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MODELLING HYSTERESIS WITH MEMRISTORS

[1]Sándor Csikós, [1]Ádám Bálint, [1]József Sárosi

[1]University of Szeged Faculty of Engineering Mechatronics and Automation Institute, Moszkvai krt 9, 6724, Szeged, Hungary,
e-mail: csikos-s@mk.u-szeged.hu

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ABSTRACT

In the realm of electronics, the foundational passive components—resistors, inductors, and capacitors—are well-established. However, in 1971, Leon Chua introduced a theoretical fourth element, the memristor, identified by its distinctive characteristic of memristance and its manifestation in a pinched hysteresis loop. This intriguing property suggests potential applications beyond conventional electronics, particularly in modelling hysteresis phenomena across various domains. This paper delves into the exploration of memristance as a mathematical framework for simulating hysteresis in electrical and mechanical systems. We commence by elucidating the theoretical underpinnings of memristance and its hysteresis behaviour, followed by a comprehensive overview of existing hysteresis models. Subsequently, we propose a novel approach that leverages the memristor model to offer enhanced insights and predictive capabilities for hysteresis in these systems. Through analytical examination and simulation studies, we demonstrate the versatility and applicability of the memristor model, underscoring its potential as a universal tool for hysteresis modelling. This research not only broadens the understanding of memristive properties but also opens new avenues for cross-disciplinary applications, ranging from electronic circuit design to mechanical system analysis.

Keywords: memristor, hysteresis, modelling

1. INTRODUCTION

The field of electronics has long relied on three fundamental passive components: resistors, inductors, and capacitors. These elements are essential to the design and functioning of electronic circuits, providing predictable and reliable behaviours. However, the discovery and theoretical introduction of the memristor by Leon Chua in 1971 marked a significant milestone, proposing a fourth fundamental component [1], [2] that could revolutionize our understanding and application of electronic systems. In 2008 the memristor had been developed by HP labs. The device was made up of a film of TiO_2 and a film of TiO_{2-x}, where oxygen vacancies act as mobile +2 dopants sandwiched between 2 platinum electrodes creating a metal-oxide-metal cross point device. The device exhibited the characteristics described by Leon Chua, a pinched hysteresis loop that was frequency dependent and the resistance of the device was changed by the direction of the current [3]. Since the 2008 discovery memristive systems have been used to describe neural networks [4], [5], [6], [7].

The term "memristor" combines "memory" and "resistor," reflecting the component’s ability to change resistance depending on the applied current direction. Without an applied current, the resistance remains unchanged, meaning the memristor remembers its resistance state. The resistance of a memristor can vary between maximum (R_{ON}) and minimum (R_{OFF}) values, and this change is not necessarily linear. Several studies have discussed the equations describing the memristors behaviour [8], [9], [10], [11], [12], [13], [14], [15]. The memristor can be described as charge controlled or flux controlled as described in equations 1-4.

$$\varphi = \hat{\varphi}(q) \tag{1}$$

$$q = \hat{q}(\varphi) \tag{2}$$

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$$U(t) = \frac{d\varphi(t)}{dt} = \frac{d\hat{\varphi}(q)}{dq}\frac{dq}{dt} = R(q)I(t) \quad (3)$$

$$I(t) = \frac{dq(t)}{dt} = \frac{d\hat{q}(\varphi)}{d\varphi}\frac{d\varphi}{dt} = M(\varphi)I(t) \quad (4)$$

Where $\hat{\varphi}(q)$ and $\hat{q}(\varphi)$ are continuous and piecewise differentiable functions with bounded slopes [16]. In this study, we propose a novel modelling approach that explores the mechanical analogue of the memristor, demonstrating its hysteresis behaviour through simulation. Our method bridges electrical and mechanical domains, providing a framework to enhance the understanding and predictive capabilities of hysteresis phenomena mechanical systems.

