mermin-Peres magic square game as a benchmark.

As we write in conclusion in more detail, the current quantum hardware is closer to small-scale business usage step-by-step, but its reliability is limited. In the Mermin-Peres magic square game, the quantum strategy has a slight advantage over the traditional one, but this effect cannot be measured by us on the tested hardware because of the noise and errors. We think it is inevitable that QEC techniques will be improved and implemented into quantum computers to develop reliable solutions. Otherwise, benchmarking quantum computers is vital because regular simulation methods on digital computers cannot test the more evolved hardware.

The rest of this article is organized as follows: the next section reviews the main directions of QECs, mainly focused on code-based techniques, such as repetition, stabiliser, topological, and surface codes. We mention a particular but promising method, namely dynamical decoupling, which is used for stabilizing the physical properties of qubits. In the third section, we introduce the Mermin-Peres magic square game, which can be used as a benchmarking tool for different quantum computers. After the game rule summary, we present and discuss our experimental results in Section 4, and Section 5 presents the conclusions.

# 2 Quantum Error Correction Techniques

Error correction (EC) is a crucial research area in quantum computing, and it plays a vital role in creating reliable and precise calculations. The qubits are easily affected by environmental noise and decoherence, which cause errors. The classical EC methods, where multiple copies of the original bit are used in the error correction, are unsuitable in quantum computing (QC) because of the no-cloning theorem. Creating an independent and identical copy of an arbitrary unknown quantum state proved impossible. The theory originated from Wootters, Zurek, and Dieks in 1982 [11] from the no-go theorem authored by Park in 1970 [12].

The no-cloning theorem [13] creates a little difficulty in constructing redundant copies of qubits and indicates indirect solutions. Firstly, Peter Shor proved a method to represent one qubit with multiple entangled physical qubits [14]. He developed the 3-Qubit code and 9-Qubit code, where each logical qubit was coded on 3 and 9 physical qubits, respectively. These codes have limited capability, but they prove some resistance against different error types.

A single-qubit error can be a bit-flip error, sign-flip (phase error) or both. Classical error-correcting uses the syndrome measurement to diagnose corrupted bits and applies the operation to correct them. These techniques are also usable by quantum error correction, but it has to use a multi-qubit operation in a way that should not disturb the qubit state. As we know, the direct measurement collapses the qubit state. Thus, the correction method should identify the erroneous qubits and apply the Pauli operators to the error type without it. That is only possible if the quantum syndrome measurement contains information about the kind of error but not the information stored in the qubit.

Not all error correction techniques are suitable for all quantum hardware; however, some methods are transferable to other systems, and a rapid pace of development can be seen in this area.

## 2.1 Quantum Error Correction Code (QECC)

In quantum tech, quantum error correction plays a crucial role. It is essential for building functional quantum computers and quantum networks. At their core, these quantum error-correcting codes mirror classical versions in terms of their operation and effectiveness. QECCs correct errors by returning a decohered quantum system to its original, undamaged state.

### 2.1.1 Repetition Code

Repetition codes are also known as Repeat-Accumulate codes or simple majority vote codes. They represent an early class of classical error-correcting codes that have also found applications in quantum systems. A repetition code encodes a qubit on multiple qubits, preferably on an odd number. A simple majority vote is taken over each group of repeated bits to decode the message. If there is a single error, it will be corrected without affecting the intended data. However, repetition codes cannot correct multiple errors efficiently and are generally considered less effective compared to more advanced quantum error-correction codes like Calderbank-Shor-Steane (CSS), Stabilizer, and Topological codes. Nevertheless, they still play a valuable role in studying quantum error correction and provide insights into fundamental concepts such as error syndromes and encoding strategies.

**The three-qubit Repetition Code** or "threshold code" encodes a single qubit into three physical qubits by repeating the original qubit twice, first proposed by Asher Peres in 1985 [15]. Each physical qubit carries the same state as the original qubit. A majority vote is taken among the three physical qubits to determine the state of the encoded qubits. If all three qubits are in the same state, the encoded qubit is also assumed to be in that state. However, an error is detected if even one qubit differs, but no correction is possible. Despite their limitations, three-qubit repetition codes are still helpful in demonstrating fundamental concepts in quantum error correction.

