MATHEMATICAL MODEL FOR PREDICTING RICE YIELD IN THE PRESENCE OF RICE BLAST DISEASE AND ITS CONTROL MEASURES

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Abstract. Rice is one of the major foods for over half of the world's population. Enhancing rice yield is crucial to achieving and sustaining food security, but rice production faces numerous challenges. One of the challenges is rice blast disease, which can lead to over 50% yield loss if not properly controlled. We developed a mathematical model to study the dynamics of rice yield, taking into consideration rice blast disease and its control measures. The essential epidemiological features of the model, such as the basic reproduction number, equilibrium points, etc., are determined and analysed. Numerical illustrations are conducted using the rice yield data from Nigeria as a case study. Model fitting and parameter estimation are carried out using real rice yield data from Nigeria. The effects of the possible control measures on rice yield are analysed. Useful recommendations on how to improve rice yield in the presence of rice blast disease, considering control measures, are presented for better management.

Keywords: *epidemiology, stability analyses, simulations, model fitting, parameter estimation*

Introduction

Rice is a major staple food consumed by over 50% of the world population, and consequently contributes significantly to total caloric intake (Katsantonis et al., 2017; Simkhada and Thapa, 2022; Abeysekara and Rathnayake, 2024). Its production and consumption patterns have a significant impact on global food security (Chauhan et al., 2017; Abeysekara and Rathnayake, 2024). Asia is the largest producer and consumer, with countries like China, India, and Indonesia as major markets, but Sub-Saharan Africa is experiencing an increase in both per capita and total rice consumption, according to a paper published on global rice consumption trends (Chauhan et al., 2017; Joseph et al., 2023).

Despite rice playing a crucial role in achieving and sustaining global food security, its production is being affected by several factors (Chauhan et al., 2017). Some of the factors that are affecting the effective production of rice, especially in developing countries, include pests and diseases, climate change, labour, seed, availability of fertile land, fertilizer, socio-economic factors, etc. (Bissah et al., 2022; Diaz et al., 2022; Salam et al., 2024). These factors can affect both the quantity and quality of rice yield.

Several diseases affect rice yield, but one of the major diseases that can lead to over 30% loss of yield is Magnaporthe grisea, also known as rice blast fungus (Wilson and Talbot, 2009; Faivre-Rampant et al., 2013; Chauhan et al., 2017). A fungus named *Pyricularia orizae* is the cause of rice blast disease (RBD). The life cycle of rice begins with a three-celled conidium spore landing on a rice leaf, germinating into a germ tube, and forming a specialized penetration structure that generates turgor pressure to penetrate the leaf. Inside the leaf, the fungus grows as invasive hyphae, eventually causing lesions and switching to necrotrophic growth to produce new conidia, completing the infection cycle and allowing for secondary spread (Garcia et al., 2024). The fungus survives on crop debris and can infect plants at any growth stage.

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RBD is spread via windborne spores, on infected plant material and grain, and in water. Spores can also be transmitted through indirect contact with people and machinery working on an infected crop (Ding et al., 2024). The disease can be controlled through the implementation of any or a combination of the following control measures: chemical control (use of appropriate fungicide), mechanical control, biological control, and use of resistant varieties of seed (Katsantonis et al., 2017; Simkhada and Thapa, 2022; Bochalya et al., 2024). A rice plant variety becomes resistant to rice blast disease when it carries specific genes that allow it to defend against the pathogen that causes rice blast disease otherwise it is susceptible. Effective fungicides for rice blast disease include systemic options like tricyclazole, which inhibits melanin production in the fungus, and other effective chemicals like (azoxystrobin and difenoconazole), and (azoxystrobin and tebuconazole) combinations. Other fungicides that have shown efficacy include hexaconazole, carbendazim, propiconazole, and fluopyram (Kongcharoen et al., 2020).

