Federated Filtering Revisited: New Directions to Distributed Systems Estimation and Filtering – a Case Study

A. Edelmayer†, M. Miranda‡

†Multidisciplinary School of Postgraduate Studies, Department of Informatics Széchenyi University, H-9026, Egyetem tér 1, Győr, Hungary.

‡Laboratory of Unit Operations, Department of Chemical Engineering, Faculty of Engineering, University of Los Andes, 5101 Mérida, Venezuela.

Abstract: The paper discusses the application of decentralized filtering to state estimation of distributed nonlinear systems. The proposed filter is operated in a novel federated architecture in an attempt to adaptively balance estimation accuracy and sensor fault tolerance in the presence of both state and measurement uncertainty when the observations of the system are subject to a variety of sensor faults. The fault management logic built into the federated structure is accountable for the control of the distributed filter architecture to achieve sensor fault accommodation in a seamless way. The performance of the traditional centralized filtering is contrasted with the proposed federated idea, and its ability to trade-off between fault tolerance and estimation accuracy are examined. Benefits from the application of the method to a chemical distillation process are demonstrated by computer simulations.

Keywords: Chemical process control, Dependable control systems, Distributed filtering, Extended federated Kalman filtering, Multi-component distillation, Sensor fault tolerance.

1. Introduction

Due to the high requirements of product quality, increased safety, the minimization of impact on the environment and reduction of manufacturing costs the importance of utilizing advanced control that make use of dependable technology have became evident in various fields of the industry in recent years. In order to achieve the system's technical goals the principles, such as the continuity in operation, the assurance of the continual availability of the primary system's functions for long-term operational safety are becoming the prevailing design factors. This is true not only in the classical fields of dependable technologies such as aviation, chemistry and nuclear industry, which are often referred to as safety-critical, traditionally, but in a widening array of commercial applications as well.

More and more frequently, one of the main objectives of the design is, therefore, to create engineering structures in which equipment faults can be detected and removed from the system, quickly and reliably, in such a way that system functionality be continuously maintained over time: fault reconciliation is the central feature in dependable control systems.

A fundamental problem of control in dependable systems is to solve *detection* and *estimation* problems reliably, to supply trustworthy data for controllers. In many technologies, process variables that determine overall performance of the system (product quality) cannot be measured directly, but can be estimated/inferred in real time relying on the available measurements. In various structures of model predictive control (MPC), for example, which are used in the chemical industry in an increasing variety (see e.g., [10, 1]), controller operation is mainly based on the estimated values of process variables.

As a standard solution alternative to the state estimation problem, centralized filters have traditionally been used in the industry for many decades. In this approach a single filter is used for estimating the complete set of the state variables of the system. Centralized solutions, however, especially in large-scale industrial applications, as typically represented by chemical process systems, suffer from the size (dimension) of the problem and are prone to both sensor and implementation failures. Reduced order filters are not always reliable, they tend to have poor estimation accuracy, and even instability under certain circumstances. Centralized approaches, moreover, cannot answer the call of the safety and dependability requirements as they lack fault tolerance when faults in the operation of the system or sensors occur.

It was recognized early that the partitioning of the global estimation problem into smaller local problems and distribution of the overall computation burden among a set of local filters have obvious advantages in large-scale systems. An alternative solution is, therefore, to decompose the global estimation problem into several smaller problems by accommodating the solution to the physical structure of the system and assign a separate, autonomous filter to each of the subsystems. These filters treat the process information locally at the component or subsystems level using local information of the system dynamics. Then, a master or fusion filter is used for composing the local results in a global estimation.

Previous efforts on distributed filtering have concentrated on either decentralized filters on centralized or hierarchical topologies or essentially centralized filters on decentralized topologies based on the work of [9, 18, 21]. Several decentralized solutions have been developed to decentralize the filter algorithm, topology, and services through tradeoffs of computation and memory that minimize communication. These implementations assume a truly decentralized architecture requiring no central facilities. Since Kalman filter has been an excellent means for exploring decentralized system trade-offs because of its optimality and linearity, the majority of solutions tend to rely on derivations of the linear

Kalman filter, see e.g., [20].

In this paper, a particular dependable filtering solution for the enhancement of filter performance and sensor fault tolerance by means of distributed filtering is proposed and investigated. Namely, in addressing the problems of both fault tolerance and estimation accuracy in nonlinear systems the application of the idea of federated filtering is investigated.

The structure of the paper is the following. Section 2 gives a short summary of traditional filtering solutions, generally used in distributed systems. The underlying theory for centralized and decentralized designs is reviewed. Section 3 gives special attention to a particular form of decentralized filters, i.e., federated filters, which is then followed by the extension of the classical federated idea. The basic features and theory of the federated solution are briefly reviewed. The necessary theory of the design of the federated filter is mainly focused on the presentation of the fault reconciliation and fusion algorithm. The federated structure relies on Extended Kalman Filter (EKF) as state estimator where the core filtering algorithm is considered the nonlinear extension of the standard linear filter. As both Kalman filters and EKFs are considered to be standard techniques of control theory by now, technical details only necessary for understanding the recent problem formulation are mentioned here. For more information, the interested reader is referred to the literature, see e.g., [4], [13], and also the references therein. The novel solution benefits from the adaptive filter management technique, which is based on the continuous health monitoring of the input channels. This is done to balance estimation accuracy and sensor fault tolerance in the presence of both state and measurement uncertainty when the observations from the system are subject to a variety of sensor faults.

The presentation of the main results of this paper is embedded in the discussion of an application example associated to the state estimation of a multi-component distillation process. The simulation example, demonstrating and comparing the features and performance characteristics of various filtering alternatives is presented in Section 4.

2. Central vs distributed filtering

2.1. Centralized filtering

As a standard solution alternative to the state estimation problem, centralized filters have traditionally been used in the industry for many decades. In this approach a single filter with universal access to all sensors is used for estimating the complete set of the state variables of the system (Figure 1).

Centralized solutions, however, especially in large-scale industrial applications, suffer from the size (dimension) of the problem and are prone to both sensor and implementation failures. Reduced order filters are not always reliable, they tend to have poor estimation accuracy, and even instability under certain circumstances. Centralized approaches, moreover, cannot answer the call of the safety and dependability requirements as they lack fault tolerance when faults in the operation of the system or sensors occur. The development of decentralized filtering, therefore, has received increasing attention during the past few years.