### 2.1.2 Stabilizer Code

These codes rely on measuring these observables to detect and correct errors in quantum systems, providing an efficient method for maintaining the integrity of quantum data. The stabilizer codes use ancilla qubits, which are appended to qubits we want to protect. The encoder circuit rotates the global state into a subspace of a larger Hilbert space by Pauli operators. Their simplicity and effectiveness have made stabilizer codes essential for developing quantum error correction and fault-tolerant quantum computing.

Experimental results were proven by Reed et al. [16] in 2012, who tested three-qubit phase and bit-flip error codes in superconducting quantum computers. In 2011, Egan et al. used Bacon–Shor logical qubit coding on a 13-trapped ion qubits computer, halving the error rate from 0.6 to 0.3 per cent [17].

The most common examples of stabilizer codes are

- **Shor Error Correcting Code (SECC)** is a pioneering quantum error-correction code introduced in 1995 by mathematician Peter Shor [14]. SECC represents one of the earliest attempts to develop practical quantum error correction algorithms based on topological principles. Although not as efficient or widely adopted as later codes like Calderbank-Shor-Steane (CSS) codes or surface codes, SECC holds historical significance for laying the groundwork for further advancements in quantum error correction.
- **Calderbank-Shor-Steane (CSS) codes** are a specific type of quantum error-correcting code named after their inventors, Robert Calderbank, Peter Shor and Andrew Steane [18], [19], [20]. CSS codes combine the benefits of classical linear and topological codes, allowing them to protect quantum information against various errors. The structure of CSS codes relies on using two commuting sets of Pauli matrices called stabilizers, which can detect and correct errors that involve a limited number of qubits. Combining multiple CSS codes allows more complex quantum computations to be reliably performed. One of the most famous examples of CSS codes is the Shor code, which protects against phase-flip and bit-flip errors on a grid of qubits. Overall, CSS codes represent a critical milestone in developing quantum error correction, paving the way for more extensive, robust quantum computing systems.
- **Steane code** was developed by Robert W. Steane in 1996. Steane code is a type of CSS code explicitly designed for systematically correcting bit-flip errors [20]. It employs seven qubits to encode a logical qubit and combines Pauli X and Z stabilizers. Steane code significantly improves over simple repetition codes because it can correct multiple errors within a logical qubit.

### 2.1.3 Topological Code

Topological codes are a type of quantum error-correction code that uses the principles of topology to protect quantum information from errors. Unlike traditional codes, which typically address single-qubit errors, topological codes aim to correct multi-qubit errors. They use quasiparticles, topologically protected excitations that encode quantum states in a manner resistant to local perturbations. With topological codes, quantum information remains stable even when individual qubits are affected by external influences or noise. Additionally, topological codes exhibit excellent fault tolerance properties due to their ability to handle multiple qubit errors.

Such exotic quasiparticles are non-Abelian anyons in quantum physics. When interacting, they exhibit non-trivial statistical properties, which can be harnessed to perform complex quantum computations and implement fault-tolerant quantum error correction using topological codes. Their name comes from Abelian and non-Abelian groups in mathematics, reflecting their distinct behaviour in quantum systems. In 2021, researchers from the University of Innsbruck used a ten qubits ion-trap quantum information processor and created for the first time two entangled logical qubits by topological quantum error-correction [21].

The most common examples of topological code are

- **Color codes** are part of the Topological codes, and they encode quantum information in a special, two-dimensional form using multiple physical qubits. This family of codes were proposed by Bombin and Martin-Delgado [22]. Postler et al. performed 2021 a logical two-qubit controlled-NOT gate between two instances of the seven-qubit colour code, which paved the road towards fault-tolerant universal quantum computing [23].
- **Toric codes** represent an extension of color codes that operate on a three-dimensional lattice of qubits arranged in a torus shape [24]. Like colour codes, Toric codes rely on manipulating stabilizer operators to encode and decode quantum information. Their main strength lies in their excellent fault tolerance properties, enabling them to correct multiple qubit errors with minimal additional overhead. Toric codes have been shown to offer promising avenues for realizing scalable, fault-tolerant quantum computers.