Mathematical models have been successfully applied to study real-life problems in epidemiology, food insecurity, etc. (Van Maanen and Xu, 2003; Collins and Govinder, 2014; Collins and Duffy, 2016, 2018; Kirtphaiboon et al., 2021; Tabonglek and Khan, 2023). By developing an appropriate mathematical model for the problem, possible predictions on the future dynamics of the problem, which is crucial for better policy implementation and better management, can be obtained. For instance, Tabonglek et al. (2022) use a mathematical model to study rice blast disease dynamics, taking spore dispersion due to climate factors (rain, air temperature, and relative humidity) into consideration. Research by Kirtphaiboon et al. (2021) used a mathematical model to study rice blast disease dynamics by taking into account tropical climate conditions. Specifically, they use a dynamical mathematical model to simulate the severity of rice blast disease as the climatic conditions change. Another research by Tabonglek and Khan (2023) extended the rice blast mathematical model by Tabonglek et al. (2022) to study the severity of rice blast disease. Other important studies on rice blast disease include those by Katsantonis et al. (2017); Ding et al. (2024), and Kim et al. (2020). Undoubtedly, these studies have made an important contribution to understanding the dynamics of rice blast disease using a mathematical model. However, none of them have considered a mathematical model to analyse rice yield due to rice blast disease using real data on rice yield as a case study. This paper aimed to fill the knowledge gap by formulating a mathematical model to analyse the dynamics and control of rice blast disease and its effects on rice yield, using rice yield per Hectare in Nigeria for illustration. The results of this study are expected to improve understanding of how to enhance rice yield in the presence of RBD.

Methods

Model development

The mathematical model for the rice yield in the presence of RBD and control is made up of two parts. The first part of the model is the mathematical model describing the population dynamics of rice plants in the presence of RBD and its control measures. This first part is an epidemiological model, and thus, the basic assumptions of the epidemiological models for plant diseases will be considered. The second part of the model involves developing a mathematical model that estimate rice yield from (first part of the model) the rice blast epidemiological model.

Rice blast disease model

The mathematical model development for rice plant population dynamics under the attack of RBD is derived based on the following assumptions. RBD starts from the canopy host, which can be healthy or infected. Let S(t) represent the population of susceptible (healthy) rice plants at time t, and I(t) denote the population of infected rice plants at time t. To reduce the infected rice plants, we consider chemical control by letting T(t) denote the population of treated rice plants at time t. The susceptible rice increases by growing logistically at a growth rate of r to a carrying capacity K and decreases through contact with I(t) at a rate β. The I(t) can die, or removed, or destroyed at a rate μ. Incorporating other control measures simultaneously will reduce the disease and enhance rice yield as follows. Fertilizer application and irrigation enhance the S(t) at a rate ρ_1. Implementing the use of resistance varieties reduces the contact rate of the disease at a rate ρ_2. The I(t) can be reduced using a mechanical control method at a rate ρ_3. Treatment of I(t) using appropriate chemical control at a rate σ can also be implemented, and T(t) can recover at a rate γ. Taking these assumptions into consideration, the following RBD model is obtained

$$
\begin{aligned}
\frac{dS}{dt} &= (1+p_1)r_1 S\left(1-\frac{S}{K}\right)-(1-p_2)\beta SI+\gamma T,\\
\frac{dI}{dt} &= (1-p_2)\beta SI-(\mu+\rho_3+\sigma)I,\\
\frac{dT}{dt} &= \sigma I-(\mu+\gamma)T.
\end{aligned}
\qquad \text{(Eq.1)}
$$

The meaning of variables and parameters are presented in *Table 1*.

Table 1. *Variables and parameters for model Eq. (1)*

Variables	Meaning	Units
N	*Total population of rice plants*	*Population Ha−1*
S	*Susceptible rice plants*	*Population Ha−1*
I	*Infected rice plants*	*Population Ha−1*
T	*Treated rice plants*	*Population Ha−1*
Parameters	**Meaning**	**Units**
K	*Carrying capacity of rice plant population*	*Population Ha−1*
β	*Contact rate of S(t) with I(t)*	*Ha Year−1 Population−1*
r	*Intrinsic growth rate of rice plants*	*Year−1*
μ	*Mortality rate of infected rice plants*	*Year−1*
ρ_1	*Enhance the growth rate of S(t) due to fertilizer and irrigation*	*Dimensionless*
ρ_2	*Rate of reduction of β due to the use of resistance varieties*	*Dimensionless*
ρ_3	*Rate of reduction of I(t) by mechanical control*	*Dimensionless*
σ	*Rate of treatment of I(t) using chemical control*	*Year−1*
γ	*Recovery rate of T*	*Year−1*

