

The Positive Net Present Value of Loss-making Projects: Economic Content of the Two Internal Rates of Return

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SUMMARY

This paper examines the economic content of the positive net present value of a project type that is loss-making and has two internal rates of return. The most important finding is that the economic content of a positive net present value is false in such cases. The financial source of the missing amount to reach the level of business efficiency is a false interest income generated by the method. In such cases, the two internal rates of return are also derived from false interest income. The revealed and mathematically proved causality relationships usually prevail in some form in the case of other types of non-conventional cash flows as well.

Keywords: net present value; internal rate of return; non-conventional cash flow; investment project evaluation; loss-making project

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INTRODUCTION

The literature does not address the economic content of the net present value (NPV). This content is clear only in the case of conventional (typical) investment projects. In this case, the NPV is the sum of the surplus profit generated above the required one, discounted for the present time (Illés 1990, 2012). The critical NPV value for business efficiency is zero. At this point, the payback requirement is met, because here all expenditures and the sum according to the required rate of return are returned. (There are many methods for exploring economic efficiency. In this field, the main question is the correct measuring of the return requirement fulfillment and the level of its over-fulfillment: Illés 2019a).

The economic content of the NPV is usually confused in the case of non-conventional investment projects. (If the sign of a given cash flow line changes more than once, then this is referred to as a non-conventional cash flow line. The label for this varies in the literature. The most commonly used variants of this are “non-normal”, “non-regular”, “non-typical”, “non-conventional”, “unconventional”, and “unorthodox”.) In the cases of non-conventional investment projects, even

loss-making projects can have a positive NPV. This means that there are special projects that do not repay even the nominal value of the invested capital; nonetheless, according to the NPV rule, these are appropriate (Illés 2016).

According to the literature, all independent projects with positive NPV are considered to be appropriate. In this context, the possibility of loss does not arise. (As is well known, loss is usually disadvantageous for business.)

Literature review

The possibility of positive NPV of loss-making projects can only occur for non-conventional cash flows. Many variants of *non-conventional* cash flow can be found in the literature (by date: Mao 1969; Arnold & Hope 1990; Van Horne & Wachowicz 2008; Ben-Horin & Kroll 2012; and so on). This type of project can have multiple internal rates of return (IRRs). Samuelson (1937) was the first to indicate this connection (Bey 1998). Nowadays it is well-known that the maximum number of IRRs can be as many as the number of sign changes in the cash flow line.

In a significant part of the accessible publications, there are examples concerning non-conventional cash flows where the initial investment is relatively low and a high surplus income occurs at the end of the first year as compared to the initial investment, then the second year also finishes with a similarly high expenditure surplus. In these examples, the cash flow row has two IRRs.

A realistic example of this investment project type first appeared in the article of Loire and Savage (1955). This often-cited example deals with the possibility of a pump replacement, which allows the remaining two years of the original project to be reduced to one year. The cash flow line of the pump replacement project: -\$1,600; +\$10,000; -\$10,000. The two IRRs are 25% and 400%. Although this type of example appears in several studies, not mentioned is the significant fact that these projects do not recover even the face value of the expenditures, that is, these are loss-making.

In this question, the difference between theoretical and applied economic science can be very large. This is

reflected – for instance – in the paper of Ben-Horin and Kroll (2012: 114-115): “From a practical point of view an investment project should have a positive NPV at zero cost of capital... Such a requirement does not follow from any rational principle, but it reflects a basic prerequisite for most practitioners.” Accordingly, by economic theory, there is not a problem if a project is loss-making. Namely, at the zero cost of capital, the negative NPV shows the nominal amount of the total loss. (Details are given later.) Unlike this, according to the applied economic sciences, the return of the expenditures is one of the basic requirements in business. Consequently, at the zero cost of capital, a non-negative NPV is a fundamental rational guiding principle in practice. Starting from the quoted statement, it can also be concluded that practitioners generally refrain from those theoretic conceptions which do not correspond to the business logic.

Table 1 shows some examples from the literature. (The publications do not mention the loss.) The list follows chronological order.