2. MATERIALS AND METHODS

The equations for modelling electronic components can be derived from the base quantities, these are charge q with units [C] and magnetic-flux φ with units of [Wb]. Taking the derivative with respect to time we get dφ/dt with units of [V] noted as U and dq/dt with units of [A] noted as I. From the derived units building up the equations of the base elements of resistance R, inductance L, capacitance C and memductance M follows a simple pattern. Resistance has a unit of [Ω] and is calculated by U/I ([V/A]) which is [(Wb/s)/(C/s)] and this simplifies to [Wb/C]. The equations for the voltage and current of a resistor can be seen in equations 5 and 6. Inductance has a unit of [H] and is calculated by ((Ut)/I) ([Vs/A]) which is [(Wbs/s)/(C/s)] and this simplifies to [Wbs/C]. The equations for the voltage and current of a inductor can be seen in equations 7 and 8. The equation to calculate the energy stored in an inductor can be seen in equation 9. Capacitance has a unit of [F] and is calculated by ((It)/U) ([As/V]) which is [(Cs/s)/(Wb/s)] and this simplifies to [Cs/Wb]. The equations for the voltage and current of a capacitor can be seen in equations 10 and 11. The equation to calculate the energy stored in an capacitor can be seen in equation 12. Memductance has a unit of [S] and is calculated by I/U ([A/V]) which is [(C/s)/(Wb/s)] and this simplifies to [C/Wb]. The equations for the voltage and current of a memristor can be seen in equations 13 and 14.

$$U_R(t) = RI_R(t) = R\frac{dq(t)}{dt} \quad (5)$$

$$I_R(t) = \frac{1}{R}U_R(t) = \frac{1}{R}\frac{d\varphi(t)}{dt} \quad (6)$$

$$U_L(t) = L\frac{dI_L(t)}{dt} = L\frac{d^2q(t)}{dt^2} \quad (7)$$

$$I_L(t) = \frac{1}{L}\int U_L(t)dt = \frac{1}{L}\varphi(t) \quad (8)$$

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$$E_L(t) = \frac{1}{2} L I_L^2(t) \tag{9}$$

$$U_C(t) = \frac{1}{C} \int I_C(t) dt = \frac{1}{C} q(t) \tag{10}$$

$$I_C(t) = C \frac{dU_C(t)}{dt} = C \frac{d^2 \varphi(t)}{dt^2} \tag{11}$$

$$E_C(t) = \frac{1}{2} C U_C^2(t) \tag{12}$$

$$U_M(t) = \frac{1}{M} I_M(t) = \frac{1}{M} \frac{dq(t)}{dt} \tag{13}$$

$$I_M(t) = M U_M(t) = M \frac{d\varphi(t)}{dt} \tag{14}$$

Electrical and mechanical systems often share analogous behaviour, this enables the use of similar mathematical models across both fields. The equations for modelling mechanical components can be derived from the base quantities, these are displacement s with units [m] and momentum p with units of [I]. Taking the derivative with respect to time we get dp/dt with units of [N] noted as F and ds/dt with units of [m/s] noted as v. From the derived units building up the equations of the base elements of mechanical resistance b (dampening), mechanical inductance m (mass), mechanical capacitance k (elasticity) and mechanical memductance M follows a similar pattern to the electrical components. Mechanical resistance is calculated by F/v ([N/m/s]) which is [(I/s)/(m/s)] and this simplifies to [I/m]. The equations for the force and speed of a mechanical resistor can be seen in equations 15 and 16. Mechanical inductance has a unit of [kg] and is calculated by ((Ft)/v) ([Ns/m/s]) which is [(Is/s)/(m/s)] and this simplifies to [Is/m]. The equations for the force and speed of a mechanical inductor can be seen in equations 17 and 18. The equation to calculate the energy stored in a mechanical inductor can be seen in equation 19. Mechanical capacitance has a unit of [m/N] and is calculated by ((vt)/F) ([(m/s)s/N]) which is [(ms/s)/(I/s)] and this simplifies to [ms/I]. The equations for the force and speed of a mechanical capacitor can be seen in equations 20 and 21. The equation to calculate the energy stored in a mechanical capacitor can be seen in equation 22. Mechanical memductance has a unit of [I/m] and is calculated by v/F ([(m/s)/N]) which is [(m/s)/(I/s)] and this simplifies to [I/m]. The equations for the force and speed of a mechanical memristor can be seen in equations 23 and 24.