Rice yield model

The total quantity of rice yield Y(t) in Tonne Per Hectare (Tonne/Ha) depends on the total population of rice plants N(t) per Hectare. Mathematically, we can say that the quantity of rice total yield Y(t) per Hectare is proportional to the total population of rice plants N(t) per Hectare. Since the total population of rice plants N(t) per Hectare comprises susceptible rice population S(t), infected rice population I(t), and treated rice population T(t), and each has a different yield rate depending on the severity of the disease on the plant. The total quantity of rice yield Y(t) (Tonne/Ha) will depend on the proportion of S(t), I(t), and T(t) per Hectare. Therefore, the mathematical model for the quantity of susceptible rice yield Y (Tonne/Ha) can be described as

$$Y_1 = \alpha_1 S \qquad \text{(Eq.2)}$$

where α1 is a proportionality constant that converts the susceptible rice population S(t) to susceptible rice yield $Y_1(t)$ Tonne per Hectare. Similarly, the mathematical model for quantity of infected rice yield $Y_2(t)$ (Tonne/Ha) and quantity of treated rice yield $Y_3(t)$ (Tonne/Ha) can be described as

$$Y_2 = \alpha_2 I, \qquad \text{(Eq.3)}$$

$$Y_3 = \alpha_3 T, \qquad \text{(Eq.4)}$$

where α_2 and α_3 are proportionality constants that converts the infected rice population I(t) and treated rice population T(t) respectively to infected rice yield $Y_2(t)$ (Tonne/Ha) and treated rice yield $Y_3(t)$ (Tonne/Ha). From *equations* (*Eq.2)–(Eq.4*), the total quantity of rice yield Y(t) (Tonne/Ha) can be determined as

$$Y = Y_1 + Y_2 + Y_3 \qquad \text{(Eq.5)}$$

From *equation (Eq.5)*, it can be seen that to determine rice yield Y(t), it suffices to determine the variables (S, I, T) and the proportionality constants (α_1, α_2, α_3). Therefore, the analysis of model (*Eq. 1*) is very crucial in determining rice yield Y.

Analyses and results

Analysis of rice blast disease model (Eq.1)

The analysis of the rice blast disease *model* (*Eq.1*) will be conducted by determining and analyzing the features of the *model* (*Eq.1*). The results of the analysis will be helpful in understanding the dynamics of the rice blast disease (RBD). For simplicity, the following assumptions are made in the *model (Eq.1)*: $k_1 = (1+\rho_1)r,\ \ k_2 = (1-\rho_2)\beta,$ $k_3 = \mu + \rho_3 + \sigma,\ \ k_4 = \mu + \gamma.$

Equilibrium points of the model (Eq.1)

Model (Eq.1) has three distinct equilibrium points, and they are presented as follows: a trivial equilibrium given by

$$(S^{o}, I^{o}, T^{o}) = (0, 0, 0), \tag{Eq.6}$$

a disease free-equilibrium (DFE) given by

$$(S^{0}, I^{0}, T^{0}) = (K, 0, 0), \tag{Eq.7}$$

and an endemic equilibrium (EE) given by

$$(S^{*}, I^{*}, T^{*}) = \left(\frac{k_2}{k_1}, \quad \frac{k_1 k_3 k_4 (K k_2 - k_3)}{k_2^2 K(\mu\,\sigma + (\mu + \rho_3) k_4)}, \quad \frac{\sigma\, I^{*}}{k_4}\right). \tag{Eq.8}$$

The basic reproduction number (R_0)

The basic reproduction number (R_0) is a threshold quantity in epidemiological models that helps to determine the rate of spread of an infection in a given population. The R_0 of *model (Eq.1)* is computed using the next generation matrix method (van den Driessche and Watmough, 2002) and is given by

$$R_0 = \frac{k_2 K}{k_3} = \frac{(1 - \rho_2)\beta K}{\mu + \rho_3 + \sigma}. \tag{Eq.9}$$

The value of R_0 indicates if an outbreak will be persist or eradicated in the population. Specifically, if $R_0 > 1$, the outbreak persist, while if $R_0 < 1$, the outbreak is eradicated (Tien and Earn, 2010). From *equation (Eq.9)* the endemic equilibrium exists only if $R_0 > 1$.

Stability analysis of model (1)

The stability of *model (Eq.1)* about its various equilibrium points describes the dynamics of the model (Liao and Wang, 2011). Thus, the results of stability analyses of *model (Eq.1)* about the equilibrium points are presented in Theorem 1.

Theorem 1. (i) *Model (Eq.1)* is unstable about the trivial equilibrium.