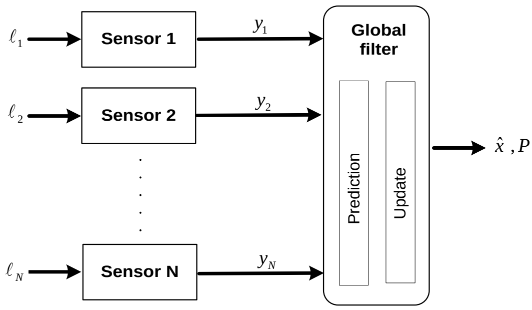

Figure 1: *Traditional centralized filter architecture.*

2.2. Decentralized filtering

It was recognized early that the division of the global estimation problem into smaller, partial problems and the distribution of the overall computation burden among a set of (local) filters have obvious advantages in large-scale applications. A solution, which is an alternative to the centralized one, decomposes the global estimation problem into several smaller problems. This is done by accommodating the filter architecture to the physical structure of the system by assigning one ore more separate, autonomous filter to each subsystems. These filters treat the process information locally at the component or subsystems level using local information of the system dynamics. Then, the estimation of sensor dedicated local filters are fused in a master or fusion filter for composing the local results in a global estimation, see Figure 2. Another distinguishing feature of decentralized filtering is the inclusion of reference sensors in the structure. The following two features distinguish the reference sensor from the local sensors:

- The reference sensor acts as a fundamental sensor in the system, and its data is directly observable for both the master filter and any of the local filters. The data from the local sensors is dedicated to the corresponding local filters only.
- The reference sensors are assorted equipments, which are usually more dependable and have higher data rate than others; thus their data is often used for the initialization

of some of the local filters. These are accountable to compensate erroneous sensor readings relying on the benefits of the decentralized architecture. The number of reference sensors included in the distributed architecture is a design issue dependent of the particular application.

Previous efforts on distributed filtering have concentrated on either decentralized filters on centralized or hierarchical topologies or essentially centralized filters on decentralized topologies based on the work of [9, 18, 21]. Several decentralized solutions have been developed to decentralize the filter algorithm, topology, and services through tradeoffs of computation and memory that minimize communication. These implementations assume a truly decentralized architecture requiring no central facilities. Since Kalman filter has been an excellent means for exploring decentralized system trade-offs because of its optimality and linearity, the majority of solutions tend to rely on derivations of the linear Kalman filter, see e.g., [20].

As a result of this decentralization technique, the computational load regarding single filter's implementation, can be significantly reduced. This property makes the solution viable in resource constrained embedded implementations.

2.3. Federated filtering

As a special case of decentralized filtering the so called federated filtering, that was originally proposed in [5, 6, 7], was developed.

Federated filters are distributed filters which consist of several local filters (LFs) and a master (or fusion) filter (MF). There is a particular LF assigned to a particular subsystem. LFs work in parallel and their solutions are periodically fused by the MF. As the local

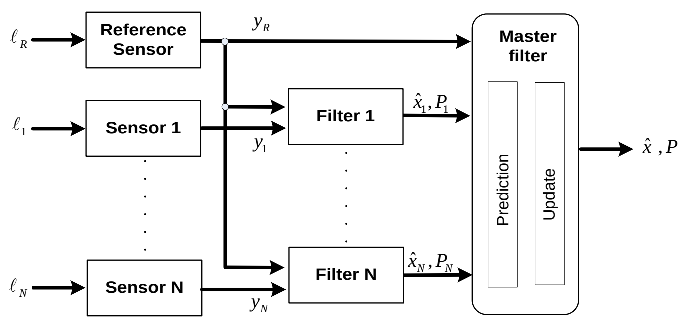

Figure 2: *Decentralized filter architecture.*

estimates are usually suboptimal, information feedback from the master filter to each local filter is necessitated. The distinctive property of the federated filter is that this feedback relies on the information sharing principle that was originally proposed in [5, 6, 7].

Although, decentralized filtering schemes, including those the filters are aligned in the federated architecture, have been recognized effective in the practice, their potential to enhance estimation performance and fault tolerance have not been widely investigated and fully realized. In this paper a novel idea to avoid filter degradation in various failure modes of the system is proposed. To maintain filter functionality and performance, an adaptive technique to influence the information sharing property as well as changing the filters' operating mode of the federated architecture is suggested. This is done by means of a sensor redundancy management applied to the fusion rule of the MF. This concept can be considered an extension of the classical federated idea.

The federated filtering scheme is categorized as a decentralized one, but the use of the information feedback via the sharing factors β_i, which determine the distribution of the information provided by master filter around local filters, distinguishes it from the classical decentralized idea. The principle of information sharing is based on the axiom of information conservation, i.e., $\sum_i^n \beta_i = 1$, where n is the number of the sharing filters. This property is discussed further in the following part.

The existence of the feedback, however, has non-desirable side effects on filters' operation. If, for example, an undetected fault, in any sensor loop of the system occurs, due to the closed-loop nature of the federated filtering process, the other filters in the network may be polluted by the fault propagated through the information network of the federated topology. This feature, however, can be used in a variety of useful ways to improve filters performance through the extension of the classical federated idea.

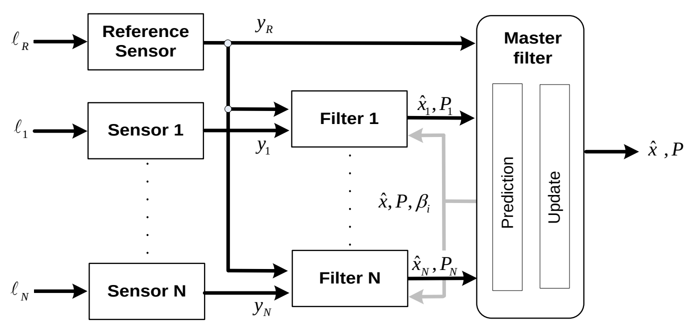

Figure 3: *Federated filter architecture.*

In the next sections of the paper the idea of this extension, and the operational benefits are shown in the framework of the solution of a nonlinear state estimation problem, which is considered part of a process control application frequently used in the chemical industry.

3. Extension of the federated filter architecture

For state estimation in chemical process control applications a wide variety of Kalman filters have now been used and developed, from Kalman's original linear formulation, now called the simple Kalman filter (from which the so-called information filter and a variety of square-root filters were derived) to nonlinear problems such as the extended and unscented Kalman filters. Whilst the techniques and theory of federated filters (especially those based on the Kalman filter) are relatively well developed for linear systems, experience subjecting nonlinear applications are not yet widely available. A rare exception can be found in [2].

The Unscented Kalman Filter (UKF) addresses of the nonlinear estimation problem by using a deterministic sampling technique and a nonlinear (the so-called unscented) transformation. These filters provide an optimal solution to the (linear) estimation problem by minimizing the disturbance (noise) effects using a quadratic cost function assuming the disturbances are stochastic processes with jointly Gaussian distribution. There are non-Gaussian extensions of Kalman filtering available too.

