

GRAPH THEORY APPLICATION FOR INVESTIGATING AGROECOSYSTEMS EFFECTED BY EXTREM WEATHER CONDITIONS

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Abstract. An agro-ecosystem is directed by the interactions among the populations living in it and depending on many abiotic factors. It is needed to investigate as many factors as possible. People are also interested in the effects of predicted climate change, experienced climate variability and frequently present extremal weather conditions nowadays. Elements of the system have direct and indirect influence on each other. Indirect or hidden types of interactions can not be expressed as different kinds of material flow. It would be also nice to combine the optimisation of the proficiency and environmental protection together with the forecast risk, damages and profit. When extending the already existing models, they become more complex and immense, so simulation and monitoring are not enough for examining and describing the whole interaction process. Application of graph theory - using well known graph theory theorems and having the help of computers - is especially powerful for controlling huge systems that are difficult to survey by other existing methods. Using informatics and electronics agricultural production can be controlled through a complex system, which integrates biological, technological and economical factors.

Keywords: *agriculture, climate, ecosystem, food web, graph, model*

Introduction

An agro-ecosystem is a highly complex system of nature, several different models have already been introduced to investigate it. In ecological research we can distinguish three main trends [14]: describing synbiological patterns and trying to explore their background patterns [22][32][37][38][40][41]; focusing on some alternative hypotheses of a hypotheses system through firmly controlled experiments [4][35][36]; and theoretical modelling by mathematical description, when only a fragment of the available knowledge is used [15][26]. These approaches have many advantages, but one of their disadvantage is that it limits the systems complexity.

The interactions in a food web and the structure of a food web have already been studied [15][16][17]. Systems of soil-plant-weather models were also constructed [8][24][26], they were examined empirically, as well [5]. To analyse how the examined agro-ecosystem is functioning, correct simulation models of the complex food web system and continuous monitoring of the processes [1][6][7] were needed. During the simulation there were examined also extremal events [13]. When extending the model, it integrates biological, technological and economical factors and joins the natural circumstances, so it becomes very complex. Agricultural production meeting several requirements such as „precision and sustainability” became more and more in focus. Nowadays investigation of influences explained by climate change and climate variability is of interest and needed, too [33]. Simulation and monitoring are not

enough. We need to ask for the help of other methods. As in the literature there are plenty of excellent models which describe certain parts of processes quite exactly [9][23][27], our aim was to create a model which describes the whole interaction process: tracking and tracing the effect of an element in an agro-ecosystem. It is also useful in case detailed data are missing, or when extending the model in case more complex data are available. These days discrete mathematics, algorithms and graph theory are very often used in different areas of mathematics, physics, chemistry, molecular biology, economy and other sciences. [3][18][25] and even in ecology [29][42][43]. Informatics and electronics are more and more often applied in different branches of sciences and in agriculture, as well [17][39]. We show that for investigation of several indirect or hidden types of interactions that can not be directly expressed, well known graph theory theorems can be used. There are several softwares for designing and testing networks: e.g. PIGALE (Public Implementation of a Graph Algorithm Library and Editor), Algorithmic Solutions Problem GmbH: LEDA [10], a program dealing with different kinds of e.g. travelling salesman problems [11], an open source project LEMON: Library of Efficient Models and Optimization in Networks [12], which does combinatorial optimization for problems working with graphs and networks using graph theory algorithms in available routines and having a possibility in it of writing programs for a given problem. We only need to translate our question into a mathematical language and use algorithms describing the situation and finding the solution of the given problem. Graph theory can provide us a powerful tool to model several indirect or hidden interactions. These interactions that we integrate in our model are not contained in most existing models or not considered together in the same model. Multidisciplinary sciences are unfortunately not very popular, but very much needed in future research.