$$F_b(t) = b v_b(t) = b \frac{ds(t)}{dt} \tag{15}$$

$$v_b(t) = \frac{1}{b} F_b(t) = \frac{1}{b} \frac{dp(t)}{dt} \tag{16}$$

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$$F_m(t) = m\frac{dv_m(t)}{dt} = m\frac{d^2s(t)}{dt^2} \tag{17}$$

$$v_m(t) = \frac{1}{m}\int F_m(t)dt = \frac{1}{m}p(t) \tag{18}$$

$$E_m(t) = \frac{1}{2}mI_m^2(t) \tag{19}$$

$$F_k(t) = \frac{1}{k}\int v_k(t)dt = \frac{1}{k}s(t) \tag{20}$$

$$v_k(t) = k\frac{dF_k(t)}{dt} = k\frac{d^2p(t)}{dt^2} \tag{21}$$

$$E_k(t) = \frac{1}{2}kF_k^2(t) \tag{22}$$

$$F_M(t) = \frac{1}{M}v_M(t) = \frac{1}{M}\frac{ds(t)}{dt} \tag{23}$$

$$v_M(t) = MF_M(t) = M\frac{dp(t)}{dt} \tag{24}$$

The analogies can be seen between charge and displacement, magnetic-flux and momentum, voltage and force, current and speed, electrical resistance and mechanical resistance or dampening, electrical inductance and mechanical inductance or mass, electrical capacity and mechanical capacity or elasticity. Therefore it is reasonable to postulate that a mechanical memristor could also exist and that its behaviour would be similar to that of an electric memristor. Namely that the dampening would be dependent on the displacement and bound between a minimum and maximum value and the hysteresis affect would be inversely proportional to frequency. An example of a mechanical memristor can be seen in Figure 1.

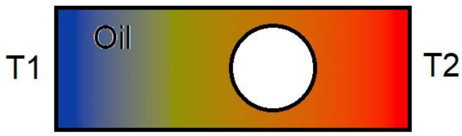

Figure 1. An example of a mechanical memristor

The example depicts a cylinder filled with oil. Two temperature potentials T1 and T2 are applied to the two ends where T1 < T2 producing a temperature gradient in the oil that influences the viscosity of the oil

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through the cylinder. Because of the temperature gradient in the oil the ball moving within the cylinder experiences different damping coefficients depending on its position within the cylinder.

3. RESULTS AND DISCUSSION

Since the equations are analogous for the mechanical and electrical cases, it is possible to model the behaviour of mechanical systems using the Simscape electrical elements by substituting current for speed and voltage for force. Figure 2 depicts a system modelled in Simulink where two memristors are in series and driven by a current source. The current source outputs 0.05 A for 1s and -0.05 A for 1s. By interpreting this in a mechanical sense we get a source of speed that moves the system at 0.05 m/s for 1s and at -0.05 m/s for 1s moving the system 0.05 m before returning to the starting position.