If no information about the statistics and distribution of the noise is available, the aforementioned methods are not directly applicable. Still, however, the filtering problem can be posed e.g., as an H_∞ estimation problem and the local, as well as master filters, can be implemented as H_∞ filters.

In the following part of the paper, a novel idea to avoid filter degradation in various failure modes of the system is proposed. The filter is based on the standard nonlinear extension of the well-known linear Kalman filter (i.e., EKF), which is used in a particular structure for the solution of the distributed nonlinear state estimation problem. This is done to derive a distributed filtering solution to achieve better estimation accuracy, which is able to tolerate sensor faults.

3.1. Formulation of the distributed filtering problem

In this paper we are concerned with nonlinear dynamical systems described in the nominal representation by ordinary differential equations subject to noise

$$\begin{aligned} \dot{x}(t) &= \phi(x(t),u(t),w(t)), \\ y(t) &= h(x(t),v(t)). \end{aligned} \tag{1}$$

For a special case, this can be written in state space form, by means of the set of noise-corrupted state and measurement equations of the following form

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x)u_i + w, \qquad (2)$$
$$y_i = h_i(x) + v_i, \qquad 1 \leq i \leq p,$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ denote the state, the input and the output of the system, respectively, moreover, $w(t)$ and $v(t)$ are the system and the measurement noise, which are independent of each other as well as of $x(0)$.

Assume each sensor s_i measures a signal $y_i(t)$ that is corrupted by measurement noise v_i taken to be a zero-mean white noise. Let Q_i and R_i denote the covariance matrix of v_i and w_i for all i, respectively,

$$Q = \mathbf{E}\{v_k v_k^T\}, \quad R = \mathbf{E}\{w_k w_k^T\}. \qquad (3)$$

Let the local estimate and its covariance provided by the i^{th} local filter be represented by $\hat{x}_i$ and P_i $(i = 1,2,...,N)$. The filtering algorithm is considered the extension of the standard linear one. Since the system is not linear, the Riccati matrices that attempt to approximate the *a priori* and the *a posteriori* covariances for each filter are defined, respectively, as

$$P_{k|k-1} \approx \mathbf{E}\{e_{k|k-1}\, e_{k|k-1}^T\}, \qquad (4)$$
$$P_{k|k} \approx \mathbf{E}\{e_{k|k}\, e_{k|k}^T\}. \qquad (5)$$

The filters are initialized with $x_{o|o} = x_o$ and $P_{o|o} = P_o$, and then operated recursively performing a single cycle each time a new set of measurements becomes available. Each iteration propagates the estimate from the time the last measurement was obtained to the current time. The propagation process consists of two stages: update and prediction. The update equations are responsible for the feedback, i.e., for incorporating a new measurement set into the *a priori* estimate to obtain an improved *a posteriori* estimate. The *a posteriori* state estimate $x_{k|k}$ is computed as a linear combination of an *a priori* estimate $x_{k|k-1}$ and a weighted difference between an actual measurement y_k and a measurement prediction:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left[y_k - \bar{h}(\hat{x}_{k|k-1})\right], \qquad (6)$$
$$K_k = P_{k|k-1}\bar{H}_k^T \left(\bar{H}_k P_{k|k-1}\bar{H}_k^T + R_k\right)^{-1},$$

where $\bar{H}_k$ is the Jacobian matrix of partial derivatives of $h(x)$ with respect to x, that is

$$\bar{H}_k = \left(\frac{\partial \bar{h}(x)}{\partial x}\right)_{x=\hat{x}_{k|k-1}.}$$

The matrix K_k is chosen such that the filter minimizes the *a posteriori* error covariance. The covariance matrix is updated by

$$P_{k|k} = (I - K_k \bar{H}_k) P_{k|k-1}. \tag{7}$$

The prediction equations are responsible for projecting forward the current state vector and error covariance estimates to obtain *a priori* estimates for the next time step. The state and covariance matrix in the next sampling instant are estimated by

$$\begin{aligned} \hat{x}_{k+1|k} &= f(\hat{x}_{k|k}, u_k), \\ P_{k|k+1} &= \bar{F}_k P_{k|k} \bar{F}_k^T + Q_k, \end{aligned} \tag{8}$$

where $\bar{F}_k$ is the Jacobian matrix of partial derivatives of $f(x)$ w.r.t. x, as

$$\bar{F}_k = \left(\frac{\partial \bar{f}(x,u)}{\partial x} \right)_{x=\hat{x}_{k|k}, u=u_k}.$$

The following characteristics distinguish the synthesizing MF and the preprocessing LF's in the architecture. Each LF is dedicated to the measurements of local sensors. For the update process, the LF's use the update information from its local sensors only and not any others. Conversely, the MF uses the local filtered estimates $\hat{x}_i, P_i$ as quasi-observables to update the global state vector in a fusion process, sequentially.

The MF is processed at the rate equal to the rate of the LF's, which means that local outputs are subject to fusion on the next stage they are processed. The time updating solution of the MF is represented by the state estimation $\hat{x}_{N+1}$ and covariance $\hat{P}_{N+1}$, respectively. If all local estimates are uncorrelated, then the global estimate can be given as

$$P_f^{-1} = P_1^{-1} + P_2^{-1} + ... + P_N^{-1} + P_{N+1}^{-1}, \tag{9}$$

$$\hat{x}_f = P_f \left[P_1^{-1} \hat{x}_1 + ... + P_N^{-1} \hat{x}_N + P_{N+1}^{-1} \hat{x}_{N+1} \right], \tag{10}$$

where the inverse of the covariance matrix is called the information matrix. Eq. (9) shows that the global information is just the sum of that of the local systems. The global estimate $\hat{x}_f$ is a linear weighted combination of the local estimates with weighting matrices P_f^{-1}, P_i^{-1} $(i = 1, \ldots, N, N+1)$.

However, the estimates of different local filters may be correlated. In order to eliminate this correlation, the process noise and state error covariance are set to their upper bounds relying on the technique proposed in [5, 6]

$$Q_i = \beta_i^{-1} Q, \qquad P_i = \beta_i^{-1} P_f, \tag{11}$$

where $\beta_i (\geqslant 0)$ is the information-sharing factor satisfying the rule $\beta_1 + \beta_2 + ... + \beta_N + \beta_{N+1} = 1$.

For the condition that makes Eq. (9) hold, see [7]. To some extent, this method results in a conservative design because of using the upper bound of the process noise variance matrix instead of the process noise variance matrix itself. Other approaches to the solution of this problem can be found e.g., in the references [16, 19].