The Effect-Graph

First, we construct the graph of our agro-ecosystem. The vertices of the graph are the elements of the agro-ecosystem. For example in a very simple food web the cultivated plant is our central element, and the others can be its weeds, different types of pests and their predators. Elements of our extended system can also be soil, weather conditions like temperature or precipitation, agrotechniques (watering, fertilization). For describing the interactions between the elements of the system we use the edges of the graph. We put an edge between two elements if there is a relationship between them. We direct the edges from an element to the other one if the certain element has an effect on the other element. We allow to have edges between two elements in both directions. Every edge will get a weight showing how strongly the elements effect one another. The weight is a number that expresses the influence of one element to the other one and it is positive or negative depending on whether this effect acts positively or negatively on the element. On the graph theory language this way we obtain a weighted directed graph. This graph is called the effect graph of our agro-ecosystem [16][17]. We can extend our graphs, influence-diagrams to encode the whole agro-ecosystem into it, labelling the vertices by the quantities of the elements. This graph we call the extended effect graph of our system. It is also possible to set the effective start date and the effective end date (which show when are the relations activated or deactivated). If we are interested in more precise details, the elements of our graph can be considered as subgraphs of the effect-graph with the similar structure. Our center element, the cultivated plant as a subgraph can consist of the elements representing the relative water holding capacity, the

evapotranspiration of the plant, the biomass growth, etc. The soil element as a subgraph can consist of e.g. water content of soil, different nutrient content of it, temperature of different layers of the soil, the water run-off in it, the evaporation or the water holding capacity of soil. In the same way variables as temperature, precipitation and others can also be included. This way we can get a very complex structure of the agro-ecosystem.

Connections in effect-systems using graph theory

The (extended) effect graph can be analysed from different points of view, like climate change, plant protection and others. In these investigations the most important (centre) element is the cultivated plant, and in the other vertices there are its weeds, pests, needs and many other biotic and abiotic factors.

We would like to examine how the agro-ecosystem takes up a new structure with the quantity change of some elements, turning our attention to the 'centre', the cultivated plant. With the help of the graph we can analyse how the quantities of other elements of the system change from time to time or how some elements compete for „food” with each other, as well.

The starting values, input data of our model can be simply measured (temperature, precipitation, watering, fertilization, global radiation) or obtained by estimation or fitting (constants related to the plant, soil, ionic nutrient, speeds of bioprocesses) easily. For other kinds of data such as the daily growth of biomass, organic substances developed in soil we can use the outputs of other food web models [30]. For the quantity analysis we can use the effect graph itself. For every ecosystem we can

construct a vector v_0 where the coordinates of v_0 are the weights of the vertices in one point of time. In the next point of time the weight of a vertex is changing according to the weights of the elements that have effect on it. For the representation of the graph we can use a matrix which have information about the set of components and relationships

between them. Let A denote the matrix such that the a_{ij} entry of the matrix is the weight between the vertices i and j . We call it the weighted adjacency matrix of the agro-ecosystem. After the first step (i.e. day) their weights of the vertices – the available quantities of the elements of the system – are given by the coordinates of the vector $v_1 = Av_0$, so after the n^{th} step $v_n = A^n v_0$. In the $(n+1)^{st}$ step we have to raise matrix A

to the power of $(n+1)$, which is an easy mathematical exercise. If we write v_0 as a linear combination of the eigenvalues $a_{ij} : v_0 = \sum \lambda_i \underline{a_i}$, then $v_n = \sum \lambda_i^n \underline{a_i}$. For some practical reasoning the eigenvalues of this matrix are of absolute values at most 1.

For other problems we shall use the extended effect graph of the system. Without extensive precise mathematical definition several times we modify the labelling of the edges.

Examples of problems with their possible solution

- What happens if one of the factors is eliminated from the system? (ie. because of no precipitation, the watering prohibition or other conditions). One would think that in this case we have to assign a 0 to the appropriate coordinate of the weight vector. Unfortunately this is not enough. Indeed, the effects of other vertices can result nonzero entries to this element at later points of time, hence

we have to change the weights of its edges (interactions) to 0. This will result a full 0 row and column in the matrix. Eliminating this row and column we obtain a smaller matrix and we can use this for our model. In matrix calculus this is the same as multiplying our matrix by a projection.