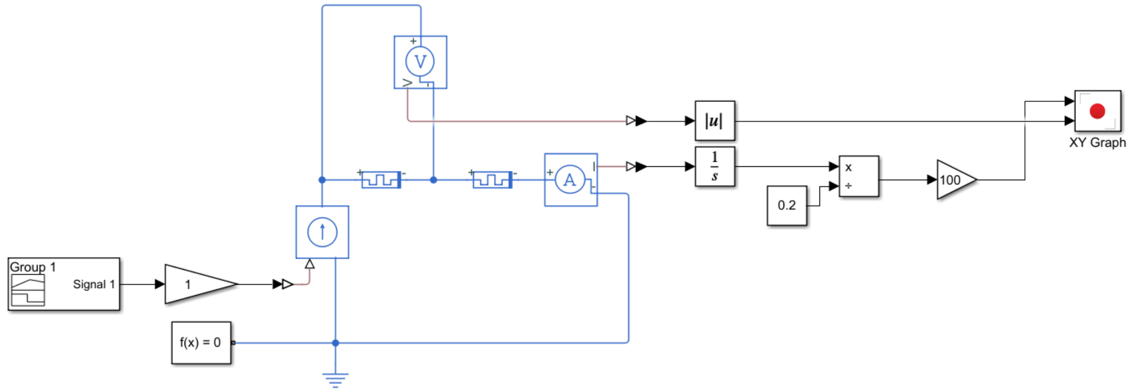

Figure 2. Simulink model to demonstrate hysteresis modelling

The first memristor is configured with R_{ON} = 0.0000001 Ω and R_{OFF} = 40000 Ω and the total charge required to change from R_{ON} to R_{OFF} is 0.04 C while the second memristor is configured with R_{OFF} = 0.0000001 Ω and R_{ON} = 40000 Ω. The integrator after the current meter calculates the charge applied to the system this translates to displacement. In this model it is assumed to be a system with a maximum charge of 0.2 C, dividing by 0.2 and multiplying by 100 gives a percentage value. Figure 3 depicts the results of a 2s simulation.

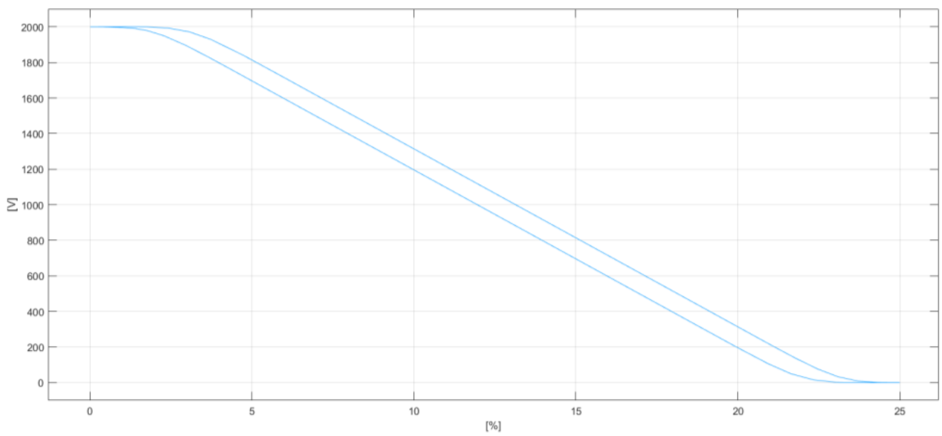

Figure 3. Results of hysteresis modelling

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Interpreting the figure as a mechanical system that has a total length of 0.2 m and can contract a maximum of 25% it is visible that maximum force is exerted at 0% contraction and minimum force is exerted at 25% contraction. Depending on the direction of motion the exerted force varies for a set contraction. The simulation results depicted in Figure 3 clearly exhibit a hysteresis loop of memristive systems. The mechanical interpretation suggests that maximum force occurs at zero contraction, decreasing as contraction increases. This behaviour confirms that damping in the mechanical analogue is non-linear and dependent on position. These findings highlight the potential of using memristor models to predict mechanical hysteresis phenomena, broadening their applicability across disciplines.

4. CONCLUSIONS

In this study, we proposed a novel modelling approach to simulate hysteresis phenomena using memristor analogues. By drawing analogies between electrical and mechanical systems, we demonstrated the viability of a mechanical memristor through simulations. These findings lay the groundwork for future research into memristors with use in mechanical systems, including in energy dissipation and dynamic system modelling. Using the current memristor model in simscape limits the model to a linear relation between R_{ON} and R_{OFF}. By modifying the model it is possible to achieve a wider array of possible curves. This is possible since the flux and charge based memristor equations only need to be continuous and piecewise differentiable. This flexibility is crucial for adapting the model to various real-world scenarios, enhancing its applicability and predictive accuracy.

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