For the structure and organization of the filter, see the architecture diagram Figure 1, where y_i stand for the sensor measurements, $\hat{x}_i$ for the state estimates and β_i for the information sharing variables. P_f denotes the covariance matrix provided by the fusion filter. The innovation series of the local filters are subject to statistical testing. The Δ blocks provide residuals ε for detecting changes in the statistics of the measurement data, upon which the determination of the measures of the covariance information sharing β_i is relied.

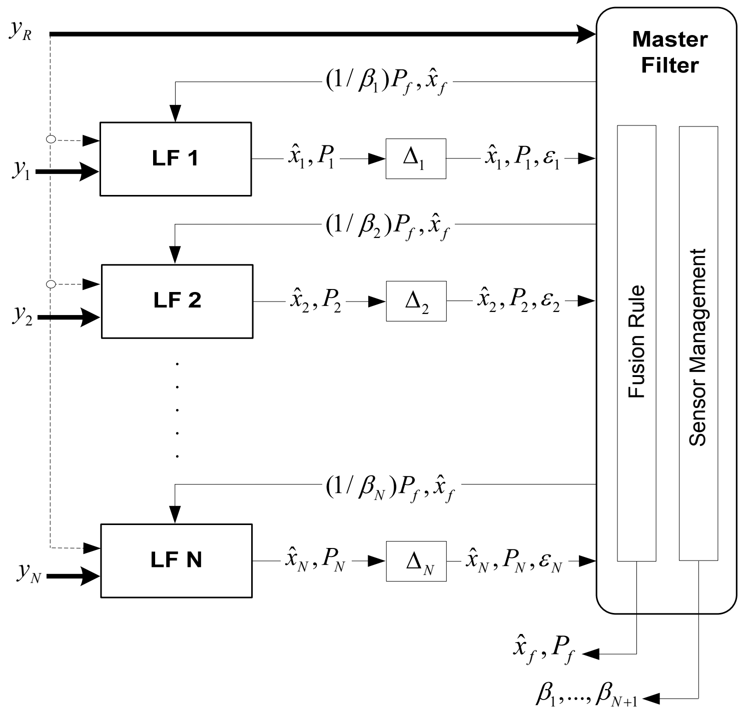

Figure 4: *The extended federated filter architecture.*

The fused estimation of the MF is, usually, based on measurements provided by reference sensors (see y_R in Figure 1). Reference sensors act as fundamental sensors in the system, which are usually more dependable and have higher data rate; thus their data is often used for the initialization of some of the local filters. These are accountable to compensate erroneous sensor readings relying on the benefits of the decentralized architecture, as will be shown later.

This structure of filters can be operated in two different operating modes, depending on how the fused process data is exploited by the local filters. These modes of operation are, usually, referred to as *reset* and *no-reset* modes. When the *reset* mode is used, the master and local filters are reset by the global solution, i.e., $\hat{x}_i = \hat{x}_f$, and $P_i = \beta_i^{-1} P_f$, for $i = 1, \ldots, N, N + 1$, meaning that a continuous information feedback from master filter to local filters is present. In *no-reset* mode this information feedback does not exist: each LF keeps its process information ($\hat{x}_i$,P_i) produced locally, thus the MF retains none of the fused data and the global fused estimation ($\hat{x}_f$,P_f) has no effect on any of the local estimations.

Obvious advantages and disadvantages associated with each of the resetting modes are known. The federated filter operated in reset mode is expected to provide better estimation accuracy, while in the no-reset mode a better tolerance of sensor faults. The justification of this assumption is one of the main objectives of our analysis presented in the next section.

4. Application to multi-component distillation

In this section we consider the distributed state monitoring problem of a distillation process in which a ternary methanol, ethanol and propanol mixture is fractionated, assuming the process is described by the system model (1)-(2).

Several authors have studied this topic. [3] uses an EKF to infer the outlet compositions of a ternary distillation column. [15] applied EKF to estimate the state of a batch distillation.

4.1. Objectives

The objective of effective monitoring provides state information reliably, with required accuracy. In large distributed systems containing a large number of components, the occurrence of sensor faults which may affect control actions is an increasing concern. Sensor failures causing measurement drop-outs, sensor stuck as well as increased noise and measurement uncertainty will, obviously, impose a practical limitation on the estimation accuracy. Federated filtering provides a variety of possibilities to compensate these problems and improve filtering performance.

In order to develop the idea further and demonstrate the application of the theory described in the previous sections, a simulation example regarding the state estimation problem of a multi-component distillation process which fractionates ternary methanol, ethanol and propanol mixture is presented.

The framework that combines sensor fault detection with isolation and fault accommodation through filter reconfiguration is outlined. The challenge is to engineer an operational strategy of the filter's architecture in an attempt to make a tradeoff between best estimation accuracy and sensor fault tolerance in every possible time. For this purpose, the classical federated architecture is extended with a sensor fault detection and a sensor management logic. The effect of the selection of the operating mode and the proper choice of the information-sharing factors are investigated. Filter operation is demonstrated through process simulations.

It is pointed out that the key to this solution is to resolve how the total information generated by the local filters is divided among the individual filters by a proper assignment of the sharing factors β_i and also, how the proper resetting policy of the fusion filter is to be achieved for the best possible estimation accuracy, or in case of a failure, for higher fault tolerance.

4.2. The multi-component distillation process

Multi-component distillation is the separation of a mixture of chemicals of, usually, 3 or more components. Nearly all commercial distillation recently used in the chemical industry is multi-component distillation. multi-component distillation represents a special class of dynamical systems in which operational safety is a primary design factor. The usually complex chemical process can be decomposed of a set of subsystems having coupled dynamics. By the term coupled, the mutual dependence of the elements of the global system state and the correlated system and measurement noise is meant.

Both the control and estimation of an equipment like this represent an engineering challenge that requires the application of the sophisticated methods of advanced control theory. Some of these systems, for instance, are controlled by MPCs that necessitate the availability of accurate data obtained from state estimation. The reliability of individual sensors is frequently inadequate to satisfy the stringent reliability requirements of this type of systems. Therefore, in many real applications an array of redundant sensors is employed to achieve the required dependability level relying on the principle of hardware (sensor) redundancy. Frequently, the redundant sensor network is extended with analytic sensor fault detection and isolation methods to detect sensor malfunctions.

Due to the availability of the redundant sensors, the estimation of a particular state of the global state vector of the system can be accomplished relying on several different redundant measurements. Estimations based on a centralized architecture may provide estimation performance differing considerably for each rearranged sensor layout. Sensor assignment, therefore, is one of the main concerns of experiment design. It is shown in this paper, how the solution of federated filtering exploits the principle of redundancy to improve both estimation accuracy and sensor fault tolerance in a smart reconfigurable structure at the same time.