### 2.1.4 Topological Stabilizer

One of the best-known family of topological stabilizers is the surface codes. They encode quantum information onto the surface of a material using a two-dimensional lattice of qubits. The topological protection offered by this arrangement allows the surface codes to mitigate errors caused by qubit interactions and external disturbances, making them a promising candidate for building large-scale, fault-tolerant quantum computers. Examples of implementing the surface code involve

arranging qubits in a two-dimensional grid on the surface of crystalline solid or superconducting circuits.

Surface codes use the concept of "surfaces" or "planes" of stabilized qubits to encode quantum information and protect it from errors. The primary advantage of surface codes is their ability to correct arbitrary errors using only local measurements, making them particularly attractive for building large-scale, fault-tolerant quantum computers. Additionally, surface codes exhibit excellent error thresholds, enabling the implementation of robust quantum circuits. In February 2023, Google claimed they reached about 3% logical gate error rate by using surface code on superconducting qubits computer [25]. In parallel, they found that more qubits used for error correction caused a slighter error rate.

### 2.1.5 Low-Density Parity-Check (LDPC) Code

LDPC codes are a class of error-correcting codes that have garnered considerable attention due to their strong capabilities and potential applications in quantum computing [26]. In contrast, to traditional block codes, which employ fixed-size codewords, LDPC codes use sparse parity-check matrices, where each row represents a single check constraint involving only a few qubits. This property results in a lower density of checks, hence the name low-density parity-check codes.

The primary advantage of LDPC codes lies in their ability to correct long error sequences that affect multiple qubits effectively. They achieve this by organizing the qubits into layers, each acting as a separate check constraint. When an error occurs, the syndrome measurement at one layer allows the identification of erroneous qubits, which can be corrected before propagating further to other layers. Consequently, LDPC codes exhibit excellent error correction performance, making them attractive candidates for improving the resilience of quantum systems against noise and errors. Moreover, LDPC codes have already demonstrated success in classical communication systems [26] and could potentially find similar applications in quantum error correction. Researchers continue to explore the potential of LDPC codes for enhancing the reliability and scalability of quantum technologies.

## 2.2 Dynamic Decoupling

Dynamic decoupling (DD) is a powerful technique to reduce unwanted interactions' effects on quantum systems, particularly those induced by magnetic fields [27]. The primary goal of dynamic decoupling is to preserve the coherence of a quantum state by applying a sequence of pulse sequences called "pulsed rotations" at regular intervals.

The basic idea behind dynamic decoupling is to apply a series of short pulses, often called "π pulses," around the clock cycle at a frequency equal to or faster than the

rate at which undesired interactions occur. This rapid application of pulses creates a protective shield around the quantum system, effectively cancelling the detrimental effects of external perturbations during the intervening periods between pulses. Dynamic decoupling has broad applicability in various areas of quantum science and engineering, including nuclear magnetic resonance spectroscopy, quantum communication and quantum computing. Its ability to suppress dephasing and other sources of error makes it a valuable tool for extending the lifetime and improving the performance of quantum systems.

# 3 Mermin-Peres Magic Square Game

In 1990, David Mermin and Asher Peres, two quantum physicists, independently developed the Mermin-Peres magic square (MPMS) game [28], [29]. In quantum thought experiments, two participants are typically called Alice and Bob. Their task is to fill in a three-by-three numerical grid known as a "magic square", where each element of the grid is given a value of either +1 or -1. In each round, a referee called Charlie randomly selects a row for Alice and then gives Bob a column (there are nine possible combinations of rows and columns). Alice and Bob send their choice simultaneously without any communication between them. Charlie needs to know the values of +1 or -1 from the players to enter each of their three grid cells. Accordingly, the product of all the entries in a row equals +1, and all the entries in a column equals -1, besides the intersection of them, should be the same number. The formal description of the MPMS game is as follows:

The players Alice and Bob may agree on a prior strategy in advance but cannot communicate once the game starts.