(ii) *Model (Eq.1)* is locally stable about the DFE provided that $R_0 < 1$.

(iii) For $R_0 > 1$, *model (Eq.1)* is locally stable about the EE.

The proof of Theorem 1 is established in Appendix section. The epidemiological implications of this stability results of Theorem 1 is as follows: (i) The instability of *model (Eq.1)* about the trivial equilibrium implies that it will be difficult for both the susceptible and infected rice plants to go into extinction. (ii) The stability of *model (Eq.1)* about the disease free equilibrium shows that RBD could be eliminated from the system using control measures if the $R_0 < 1$. (iii) The stability of model (1) about the endemic equilibrium implies that if the control measures are not effective enough to reduce the R_0 below unity, the disease will persist provided the $R_0 > 1$.

Numerical simulations results

In this section, further analysis on *models (Eq.1) and (Eq.5)* is carried out using numerical simulations. A case study of rice yield in Nigeria is considered in the simulations. Nigeria is one of the countries that is currently experiencing poor rice yield (Tonne/Ha). The rice yield (Tonne/Ha) in Nigeria from 2014 to 2024 extracted from the

International Production Assessment Division (IPAD), Foreign Agricultural Service, US Department of Agriculture, presented in *Figure 1*, is used in the simulation (IPAD, 2025). The parameter values used in the numerical simulations are presented in *Table 2*.

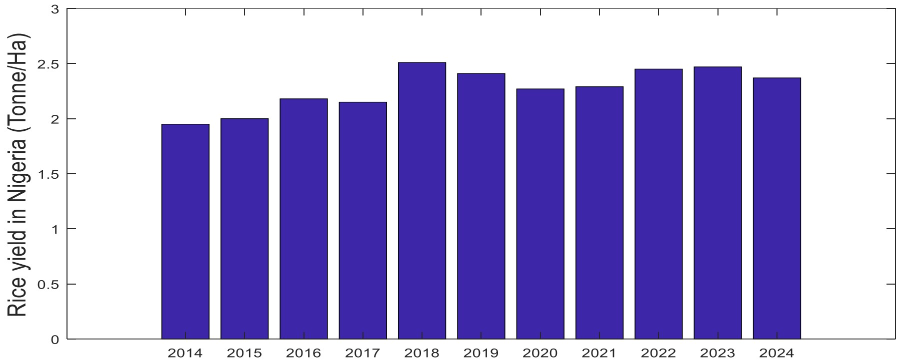

Figure 1. *Bar chart illustrating rice yield in Nigeria (Tonne/Ha) from 2014 to 2024 (IPAD, 2025)*

Table 2. *Parameter values used for numerical simulations*

Parameter symbols	Value	Source
r	*0.16*	*Estimated*
K	*10.0*	*Estimated*
ρ_1	*0.9899*	*Estimated*
ρ_2	*0.2891*	*Estimated*
ρ_3	*0.5781*	*Estimated*
β	*0.6813*	*Estimated*
γ	*0.0011*	*Estimated*
μ	*0.0010*	*Estimated*
σ	*0.0974*	*Estimated*

Model fitting and parameter estimation

To use *model (Eq.5)* to study rice yield in Nigeria, we fitted the model to the data on rice yield in Nigeria from 2014 to 2024. The result of model fitting is presented in *Figure 2* while the parameter values estimated from the model fitting are given in *Table 2*. *Figure 2* shows that *models (Eq.1) and (Eq.5)* give a reasonable representation of rice yield in Nigeria and therefore can be used to study the dynamics of rice yield in Nigeria and also make possible future predictions of rice yield in Nigeria.

Model predictions

Using the estimated parameter values, a possible long-term model prediction of rice yield in Nigeria (Tonne/Ha) is determined and is presented in *Figure 3*. From the figure, it can be seen that the overall rice yield (Tonne/Ha) is increasing, but at a much slower rate when compared with rice yield (Tonne/Ha) in some Asian countries like China. For instance, the rice yield will attain about 3.5 (Tonne/Ha) in 20 years, which is small when compared with some Asian countries that have rice yields of about 4-7 (Tonne/Ha). One of the major implications of this predicted low rice yield in Nigeria is food insecurity.

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Thus, we shall analyse in detail the factors affecting the rice yield based on our model formulation.