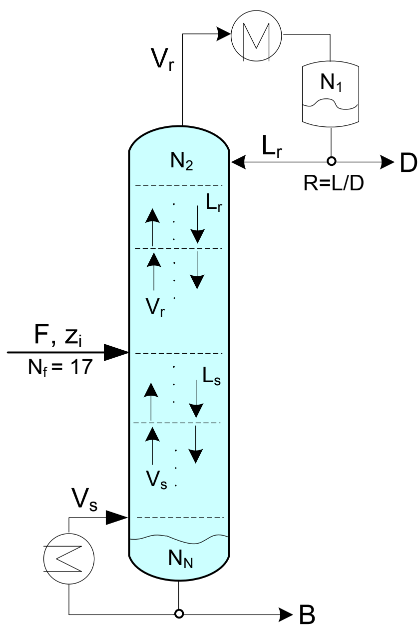

Figure 5: *Distillation column schematic.*

The process consists of 30 separate distillation stages (i.e., column trays), identified with parameter N_T, plus the reboiler and a total condenser as usual. The control-input is the reflux of liquid flow rate which acts on the plant. The sensor outputs used for control and estimation purposes are the temperature data of the column trays. It is assumed that there is one temperature measurement available for each tray. The trays (and the corresponding measurements) are numbered from the top to the bottom of the column. Due to the coupled nature of the nonlinear dynamics, these sensors provide a redundant set of measurements upon which the estimation and control of the process can be based.

The instrumentation includes temperature sensors of varying quality. There are two highly reliable reference sensors used, one at the topmost and one at the bottommost of the column.

We consider an idealized column model with N ideal vapor-liquid equilibrium (VLE) stages. These consist of $N_T = N - 2$ plates together with a condenser and the reboiler. The plates are numbered as N_j, $j = 2,...,N - 1$, where plate N_2 corresponds to the top and plate N_{N-1} corresponds to the bottom of the column.

Quantities associated with the reboiler are indexed $j = N$ and, with the condenser $j = 1$. Feed F is directed on to plate N_f of the column and is composed of a mixture of 3 components, namely, $methanol$,$ethanol$ and $1 - propanol$. Components c_i are numbered as $i = 1,2,3$. The volatilities of the components are ordered such that $\alpha_1 > \alpha_2 > \alpha_3$, where component 1 is the most volatile component (methanol), and so on. Heat input to the reboiler vaporizes some material which passes up the column as vapour. At the top of the column, the vapor is condensed, and some liquid is returned as reflux to the top of the column. The flow of liquid descending from stage j to $j - 1$, above the feed stage (rectifying section) is denoted by L_r and under the feed stage (stripping section) by L_s.

The liquid flow in the top and bottom streams are denoted by D and B, respectively. The ratio of the reflux flow to the flow in the top stream L_r/D is referred to as reflux ratio R. Similarly, the flow vapour in the rectifying section is denoted by V_r and in the stripping section by V_s. Our model assumes that the feed is saturated liquid, moreover, the liquid and vapor flow are constant, therefore:

$$\begin{aligned} L_s &= L_r + F \\ V_s &= V_r, \end{aligned}$$

where the energy equations are neglected. The mole fraction of component c_i in the liquid of stage j is denoted by $x_{i,j}$, and $y_{i,j}$ is the vapor composition which, in equilibrium with the liquid phase, is given by

$$y_{i,j} = \frac{x_{i,j}\alpha_i}{\sum\limits_{i=1}^{n_c} x_{i,j}\alpha_i},$$

where α_i-s are the relative volatility parameters.

The fractional composition of the top stream is the same as that of the recycled liquid. The vapor holdup is negligible and the liquid holdup (M_j) on each stage is assumed to be constant.

The nonlinear model of the distillation column can be represented by the following set of equipment models. The main assumptions and conditions adopted for the model development are not mentioned here, the interested reader is referred to the literature, such as e.g., [3]. The condenser is subject to

$$M_1 \frac{d(x_{i,1})}{dt} = V_s y_{i,2} - L_s x_{i,1} \left(1 + \frac{1}{R}\right) \tag{12}$$

with reflux ratio R which is, normally, used as the control input of the distillation process. A generic tray in the enriching (rectifying) section is modeled as

$$M_j \frac{d\left(x_{i,j}\right)}{dt} = V_s y_{i,j+1} + L_s x_{i,j-1} - V_s y_{i,j} - L_s x_{i,j} \tag{13}$$

for $i = 1,...,n_c - 1$ and $j = 1,...,f - 1$, where f is the tray number where the column is fed. Then, the feed tray is subject to

$$M_f \frac{d\left(x_{i,f}\right)}{dt} = V_s y_{i,f+1} + L_s x_{i,f-1} - V_s y_{i,f} - L_s x_{i,f} + F z_i, \tag{14}$$

with feed flow rate F and molar composition z_i of component i. A generic tray in the stripping section is represented by

$$M_j \frac{d\left(x_{i,j}\right)}{dt} = V_r y_{i,j+1} + L_r x_{i,j-1} - V_r y_{i,j} - L_r x_{i,j}, \tag{15}$$

for $j = f + 1,..,N_T - 1$, and the reboiler is modeled as

$$M_N \frac{d\left(x_{i,N}\right)}{dt} = L_r x_{i,N-1} - V_r y_{i,N} - L x_{i,N}. \tag{16}$$

In model equations (12)-(16) the parameter n_c stands for the number of liquid components and M_j is the molar holdup corresponding to each particular tray.

The liquid compositions are considered as state variables. For a system of n_c components ($n_c = 3$ in this case), there are $n_c - 1$ state variables at each particular tray. Thus, 64 state variables are included in the global model, altogether. The composition of the n_c^{th} component is obtained by

$$x_{n_c,j} = 1 - \sum_{i=1}^{n_c-1} x_{i,j}, \qquad 1 \leq j \leq N.$$

As the stages are considered ideal, the relationship between tray temperature and composition can, traditionally, be given by the VLE equations. For simplicity, this calculation is not shown here, for details see e.g., [15]. The realization of the EKF filters (both local and master) are based on model equations (12-16).

The temperature associated with stage j is denoted by T_j. The pressure is considered constant and known on each tray. It varies linearly up the column from p_B (in the base) to p_D (at the top).

The measurement vector is obtained as a function of the state vector. The simplifications included in the modeling process introduce modeling errors. Modeling uncertainties are expected to be compensated by the individual EKF's. The values of the nominal operating variables of the column are summarized in Table 1.