- What happens, if suddenly, unexpectedly the amount of an element of the system undesiredly increases (ie. very high temperature) such that its effect is harmful to our centre? Then, of course we would like to eliminate the effect of this item to our plant. Time to time it happens that we have no direct access to this element. Then we have to examine, how we can alter the indirect interaction between these items. In our graph this can be implemented by examining all the paths from one vertex to the other and try to terminate the flow of effects on all these paths. In graph theory language we have to disconnect the two vertices. To achieve the shortest or fastest or cheapest way one can use the Menger or Ford-Fulkerson type mincut-maxflow theorems [2][20][28] with the help of e.g. `lp_maxflow` in Lemon [12].
- In case of an expected drought an earlier harvest might be decided. How to find the cheapest and fastest possible way of increasing the yield in the remaining period of time. In our effect system this requires to find the fastest way to increase the yield with our leftover sources. In graph theory language it means finding the shortest paths with largest weights that can be done with e.g. Floyd's algorithm [20]. Here we modify the labelling. The weights will represent the quotient of the effect and the expenses of the desired interactions.
- It would be desired to control or predict all the effects in our system. How can it be done if we have access to the monitoring of a few elements of the system? We would like to choose as few elements as possible to keep a close watch on. In the graph theory language this is equivalent to find a smallest covering set of elements and we can apply Gallai's theorem [2][20][28] to achieve it.
- The reverse procedure, monitoring the effects to control the elements can be a real requirement, as well. The dual of the previous method is a straight application of König's theorem about the independent sets of edges. [2][20][28]

Discussion

In order to understand how an ecological system operates, a matrix or a food web graph is suitable either for description or for analysis of the relations. Based on the problems and their solutions mentioned above, we see that this method is very likely suitable to solve many other ones. Analysing the effecting factors standing behind ecological patterns can be of interest of human society, too. It is very important in planning the future: controlling the vulnerability helps the mitigation and the adaptation for the new circumstances. Graph theory helps us to model and follow the changes of the number of the individuals and the quality of the elements of the complex system. Nowadays apart from environment protection and public health issues the expenses of the agrotechnical methods has to be considered, as well. We do not only have to analyse if the given problem has solution, but we have to examine and decide which of the possible solutions is the most effective one. Application of graph theory in other sciences is a rapidly developing area of mathematics. It is especially powerful for controlling huge systems that are difficult to survey by the existing (manual) methods.

Today's computers can carry out the necessary calculations in a reasonable amount of time. The complexity of the algorithms we used in our paper are all "easy". The methods offer new possibilities in investigating agro-ecosystems, we presented only a few examples to demonstrate the power of graph theory in this application. Predictions can be given to alternative climate change situations not experienced before. This is important in finding the sensitive points and the role of different creatures in the stability of the system. It is a possibility of decreasing the risks caused by effects of some elements in the system. Our research is still under development and generalization.