Table 1
Some published examples of non-conventional cash flow patterns with double internal rates of return

SOURCE	UNIT	YEAR			LOSS	IRRS (%)
		0	1	2		
Solomon (1956: 128)*	\$	-1,600	+10,000	-10,000	-1,600	25 and 400
Renshaw (1957: 199)*	\$	-1,600	+10,000	-10,000	-1,600	25 and 400
Southwick (1985: 532)	\$	-125,000	+400,000	-300,000	-25,000	20 and 200
Brealey & Myers (1988: 80)	\$	-4,000	+25,000	-25,000	-4000	25 and 400
Arnold & Hope (1990: 258)	£	-2,000	+5,100	-3,150	-50	5 and 50
Emery & Finnerty (1991: 295)	\$	-10,000	+25,000	-15,600	-600	20 and 30
Shull (1993: 68)*	\$	-1,600	+10,000	-10,000	-1,600	25 and 400
Plath & Kennedy (1994: 82)	-	-16	+100	-100	-16	25 and 400
Firer & Gilbert (2004: 43)	-	-1,600	+10,000	-10,000	-1,600	25 and 400
Van Horne & Wachowicz (2008: 341)*	\$	-1,600	+10,000	-10,000	-1,600	25 and 400
Ross et al. (2010: 277)	\$	-60	+155	-100	-5	25 and 33 $\frac{1}{3}$
Bierman & Smidt (2012: 93)	\$	-100	+310	-220	-10	10 and 100
Balyeat et al. (2013: 45)	-	-60	+500	-500	-60	16.2 and 617.13
	-	-40	+500	-500	-40	9.61 and 1040.39

* Referring to the example from Loire and Savage (1955).

The NPV curve for these examples can be found in Brealey and Myers (1981: 80); Campani (2014: 4); Firer and Gilbert (2004: 43); Ross et al. (2010: 278); Van Horne and Wachowicz (2008: 342) and so on. The shape of these is the same. Each of these curves starts with a negative value range. The starting point is the sum of the loss in face value (the content of which is not mentioned). The curves cross the x-axis twice. In the zone of x-axis bounded by the two IRRs, each interest

rate results in positive NPV. Outside these borders, each of the interest rates leads to the negative NPV. The general shape of NPV curves of the examples above is illustrated in Figure 1.

According to the general academic opinion, the NPV method is suitable for evaluation in the case of projects with non-conventional cash flows as well (e.g. Southwick 1985; Brealey & Myers 1988; Arnold & Hope, 1990; Shull 1993; Johnstone 2008; Ross et al.

2010; Bierman & Smidt 2012; Banerjee 2015). The argument – if there is one – is that also in this case only one NPV can occur, as opposed to the IRR.

Despite the dominant consensus, many efforts to solve multiple-IRR problems can be found in the literature. Campani (2014:3) says: “Although the

problem of multiple IRRs is very well known by the literature, still its solution is not!” However, the real solving of this problem starts with the question as to how a positive NPV can be generated when even the face value of the expenditures is not returned by the project.