1) The referee picks a random row $r \in \{1,2,3\}$ and a random column $c \in \{1,2,3\}$
2) The referee (Charlie) sends $r$ to Alice and $c$ to Bob.
3) In a 3×3 grid, Alice writes -1 or +1 in the three cells of row $r$ so that the multiplication of the numbers is +1 and sends this to the referee.
4) In a 3×3 grid, Bob writes -1 or +1 in the three cells of column $c$ so that the multiplication of the numbers is -1 and sends this to the referee.
5) The referee checks that Alice and Bob have assigned the number to the cell in row $r$ and column $c$.
6) If this check succeeds, Alice and Bob win the game; otherwise, they lose [1].

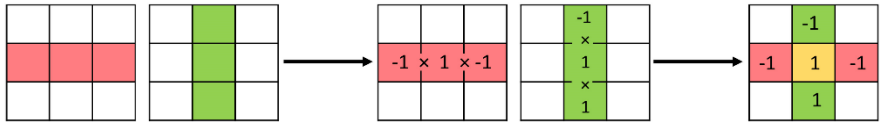


Figure 1
The Classical Mermin-Peres magic square game [2]

The classical approach says that no matter how accurately Alice and Bob estimate each time, there will always be a round in which their allocations must mismatch. This makes winning all rounds impossible. Eight wins out of nine is the best they can hope for.

Assume, however, that Alice and Bob can employ this quantum approach: rather than giving each grid element a value of +1 or -1, they give it a pair of quantum bits (qubits), each of which has a measured value of +1 or -1. By measuring the two-qubit values and computing the product of the pair, each player assigns a value to a specific grid element. Based on how Alice and Bob perform their measurements, they can elicit different results from the identical two qubits, avoiding the traditional disagreement. For each round, a specific measurement approach guarantees that all nine combinations of rows and columns satisfy the winning criterion, i.e., the products of Alice's and Bob's three inputs must be +1 and -1, respectively.

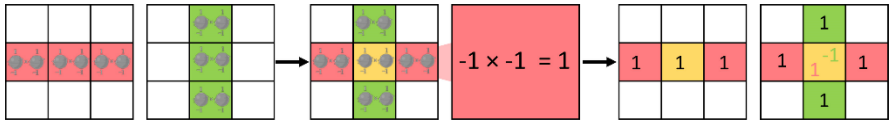


Figure 2
The Quantum Mermin-Peres magic square game [2]

However, there is a catch to this tactic. Alice and Bob must work together to determine which of their three lattice elements intersect with the other player's to determine the appropriate set of measurements. Nevertheless, this is not an issue since the MPMS measures the same three-qubit pairs one after the other. This means that the pair Bob receives can "communicate" with each other, as they have an impression of how Alice has already measured these qubits. This is not real communication between them because they do not exchange information after the game starts; instead, they use an entangled qubit pair. They created a highly correlated initial state, and their choices are derived from it, which results in a solution in this richer algebraic structure. Alice and Bob's strategy above is the so-called quantum pseudo-telepathy, demonstrated by Mermin in 1993 [30].

Therefore, it is possible to use this game to demonstrate the profound difference between quantum and classical systems and highlight the potential benefits of exploiting quantum effects in information processing. The algorithm of the MPMS game also provides an opportunity to test the computational accuracy of current

quantum computers, as the results can be easily compared with the theoretically proven 100% result.

# 4 Experimental Analysis

Our investigation focuses on the Mermin-Peres magic square game as a benchmark, where classical solutions have an approximate 8/9 (88.8%) winning probability, while quantum strategies have a perfect 100% theoretical winning rate. However, the practical advantage of the quantum strategy is relatively slight, and it could be understood as a basis for applied purposes.

Dikme et al. [9] demonstrated an experiment on five superconducting qubits on an IBMQ computer in 2022. Still, they found the computer *"…leaves much to be desired in producing accurate and reproducible results, …"* [9] while using the Mermin-Peres magic square game to measure the machine's properties. In the same year, Xu et al. [10] used a photonic setup in their experiments and published 91-97% success rates through the game as proof of the practical quantum pseudotelepathy.

Among the many manufacturers, one of the most influential is IBMQ[1], a family of quantum computers developed by IBM. These machines are designed to perform specific quantum computations beyond classical computers' capabilities. They are built from superconducting qubits and are considered the most mature technologically nowadays. This hardware is broadly available for research communities to test quantum algorithms via the cloud-based IBM Quantum Experience platform.