Table 1: Nominal operation data of the distillation column

#	Parameter name	Value
f	Feed tray (#)	17
F	Feed Flow Rate (mol/s)	1
c_1	Methanol composition ($\%mol$)	0.4
c_2	Ethanol composition ($\%mol$)	0.4
c_3	1-Propanol composition ($\%mol$)	0.2
p_D	Top pressure of the column (kPa)	97
p_B	Bottom pressure of the column (kPa)	156
M_j	Molar holdup on each tray (mol)	0.25
M_1	Molar holdup in the condenser (mol)	0.5
M_N	Molar holdup in the reboiler (mol)	1
α_1	Relative volatility of methanol	1.664
α_2	Relative volatility of ethanol	1
α_3	Relative volatility of 1-propanol	0.451

4.3. Results and discussion

In this section we present the results obtained for the solution of the nonlinear state estimation problem based on various filter layouts and process conditions. The process simulations were performed in Matlab. A structure of filters assembled in the federated architecture, resembling to the one depicted by Figure 1., containing three local and one master filter was derived for evaluation.

In our case the ternary mixture distillation process is assumed to be observable, providing that at least two temperature measurements for the filter calculations are available. For the proof of this condition, see Ref. [22]. When more than two measurements can be used, the performance of the estimator may improve. In the following part we present the case of using the minimum number of measurements, i.e., only two of the 32 temperature measurements are considered available to each of the filters.

Another relevant aspect is the proper allocation of the measurement locations along the column. The optimal allocation of temperature sensors for the estimation of distillation compositions were suggested by [11] and [14]. The rules of placing sensors on processes described by stable nonlinear dynamic systems were described e.g., in [17]. Not going into details of the sensor assignment procedure, in our case two reference sensor measurements were allocated: one to the bottom of the column, which has the largest inertia of the system, and the other to the top of the column, so as to reflect promptly every possible changes in the end-product composition.

Relying on the above considerations, the local filter estimates are based on the tray temperature measurements in grouping LF1:(y_1,y_{31}), LF1:(y_2,y_{32}) and LF3:(y_3,y_{31}), whilst the master uses reference inputs MF:(y_1,y_{32}). In normal operation conditions the normal and reference measurements are assumed to have sensor noise with variance $\sigma = 0.01$ and $\sigma = 0.001$, respectively, indicating the higher dependability of the reference sensors.

In the simulation of the complex column behavior, a rigorous distillation model, different from those of (12-16) based on Differential Algebraic Equations (DAE) was employed. For more details regarding this process modeling and simulation technique the reader is directed to the literature, see Ref. [12].

The process simulation relied on the assumption that a step change in zero time in the control signal (reflux) had been injected. As an effect, the process imposed a transient behavior where the resulted state variation was the subject of estimation. Since the composition at the top of the column is in direct relationship with the end product of the distillation (i.e., methanol), the estimation of this composition was of the upmost importance. Therefore, our filter provided estimation of this state variable.

Sensor faults may be caused by sensor drifts, step changes, scale factor errors changing the mean, incorrect calibration, etc. Correspondingly, the degraded modes of sensors can be characterized by the presence of a systematic nonzero mean, in the form of a constant jump bias, a ramp bias, or by an increase in variance of the driving noise.

In our simulations two different types of sensor failures were modeled. A sensor failure (shift) changing the mean value of the innovation sequence of the second measurement channel by a constant bias ($\mu = 1$), moreover, increased variance in the driving noise $\sigma = 10$ (instead of the normal $\sigma = 0.01$ value) was considered. For a more accurate comparison of the results, the same noise sequences were used for each simulation experiments. Experiments for contrasting centralized filter performance and the federated idea were carried out.

A comparative summary of the results can be found in Table 2 showing filter performance data corresponding to the particular filter configurations and various process conditions. The performance of individual solution alternatives is characterized in terms of the L_2 norm of estimation error $\tilde{x}$. Relative estimation error δ with respect to the estimation accuracy obtained with the centralized filter in fault-free case is also given for comparison.

Figures 3-7 show the plots of distinguished experiments. The simulation results confirm that the centralized and federated filtering operated in full reset mode with identical information sharing factors provide almost the same performance both in faulty situations and in fault free-case (see Table I). This was an expected result, because it comes directly from the theory of federation. Differences in filter performances can be observed, however, in faulty situations when, after detecting a fault, one switches the federating scheme to no reset mode with setting the corresponding β to zero (in fact a small value near to zero) in an attempt to mask out the estimation of the subjected local filter.

4.4. Sensor failure reconciliation

In the previous experiments the values of β_i as well as the operating mode of the filter were chosen manually. The requirement for fully autonomous filter operation needs the application of a sensor management logic which is based on a fault detection algorithm responsible for fault accommodation by real-time adaptation of the sharing factors.

If the system operates normally, the normalized innovation sequence of a filter is a zero mean white noise with unit covariance. A real-time detection of sensor failures affecting the mean and/or the variance of the innovation process triggers a filter reconfiguration action in the sensor management logic of the fusion filter in an attempt to keep the performance of the impaired structure as close to the optimum as possible. This includes the modification of the sharing factors and the switch-over of the reset mode of the filter.

The diagnosis can be based on the analysis of the innovation sequences of the individual filters. To detect failures changing the mean of the innovation sequence a number of statistical approaches are available that make use a threshold test on the moving window average of the output of the instrument. These tests commonly known as generalized likelihood ratio tests (GLRT), χ^2-tests and others, are appropriate for online implementation and embedding in the federated structure, according to the general scheme shown in Figure 1. For further details the reader is directed to Refs. [8] and [13] and also the

Table 2: Comparison of filtering performance

Filter Layout	Sensor Fault	β_i	$\|\tilde{x}\|_2$	δ
Centralized	no fault	–	0.0134	0
Centralized	$\sigma = 10, \mu = 0$	–	0.0192	43.2
Fed. with reset	no fault	equal	0.0135	0
Fed. with reset	$\sigma = 10, \mu = 0$	equal	0.0192	43.2
Fed. w/o reset	no fault	equal	0.0136	1.5
Fed. w/o reset	$\sigma = 10, \mu = 0$	equal	0.0291	117.2
Fed. w/o reset	$\sigma = 10, \mu = 0$	$\beta_2 = 0$	0.0160	19.4
Fed. w/o reset	$\sigma = 10, \mu = 1$	equal	0.0223	66.4
Fed. w/o reset	$\sigma = 10, \mu = 1$	$\beta_2 = 0$	0.0145	8.2