REFERENCES

- [1] Akiyama, T., Inoue, Y., Shibayama, M., Awaya, Y., Tanaka, N. (1995): Monitoring and predicting crop growth and analysing agricultural ecosystems by remote sensing, *Global Climate Change and Agriculture in the North*, 367-377
- [2] Bollobás B.: *Modern graph theory*, Springer, 2002.
- [3] C. A. van Dorp (2003): A traceability application based on Gozinto graphs, *Efita 2003 Conference*, Debrecen, 280-285
- [4] Fahim, M. A., Hassanien, M. K. & Mostafa, M. H. (2003): Relationships between climatic conditions and Potato Late Blight epidemics in Egypt during winter 1999-2001 - *Applied Ecology and Environmental Research*, 1(1-2): 159-172.
- [5] Gaál, M. (1998): Ökológiai alapú szaktanácsadási információs rendszer, *Gazdálkodás*, XLII. 50-57.
- [6] Gaál, M., Hufnagel, L. (1999): Vízi élőhelyek állapotának monitorozása poloskaközösségek alapján, *Agrárinformatika 1999*, Debrecen, 346-350
- [7] Gaál, M., Hufnagel, L. (2001): Combination of Multivariate Methods and Graphical Database Management in Service of Ecological Monitoring, *3rd EFITA 2001 agroMontpellier ENSA, Montpellier (vol 1)*, 285-290
- [8] Hansen, S., - Jensen, H.E., - Nielsen, N.E., - Svedsen, H. (1990): DAISY- Soil Plant Atmosphere System Model, *NPO-forskning fra Miljøstyrelsen Nr. A10*.
- [9] Harnos, Zs., Bussay, A., Harnos, N. (2000): Modelling climate change impacts on wheat and potato in Hungary, *Climate Change, Climate Variability and Agriculture in Europe*, 349-366
- [10] <http://algorithmic-solutions.com/enleda.htm>
- [11] <http://iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95>
- [12] <http://lemon.cs.elte.hu>
- [13] Hufnagel, L., Gaál, M. (2002): Többváltozós állapotsík rendszerek alkalmazása valós és szimulált adatsorok kezelésében, *Agrárinformatika Konferencia 2002*, Debrecen
- [14] Hufnagel, L., Gaál, M. (2005): Seasonal dynamic pattern analysis in service of climate change research (A methodical case-study, monitoring and simulation based on an aquatic insect community) – *Applied Ecology and Environmental Research*, 3(2): 107-137.
- [15] Jordán, F. (2001): Seasonal changes in the positional importance of components in the trophic flow network of the Chesapeake Bay, *Journal of Marine Systems*
- [16] Jordán, F., - Molnár, I., (1999): Reliable flows and preferred patterns in food webs. *Evol. Ecol. Res.* 1: 591-609.
- [17] Jordán, F.: Comparability: The key to the application of food web research, *Applied Ecology and Environmental Research*, 2003/1-2:1-18
- [18] Jorgensen, E., Höhle, M., Højsgaard, S. (2003): Diagnostic testing: Model estimation and decision support using graphical models, *Efita 2003 Conference*, Debrecen, 760-767