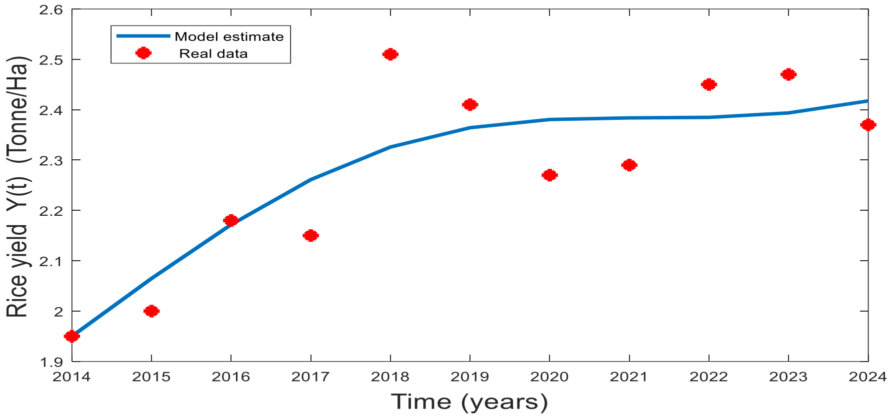

Figure 2. *Model fitting of rice yield in Nigeria (Tonne/Ha) from 2014 to 2024*

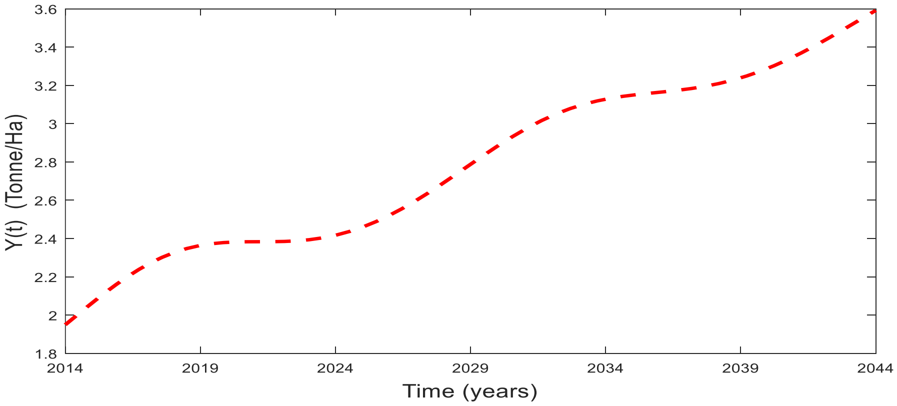

Figure 3. *Plot showing possible model prediction of rice yield in Nigeria (Tonne/Ha) using estimated parameter values*

Effect of control measures on rice yield (Tonne/Ha)

One of the major diseases that affects rice yields is RBD. This disease can reduce yield by 30% if it is not properly controlled. Here, we shall be investigating the effects of control measures on rice yield (Tonne/Ha) using *models (Eq.1) and (Eq.5)*. The effect of each control parameter is determined numerically by varying the control parameter while other parameters are kept fixed. The plot of the various values of the control parameter demonstrates the effects of the control parameter.

Mechanized fertilizer application and proper irrigation are one of the ways of enhancing rice yield. This important factor is captured in our model formulation by a parameter ρ_1, which describes the growth rate of S(t) due to fertilizer application and irrigation. The effects of ρ_1 on rice yield in Nigeria (Tonne/Ha) using the estimated parameter value are presented in *Figure 4*. The figure shows that increasing fertilizer application and irrigation increase total rice yield in the presence of RBD. Specifically, we discovered that a 20% increase in fertilizer application and irrigation leads to an increase of about 0.2 (Tonne/Ha) in rice yield in the presence of RBD. Effective fertilizer application and proper irrigation can lead to about 3.6 (Tonne/Ha) in the presence of RBD in 20 years based on our formulation. Although fertilizer application and irrigation improve rice yield (Tonnes/Ha), it is still not sufficient when compared with rice yields from some Asian countries.