A specific development framework, Qiskit, was created and maintained by engineers at IBM for programming quantum computers. This open-source software is written in Python and offers comprehensive functionality. It also integrates with popular platforms like Rigetti[2] and IonQ[3], so it is not exclusively tied to IBM machines.

The development process can happen on personal computers, and a circuit simulator tool can also be provided to test the algorithms before sending them to actual quantum hardware. It has an extensive library of quantum algorithms and documentation with a large base of learning materials. The Qiskit framework presents an environment for building quantum circuits and special visualization tools to display the manipulation and measuring of qubits.

[1] IBM Quantum Platform (https://quantum.ibm.com/)
[2] Rigetti Computing (https://www.rigetti.com/)
[3] https://ionq.com/

Our experiment was conducted on various IBMQ Quantum computers, which are part of the free plan, and we measured the winning rates at different player choices in the games. Firstly, we simulated all our executions with the in-built simulator, which imitates the logical operations of a quantum machine. It can run our personal computer, but a high-performance cloud-based one is also provided alternatively. The simulators work perfectly, producing the codes' theoretical values without gate or measuring errors. We have 100% winning ratios on both setups to validate our quantum program.

Afterwards, we executed 204 trials (every trial contains 1024 shots) on seven IBM Q superconducting seven-qubits computers, ibm_lagos, ibm_nairobi, ibm_perth, ibmq_jakarta and on five-qubits computers as ibmq_belem, ibmq_lima, ibmq_manila also. Our codes have not used all the available qubits; thus, we have left it to the transpiler of machines to allocate computing resources. The transpiler translated our Qiskit-based code to runnable for specific computers.

Despite the limitations imposed by the free plan, allowing us to submit only five jobs concurrently, we achieved notable results within approximately 3.28 seconds per trial. We accumulated a total runtime of 668 seconds. However, the total time of our experiment lasted for six days because of the long job queues on some computers. We have not conducted further tests because of the unmanageable waiting times, which sometimes reach several months.

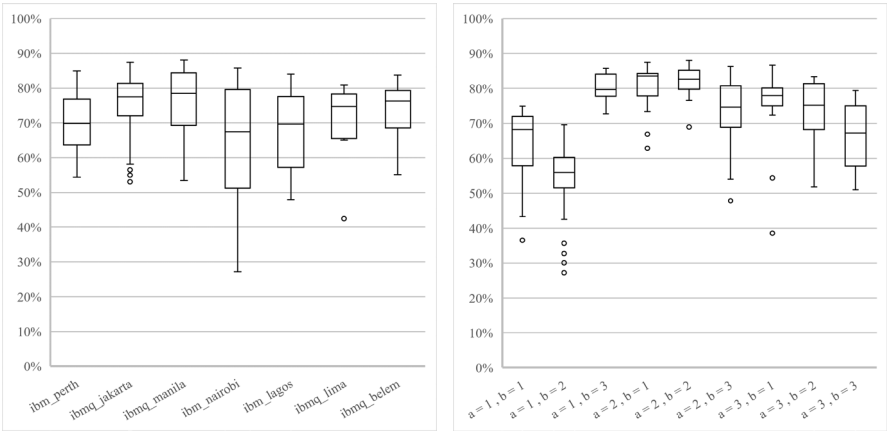


Figure 3
Winning probability distributions grouped by quantum computers and game cases (Source: prepared by the authors)

Our analysis revealed that average winning ratios ranged between 64-75% with wide standard deviations by computers, falling significantly short of the classical (88.89%) and quantum (100%) theoretical values. Our best executions resulted near but slightly below the classical 88.89% winning ratio, 88.09%, 87.50% and 87.40%, respectively.

The results draw interesting pictures if we investigate the cases of the games through the players' choices. Different operations were executed respectively to the specific lines and rows in the box. As we see in Fig. 3, the a=1 and b=2 cases were peculiarly bad compared to others. Less low ones, but significantly lower, were the a=1, b=1 and 1=3, b=3 cases. These results indicate that all the tested computers have difficulties using specific operations (or sequences) between exact qubits.