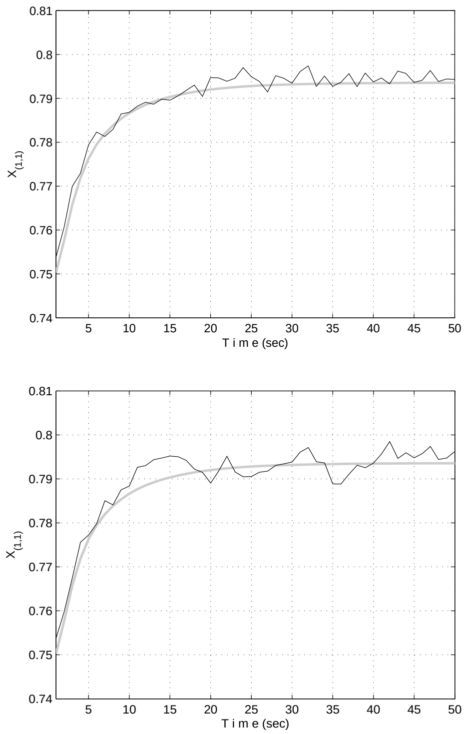

Figure 6: *Centralized estimation with no sensor fault (a), and with increased variance in the driving noise of the measurement channel (b).*

references cited thereof. In the following part of the paper we use a simple demonstrative algorithm that can be summarized as follows.

Let the median of the estimated states $\hat{x} \in \mathbb{R}^n$ in each particular time be given as $\tilde{X}_t = \{\hat{x}_i\}|_t$. Let S_i denote the error variance between estimation $\hat{x}_i$ and median $\tilde{X}$, and define the inverse of the sum of the elements of S_i by

$$\bar{S}_i = \frac{1}{\sum_{j=1}^{n} S_{ij}}. \tag{17}$$

Obviously, the magnitude of S_i characterizes the dependability of a particular local es-

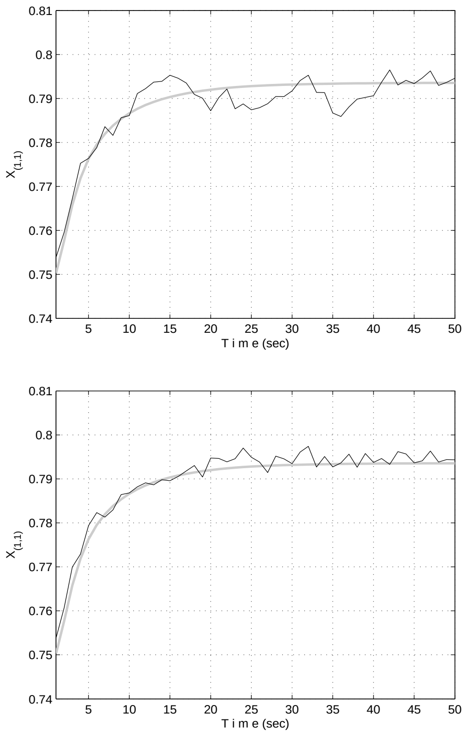

Figure 7: *Federated estimation with no reset of process information (a) assuming a nonzero mean sensor fault with increased variance in the measurement y_2 and, with full reset in no fault condition (b). The sharing factors are identical in both cases, i.e., $\beta_1 = \beta_2 = \beta_3 = \beta_M$.*

timation relative to the fused one. Under fault-free conditions measure S_i can be kept below a well chosen threshold limit, meaning that the channel is operating normally. In case a measurement channel gets faulty, the corresponding S_i will exceed the threshold quite sensitively. Therefore, eq. (17) can be taken as residual generated by each particular LF that may be used for providing information about the appearance of sensor faults (*cf.* Fig 1).

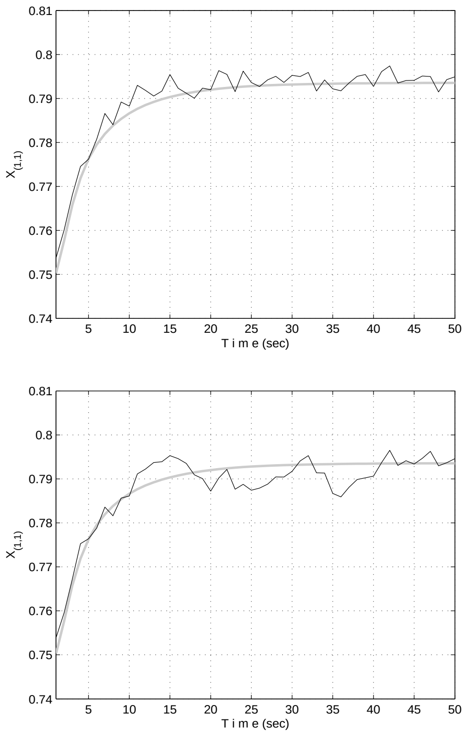

Figure 8: *Federated estimation with no reset of process information assuming increased variance of the driving noise (a), and with additional change in the mean of the driving noise characteristics of measurement y_2 (b), by setting sharing factor β_2 to zero.*

Then, based on (17), information-sharing factors β_i can be calculated as

$$\beta_i = \frac{\bar{S}_i}{\sum_{i=1}^{k} \bar{S}_i}, \tag{18}$$

where k is the total number of local filters. Realize that an actual β_i is a decreasing variable with increasing error variance S_i. The application of the median in (17) and (18) is motivated by the use of a limited number of local filters in this sample application. This approach may give good results in averaging the estimation error variances in cases when only a small number of local filters are available (i.e., $k < 10$). For $k > 10$, however, the application of the statistical mean could provide much better results.

The results of a process simulation assuming the occurrence of two different types of sensor faults are summarized in Figure 6. and Figure 7., respectively. In Figure 6/a the effect of a step change occurring in one of the temperature measurements at $t = 22$ is shown. As a results of this change that can be attributed to a sensor shift fault, the sharing factors are re-calculated: after violating the preset decision limit ($\beta_o = 0.0003$ in this case) the sharing factor β_i that corresponds to the affected local filter, becomes permanently zero (see Figure 6/b), thus excluding the effect of the faulty measurement from the consecutive estimate fusion, effectively and immediately. This results in increased estimation accuracy in the presence of sensor fault, as it is clearly demonstrated in Figure 6/c.

Figure 7. shows the same principle but with the assumption of an increased variance of the sensor noise that occurs in $t \geqslant 22$. Note that in case of some slowly developing sensor malfunctions (typically when e.g., a sensor gradually gets noisy) the particular estimation error variance does not necessarily exceed the threshold limit instantly, and the respective β_i not necessarily becomes zero. However, by adaptively changing β_i the algorithm balances the estimation weights of the individual filters continuously in accordance with the actual quality of the measurements, thus reconciling the effects of any developing sensor faults.