- [19] Junttila, O. (1995): Plant adaptation to temperature and photoperiod, *Global Climate Change and Agriculture in the North*, 251-260
- [20] Katona, Gy., Recski, A., Szabó, Cs.: *A számítástudomány alapjai*, Typotex Kiadó, Budapest, 2002
- [21] Kaukoranta, T. (1995): Impact of global warming on potato late blight: risk, yield loss and control, *Global Climate Change and Agriculture in the North*, 311-328
- [22] Kharkwal, G., Mehrotra, P., Rawat, Y. S. & Pangtey, Y. P. S. (2004): Comparative study of herb layer diversity in pine forest stands at different altitudes of Central Himalaya - *Applied Ecology and Environmental Research*, 2(2): 15-24.
- [23] Kleemola, J., Karvonen, T. (1995): Modelling growth and nitrogen balance of barley under ambient and future conditions, *Global Climate Change and Agriculture in the North*, 299-310
- [24] Kniesel L. W.G., (ed): *CREAMS A Field Scale Model for Chemicals, Runoff, and Erosion from Agricultural Management Systems*, USA Dept. of Agriculture, Conservation Research Report No. 26., 1980
- [25] Lábos Elemér (2005): Neuronhálózatok kaotikus egységekből. Szintézis és analízis feladatok, VII. Magyar Biometriai és Biomatematikai Konferencia, Budapest
- [26] Ladányi, M., Horváth, L. Gaál, M. & Hufnagel, L.: An Agro-ecological Simulation Model system, *Applied Ecology and Environmental Research*, 2(1-2): 52-79.
- [27] Ladányi, M.,-Hufnagel, L. (2003): A Fenology Model nested in an Ecosystem Model for Agroecological Processes – EFITA 2003, Debrecen, Hungary
- [28] Lovász, L.: *Combinatorial Problems and Exercises*, Akadémiai Kiadó, Budapest, 1979
- [29] Luczkovich, J. J., Borgatti, S. P., Johnson, J. C., Everett, M. G. (2003): Defining and measuring trophic role similarity in food webs using regular equivalence, *J. theor.Biol.* 220, 303-321
- [30] M. Ladányi, É. Erdélyi, A Révész: An ecosystem model to simulate agroecological processes, *Efita 2003*, Debrecen, Hungary
- [31] Máthé-Gáspár, G., Kovács, G. J. (2003): Use of simulation technique to distinguish between the effect of soil and weather on crop development and growth - *Applied Ecology and Environmental Research*, 1(1-2): 87-92.
- [32] Mehrotra, P., Kharkwal, G. & Pangtey, Y. P. S. (2004): Ecological implication of plant traits, strategies and competitive abilities of herbs - *Applied Ecology and Environmental Research*, 2(2): 1-13.
- [33] Mela, T. J. N. (1995): Northern agriculture: constraints and responses to global climate change, *Global Climate Change and Agriculture in the North*, 229-234
- [34] Őszi, B., Ladányi, M., Hufnagel, L. (2005): Population dynamics of the sycamore lace bug, *Corythucha ciliata* (Say) (Heteroptera: Tingidae) in Hungary - *Applied Ecology and Environmental Research*, 4(1): 135-150.
- [35] Patel, S. R., Awasthi, A. K. & Tomar R. K. S. (2004) Assessment of yield losses in mustard (*Brassica juncea* L.) due to mustard aphid (*Lipaphis erysimi* Kalt.) under different thermal environments in Eastern Central India - *Applied Ecology and Environmental Research*, 2(1): 1-15.
- [36] Précseányi, I. (1995): Alapvető kutatásszervezési, statisztikai és projektértékelési módszerek a szupraindividuális biológiában. (Elementary research arranging, statistical and project evaluation methods in supraindividual biology) *KLTE*, Debrecen
- [37] Rédei, D., Harmat, B. & Hufnagel, L. (2004): Ecology of the *Acalipta* species occurring in Hungary (Insecta: Heteroptera: Tingidae) (Data to the knowledge of the ground-living of the Heteroptera of Hungary, No.3) - *Applied Ecology and Environmental Research*, 2(2): 73-90.
- [38] Rifaat, H. M. & Yosery, M. A. (2004): Identification and characterization of rubber degrading Actinobacteria - *Applied Ecology and Environmental Research*, 2(1): 63-70.
- [39] Salga, P., Herdon, M., Tóth, T. (2004): Intelligent neural grid, XIII. Agrarian Perspectives Conference and Information Technology Workshop, Prague

- [40] Salga, P., Szilágyi, R., Tóth, T., Pokol, A. (2004): Application of CNN-GRID technology, Information Systems for Agriculture, Forestry and Rural Areas – enlargement of EU 10th International Conference, Seč u Shrudimi, Czech Republic
- [41] Spellenberg, I. F. (1991): Monitoring ecological change, Cambridge University Press, Cambridge
- [42] Ulanowitz, R. E. (1983): Identifying the structure of cycling in ecosystems. *Math.Biosci.*65, 219-237
- [43] Ulanowitz, R. E. (1991): Ecosystem flow networks: loaded dice, *Math. Biosci.*103, 45-68
- [44] Willows, R., Connell, R. (eds.) (2003): Climate adaptation: Risk, uncertainty and decision-making, UK Climate Impacts Programme Technical Report, UKCIP Oxford