## Conclusions

We have explored the potential of quantum computing in the context of the Mermin-Peres magic square game, highlighting both the promising theoretical advances and the practical obstacles we face. While quantum strategies guarantee a perfect win, our results also reveal areas for improvement. Engineering progress can be seen between different generations of IBMQ quantum computers. Still, our benchmark could not show a significant difference in game results and cannot prove the quantum advantage. We believe that hardware-level implementation of quantum error correction techniques is necessary to move toward fault-tolerant computing, where more physical qubits represent one logical one. The development of quantum error correction techniques is a step toward building scalable quantum networks, which could revolutionize communication and data exchange in corporate ecosystems.

Our research on quantum error correction and the Mermin-Peres magic square game aligns closely with the principles of Digital & Cognitive Corporate Reality. By addressing the accuracy and reliability of quantum computing, this research contributes to the development of robust digital ecosystems, hybrid cognitive capabilities, and secure corporate infrastructures. It also provides practical insights for businesses adopting quantum technologies, making it a valuable addition to the DCR framework.

## References

[1] A. Kő, I. Szabó, Á. B. Csapó, T. Kovács, L. Lőrincz, and P. Baranyi, “Digital & Cognitive Corporate Reality,” *Infocommunications J.*, Vol. 15, No. Special Issue, pp. 2-10, 2023, doi: 10.36244/ICJ.2023.6.1

[2] N. C. L. Ramalho, H. A. de Souza, and M. L. Chaim, “Testing and Debugging Quantum Programs: The Road to 2030,” May 15, 2024, *arXiv*: arXiv:2405.09178. doi: 10.48550/arXiv.2405.09178

[3] D. F. Zickert, *Hands-On Quantum Machine Learning With Python: Volume 1: Get Started*. 2021

[4] E. G. Rieffel and W. Polak, “An Introduction to Quantum Computing for Non-Physicists,” Jan. 18, 2000, *arXiv*: arXiv:quant-ph/9809016. doi: 10.48550/arXiv.quant-ph/9809016

[5] E. Grumbling and M. Horowitz, Eds., *Quantum Computing: Progress and Prospects*. Washington, D.C.: National Academies Press, 2019, doi: 10.17226/25196

[6] P. W. Shor, "Algorithms for quantum computation: discrete logarithms and factoring," in *Proceedings 35th Annual Symposium on Foundations of Computer Science*, Santa Fe, NM, USA: IEEE Comput. Soc. Press, 1994, pp. 124-134, doi: 10.1109/SFCS.1994.365700

[7] D. Castelvecchi, "IBM releases first-ever 1,000-qubit quantum chip," *Nature*, Vol. 624, No. 7991, pp. 238-238, Dec. 2023, doi: 10.1038/d41586-023-03854-1

[8] M. Brooks, "Beyond quantum supremacy: the hunt for useful quantum computers," *Nature*, Vol. 574, No. 7776, pp. 19-21, Oct. 2019, doi: 10.1038/d41586-019-02936-3

[9] A. Dikme, N. Reichel, A. Laghaout, and G. Björk, "Measuring the Mermin-Peres magic square using an online quantum computer," *Eur. J. Phys.*, Vol. 43, No. 5, p. 055401, Sep. 2022, doi: 10.1088/1361-6404/ac79e0

[10] J.-M. Xu *et al.*, "Experimental Demonstration of Quantum Pseudotelepathy," *Phys. Rev. Lett.*, Vol. 129, No. 5, p. 050402, Jul. 2022, doi: 10.1103/PhysRevLett.129.050402

[11] W. K. Wootters and W. H. Zurek, "A single quantum cannot be cloned," *Nature*, Vol. 299, No. 5886, pp. 802-803, Oct. 1982, doi: 10.1038/299802a0

[12] J. L. Park, "The concept of transition in quantum mechanics," *Found. Phys.*, Vol. 1, No. 1, pp. 23-33, Mar. 1970, doi: 10.1007/BF00708652

[13] G. Lindblad, "A General No-Cloning Theorem," *Lett. Math. Phys.*, Vol. 47, No. 2, pp. 189-196, Jan. 1999, doi: 10.1023/A:1007581027660