5. Conclusions

The paper advocates the principle of distributed filtering as the basis for realizing dependable filtering solutions in large-scale distributed systems, such as in multi-component distillations, which are subject to sensor faults. Namely, a design strategy for the application of a federated filter for increased accuracy estimation and sensor fault tolerance was presented. The idea consists of using dedicated local filters to particular components or subsystems and utilizing a master filter for obtaining the global estimation. Information generated locally is then divided among the distributed filters in the update process of the filter by a proper choice of the information sharing factors.

Our main motivation was to create a distributed filter to maintain estimation accuracy in every possible time in a structure able to accommodate the effects of sensor faults, effectively. It was shown how this objective could be achieved by using a novel combination of the special distributed structure and a sensor fault management logic.

The architectural and functional analysis, supported by extensive numerical simulations, indicates that the due manipulation of the sharing factors, as well as the special choice of the resetting policy of the filter are capable factors for the improvement of filtering performance and sensor failure reconciliation. This supports the hypothesis that a better estimation accuracy and sensor fault tolerance in case of various types of sensor

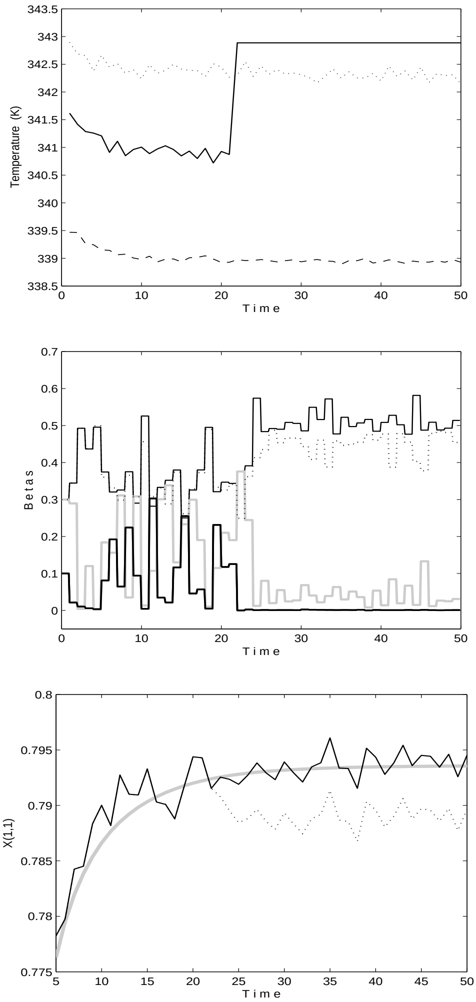

Figure 9: *a./ Step change failure in one of the temperature measurements at $t = 22$. b./ Adaptation process of β_i's. The sharing factor of the filter corresponding to the faulty sensor goes to zero within a few recursion. c./ Comparison of estimation accuracies with and without sharing factor adaptation (continuous and dotted lines, respectively).*

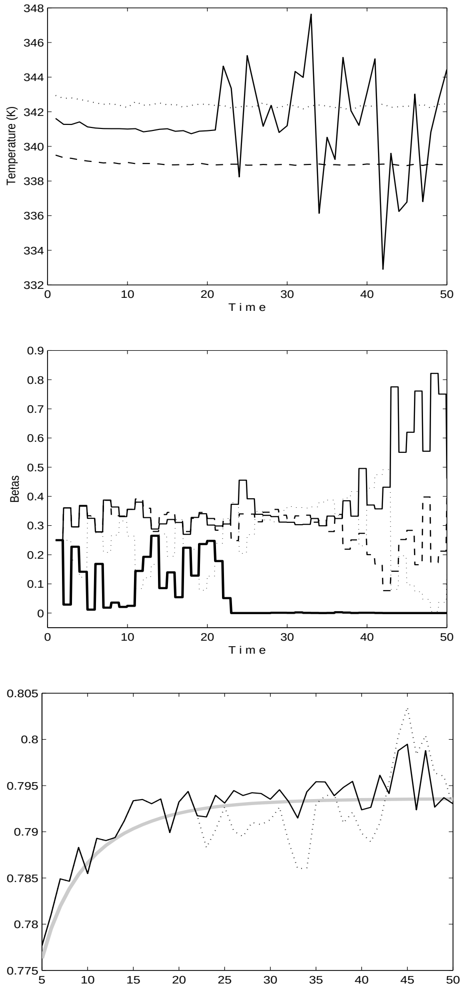

Figure 10: *a./ Increased variance in the noise of one of the temperature measurements at* $t \geqslant 22$. *b./ Adaptation process of* β_i*'s. The sharing factor of the filter corresponding to the faulty sensor goes to zero within a few recursion. c./ Comparison of estimation accuracies with and without sharing factor adaptation (continuous and dotted lines, respectively).*

degradation can be obtained by using the concept of federation.

The key to this solution is the re-organizable architecture represented by the federated idea, as well as the reconfiguration scheme that, with the aid of the utilization of the sensor fault detection and fault management logic, alters the fusion filter law to achieve better performance. Each filter within a federated group is assigned, under the control of the fault management function of the master filter, to operate in either a full-reset or a no-reset mode. Failure in any input channel can be reconciliated without significant deterioration of estimation performance, or, in case the estimation is used in a subsequent control action, without loss of equipment functionality.

6. Nomenclature

B	bottom flow rate of the distillation column (mol/s)
D	distillate flow rate (mol/s)
e	estimation error
F	feed flow rate (mol/s)
K	Kalman filter gain
L_s	liquid flow rate of stripping section (mol/s)
L_r	liquid flow rate of rectifying section (mol/s)
M_i	molar holdup at tray i (mol)
n_c	number of distillate components
N_T	total number of trays in the column
N	number of ideal VLE's of the column
N_j	the j^{th} tray of the column
N_f	the feed tray of the column
p	pressure (bar)
R	reflux ratio
$\mathbf{P}_i$	estimation error covariance assigned to the local filter i
$\mathbf{P}_f$	fused estimation error covariance
$\mathbf{Q}$	process noise covariance
$\mathbf{R}$	measurement noise covariance
T	temperature (K)
t	time (s)
v	measurement noise
w	process noise
V_s	vapor flow rate of stripping section (mol/s)
V_r	vapor flow rate of rectifying section (mol/s)
$x_{i,j}$	state variable: liquid composition of component i at stage j $(mole\,fraction)$
$\hat{x}_{j\|k}$	estimate of the state at sample time j

given the output measurements up to sample time k

y_i measurement function provided by sensor i

$y_{i,j}$ vapor composition of component i at stage j $(mole\, fraction)$

z_i feed molar composition of component i $(mole\, fraction)$

$\bar{F}_k$ the Jacobian

α_j relative volatility of component j with respect to ethanol

β_i information-sharing factor imposed by the i^{th} local filter

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