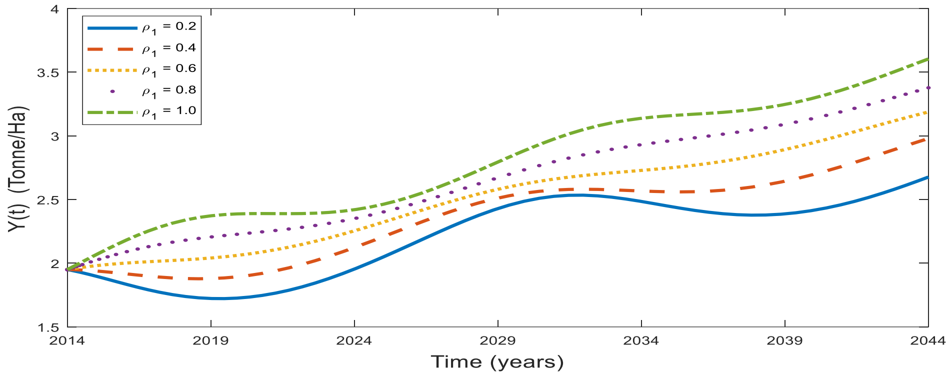

Figure 4. *Plot showing the effects of fertilizer application and irrigation (ρ_1) on rice yield (Tonne/Ha) using estimated parameter values*

One of the effective methods of reducing RBD is through the use of resistant varieties. The reduction in contact rate due to the use of resistance varieties is captured in our model formulation as ρ_2. Plot showing the effects of ρ_2 on rice yield (Tonne/Ha) using estimated parameter values is presented in *Figure 5*. The figure shows that enhancing the resistant varieties increases rice yield. Ultimately, the effective implementation of resistant varieties will lead to about 10.0 (Tonne/Ha) rice yield. The yield obtained through the effective implementation of resistant varieties is high when compared with rice yields from some Asian countries. Therefore, the effective implementation of resistant varieties is strongly recommended for achieving maximum rice yield.

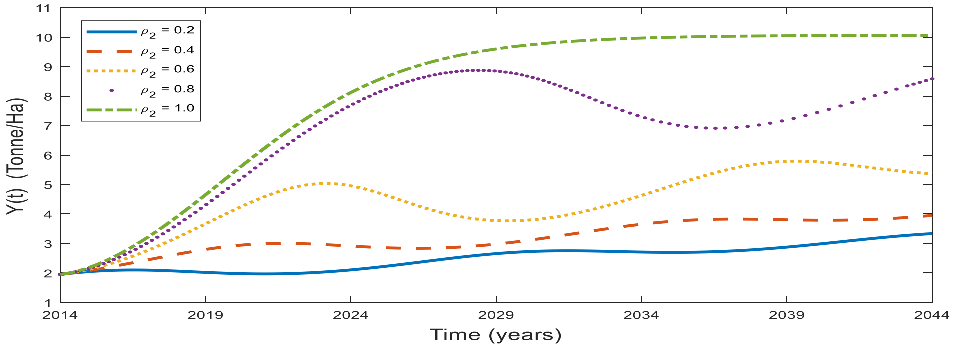

Figure 5. *Plot showing the effects of use of resistant varieties (ρ_2) on rice yield (Tonne/Ha) using estimated parameter values*

Another effective method of controlling RBD is by mechanical control. This involves removing the infected rice plants to reduce the spread of the disease. This control measure is captured in our model as ρ_3. Plot showing the effects of ρ_3 on rice yield (Tonne/Ha) using estimated parameter values is presented in *Figure 6*. The figure shows non-linear

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dynamics, but on average reveals that increasing the rate of mechanical control increases rice yield (Tonne/Ha). Effective implementation of mechanical control leads to a total rice yield of about 4.3 (Tonne/Ha). Even though mechanical control improves rice yield, there is still a need for improvement when compared with rice yield from some Asian countries.

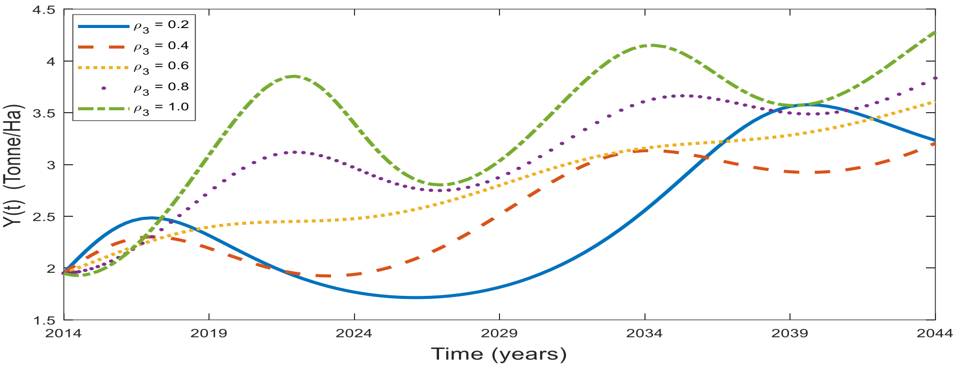

Figure 6. *Plot showing the effects of mechanical control (ρ_3) on rice yield (Tonne/Ha) using estimated parameter values*