[14] P. W. Shor, "Scheme for reducing decoherence in quantum computer memory," *Phys. Rev. A*, Vol. 52, No. 4, pp. R2493-R2496, Oct. 1995, doi: 10.1103/PhysRevA.52.R2493

[15] A. Peres, "Reversible logic and quantum computers," *Phys. Rev. A*, Vol. 32, No. 6, pp. 3266-3276, Dec. 1985, doi: 10.1103/PhysRevA.32.3266

[16] M. D. Reed *et al.*, "Realization of three-qubit quantum error correction with superconducting circuits," *Nature*, Vol. 482, No. 7385, Art. no. 7385, Feb. 2012, doi: 10.1038/nature10786

[17] L. Egan *et al.*, "Fault-tolerant control of an error-corrected qubit," *Nature*, Vol. 598, No. 7880, Art. no. 7880, Oct. 2021, doi: 10.1038/s41586-021-03928-y

[18] A. R. Calderbank and P. W. Shor, "Good quantum error-correcting codes exist," *Phys. Rev. A*, Vol. 54, No. 2, pp. 1098-1105, Aug. 1996, doi: 10.1103/PhysRevA.54.1098

[19] A. R. Calderbank, E. M. Rains, P. W. Shor, and N. J. A. Sloane, "Quantum Error Correction via Codes over GF(4)," Sep. 10, 1997, *arXiv*: arXiv:quant-ph/9608006. doi: 10.48550/arXiv.quant-ph/9608006

[20] A. M. Steane, "Error Correcting Codes in Quantum Theory," *Phys. Rev. Lett.*, Vol. 77, No. 5, pp. 793-797, Jul. 1996, doi: 10.1103/PhysRevLett.77.793

[21] A. Erhard *et al.*, "Entangling logical qubits with lattice surgery," *Nature*, Vol. 589, No. 7841, Art. no. 7841, Jan. 2021, doi: 10.1038/s41586-020-03079-6

[22] H. Bombin and M. A. Martin-Delgado, "Topological Quantum Distillation," *Phys. Rev. Lett.*, Vol. 97, No. 18, p. 180501, Oct. 2006, doi: 10.1103/PhysRevLett.97.180501

[23] L. Postler *et al.*, "Demonstration of fault-tolerant universal quantum gate operations," *Nature*, Vol. 605, No. 7911, Art. no. 7911, May 2022, doi: 10.1038/s41586-022-04721-1

[24] A. Yu. Kitaev, "Fault-tolerant quantum computation by anyons," *Ann. Phys.*, Vol. 303, No. 1, pp. 2-30, Jan. 2003, doi: 10.1016/S0003-4916(02)00018-0

[25] R. Acharya *et al.*, "Suppressing quantum errors by scaling a surface code logical qubit," *Nature*, Vol. 614, No. 7949, Art. no. 7949, Feb. 2023, doi: 10.1038/s41586-022-05434-1

[26] T. B. Iliev, G. V. Hristov, P. Z. Zahariev, and M. P. Iliev, "Performance of the Duo-Binary Turbo Codes in WiMAX Systems," T. Sobh, K. Elleithy, and A. Mahmood, Eds., Dordrecht: Springer Netherlands, 2010, pp. 161-165. doi: 10.1007/978-90-481-3662-9_27

[27] L. Viola, E. Knill, and S. Lloyd, "Dynamical Decoupling of Open Quantum Systems," *Phys. Rev. Lett.*, Vol. 82, No. 12, pp. 2417-2421, Mar. 1999, doi: 10.1103/PhysRevLett.82.2417

[28] N. D. Mermin, "Simple unified form for the major no-hidden-variables theorems," *Phys. Rev. Lett.*, Vol. 65, No. 27, pp. 3373-3376, Dec. 1990, doi: 10.1103/PhysRevLett.65.3373

[29] A. Peres, "Multiple time scales for recurrences of Rydberg states," *Phys. Rev. A*, Vol. 47, No. 6, pp. 5196-5197, Jun. 1993, doi: 10.1103/PhysRevA.47.5196

[30] N. D. Mermin, "Hidden Variables and the Two Theorems of John Bell," *Rev. Mod. Phys.*, Vol. 65, No. 3, pp. 803-815, Jul. 1993, doi: 10.1103/RevModPhys.65.803