Chemical control using an appropriate fungicide is one of the effective methods of controlling RBD. This control measure involves applying the chemical on infected rice plants to reduce the spread of the disease. This control measure is captured in our model as σ. Plot showing the effects of σ on rice yield (Tonne/Ha) using estimated parameter values is presented in *Figure 7*. From the figure, we discovered that an increase in the rate of chemical control leads to a significant increase in rice yield. Specifically, a 10% increase in the rate of chemical control leads to about 1.0 (Tonne/Ha) increase in rice yield. Effective implementation of chemical control will result in about 10.0 (Tonne/Ha) rice yield. The yield obtained through the effective implementation of chemical control is high when compared with rice yields from some Asian countries. Therefore, effective implementation of chemical control is strongly recommended for achieving maximum rice yield.

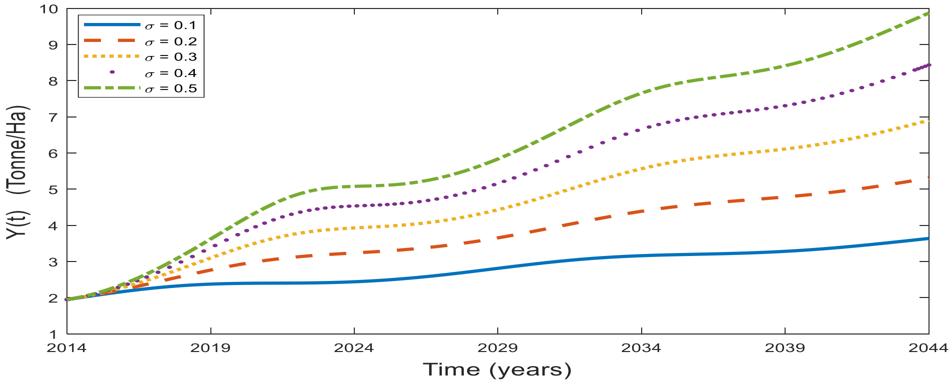

Figure 7. *Plot showing the effects of chemical control (σ) on rice yield (Tonne/Ha) using estimated parameter values*

Discussion

Rice is one of the major foods for over 50% of the world population. It contributes significantly to total caloric intake. Its production and consumption have a significant impact on global food security. Despite the major role of rice in achieving and sustaining food security, its production is being affected by several factors. Some of the factors that are affecting the effective production of rice, especially in developing countries, include pests and diseases, climate change, etc. These factors can affect both the quantity and quality of rice yield. Several diseases affect rice yield, but one of the major diseases that can lead to over 30% loss of yield is Magnaporthe grisea, also known as rice blast fungus. Understanding how to control and manage RBD is very crucial to enhance rice yield.

A mathematical model for rice yield that takes into consideration RBD and its control measures was developed. The epidemiological features of the model, such as the basic reproduction number and equilibrium points are determined and analysed accordingly. For instance, when the basic reproduction number is less than one, RBD can be eradicated in the presence of control measures, leading to enhanced rice yield. However, if the control measures are not implemented effectively, such that the basic reproduction number is greater than one, RBD persists, leading to poor rice yield.

A numerical illustration of the dynamics of rice yield using rice yield in Nigeria as a case study was conducted. Nigeria is one of the countries that currently has poor rice yields. Rice yield in Nigeria is about 2-3 (Tonne/Ha) is small when compared with some Asian countries that have about 4-7 (Tonne/Ha). By fitting the *model (Eq.1)* to real data on rice yield (Tonne/Ha), the parameters of the *model (Eq.1)* were estimated. Using the estimated parameters and model, the possible rice yield (Tonne/Ha) was predicted. This prediction is crucial for better policy implementation that will enhance rice yield.

The effects of control measures on rice yield were analysed numerically. Increasing fertilizer application and irrigation were shown to increase total rice yield in the presence of RBD. Effective fertilizer application and proper irrigation can lead to about 3.6 (Tonne/Ha) in the presence of RBD in 20 years, based on our formulation. Even though fertilizer application and irrigation improve rice yield, it is still not adequate when compared with rice yield from some Asian countries. It was also discovered that enhancing the rice-resistant varieties increases rice yield. Ultimately, the effective implementation of resistant varieties will lead to about 10.0 (Tonne/Ha) rice yield. The yield obtained through the effective implementation of resistant varieties is high when compared with rice yields from some Asian countries. Therefore, the effective implementation of resistant varieties is strongly recommended for achieving maximum rice yield. This result agrees with findings in the literature which state that resistance significantly reduces yield losses caused by the blast fungus, leading to higher crop yield and a more stable food supply (Nalley et al., 2017). For the mechanical control, we discovered that effective implementation of mechanical control leads to a total rice yield of about 4.3 (Tonne/Ha). Even though mechanical control improves rice yield, there is still a need for improvement when compared with rice yield from some Asian countries. We also discovered that effective implementation of chemical control will lead to an increase in the total rice yield to about 10.0 (Tonne/Ha). The total yield obtained through the effective implementation of chemical control is high when compared with rice yield from some Asian countries. Therefore, effective implementation of chemical control is strongly recommended for achieving maximum rice yield. This result agrees with findings by Katsantonis et al. (2017) which discovered that Chemical control of this disease remains the most effective rice blast management method.

Conclusion

Rice blast disease has been shown to be a very serious disease affecting rice plants (Kirtphaiboon et al., 2021; Tabonglek et al., 2022; Tabonglek and Khan, 2023). This study aimed to estimate the impact of various control measures on rice yield using mathematical models. The models considered in this study are new and therefore can point to further research in this area. In general, these results could aid in better policy implementation for the management of rice blast disease and other similar rice diseases, leading to improved rice yield and consequently achieving food security globally.

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APPENDIX

Proof of Theorem 1

Proof. (i) The eigenvalues of the Jacobian of *model (Eq.1)* about the trivial equilibrium are:

$$\lambda_1 = k_1,$$
$$\lambda_2 = -k_2,$$
$$\lambda_3 = -k_4.$$

Since one of the eigenvalues $\lambda_1 = k_1 > 0$, we conclude that *model (Eq.1)* is unstable about the trivial equilibrium.

(ii) The eigenvalues of the Jacobian of *model (Eq.1)* about the DFE are:

$$\lambda_1 = -k_1,$$
$$\lambda_2 = -k_4,$$
$$\lambda_3 = Kk_2 - k_3 = k_3(R_0 - 1).$$

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Obviously, $\lambda_1 < 0$, and $\lambda_2 < 0$, whereas $\lambda_3 < 0$ if and only if R0 < 1. Thus, we conclude that *model (Eq.1)* is locally stable about the DFE provided R0 < 1.

(iii) The Jacobian of *model (Eq.1)* about the EE is

$$J* = \begin{pmatrix} a_{11} & k_3 & \gamma \\ k_2 I^* & 0 & 0 \\ 0 & \sigma & -k_4 \end{pmatrix},$$

where $a_{11} = -k_1 I^* + k_1 \left(\frac{2}{R_0} - 1\right)$. Since the endemic equilibrium exists only when R0 > 1, the expression a11 can be simplify to $-k_1 I^*$ by taking R0 = 2. Consequently, the characteristic polynomial of the Jacobian matrix (Eq.1) becomes

$$a_3\lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 = 0,$$

where $a_3 = 1$, $a_2 = k_4 + k_2 I^*$, $a_1 = k_2 I^*(k_2 + k_4)$ and $a_0 = k_2 I^*(\mu\sigma + (\mu + \rho_3)k_3)$. The Ruth-Huwitz criterion for stability requires that all the coefficients of the characteristic polynomial be positive and a2a1 -a3a0 > 0. Clearly the coefficients a3 > 0; a2 > 0; a1 > 0; a0 > 0. Evaluating the second criterion gives a2a1 -a3a0 = $k_2 I^*$(k2 + $k_2 I^*$(k3 + k4) +) > 0. Since the Ruth-Huwitz criterion is satisfied, we conclude that *model (Eq.1)* is locally stable about the EE.

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