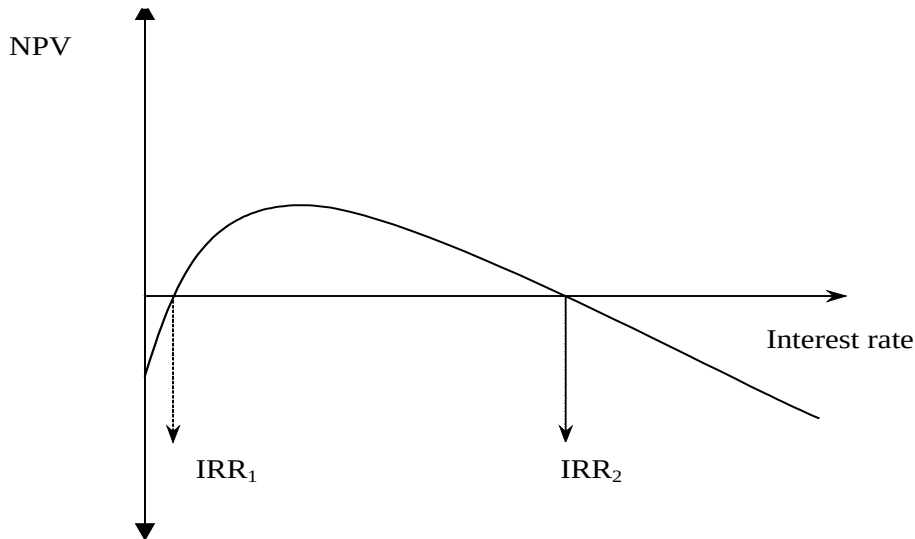


Figure 1. The general shape of NPV curves for the examples in Table 1

Purpose and method

The main objective of this paper is to explain and to prove that the positive NPV of loss-making projects can be misleading. For the sake of relative simplicity, the paper uses a special variant of the non-conventional investment projects, which have a two-year lifetime, are loss-making, and have two IRRs. As seen in Table 1, this project type is often mentioned and used in the literature.

In implementing the main objective, the paper is based on the following main content elements:

- Revealing the fundamental methodological relationships that can lead to a positive NPV for the case of the loss-making projects;
- Proving that the mechanism of the NPV method can create a false interest income;
- Proving that the two IRRs are also derived from false interest;
- Analyzing the content structure of the false interest income;
- Exploring the possibilities of business efficiency examination for such cases;
- Numerical demonstrations of the revealed relationships;
- Indications that the false interest incomes can prevail also in other types of non-conventional cash flows.

The purpose of the paper is also to demonstrate that the revealed relationships to be interpretable without having to follow the process of proof in detail. The

verbal explanations and the numerical demonstrations serve this purpose.

The main research methods are content analysis, methodological analysis, and model editing and model analysis. Logical and mathematical methods are used to confirm the findings.

This paper uses a new analytical method for examining the special NPV content. The new method became necessary because the primary discounting process makes the content examination of the payback process impossible. The follow-up of the payback process is the only way to explore the emergence and realization of return requirements and the process of surplus profit formation. (The planning activity and thought process of corporate executives also move in a forward direction in time.) The year-after-year calculation allows also the net future value (NFV) to be calculated, the discounted value of which is the NPV. Thus after the systematic exploration of content relations, the analysis can return to the well-known NPV.

The most important conditions and categories are as follows:

- The research uses an interdisciplinary approach during the exploration of the literary background. It analyzes sources in economics, finance, and management accounting (and also refers to some textbooks). The explanation and analysis use a business approach according to the real conditions and practical wording.

- In the general case, capital is a scarce resource for the company.
- The literature of investment projects evaluation frequently names the amount of the negative NPV "loss". This can be misleading. In the case of a conventional cash flow line, the negative NPV has two grades. The first is when the IRR is positive but NPV is negative. In this case, the examined project is profitable from a practical point of view, but the profit is not sufficient to fulfill the profit requirement on the required rate of return. Here the negative NPV signals the amount of missing profit at present value. In the second grade, the IRR is also negative. This means that the face value of the expenditures cannot be recovered (Illés 1990). In practice, only the unrecovered expenditures or unrecovered practical costs are called loss. (In practice, the cost is a part of the expenditure, related to a specific period or a specific matter.) The paper uses the practical variant of the expression of loss.

THE BASIC CONTENT RELATIONSHIPS

The relevance of the nominal profit value calculated for the whole lifetime

In the case of conventional cash flows, the NPV calculated with zero interest rate shows the nominal profit sum for the whole lifetime. This profit amount is the source that can serve as a cover for the profit requirement of capital tied up at all times. Moving from the zero interest rate a positive NPV is given, as long as the cumulative profit requirement according to the interest rate is less than the nominal amount of profit. The interest rate at which the two sums are equal is the IRR itself. By further raising the interest rate, NPV will be negative, indicating that the sum of the whole profit sum no longer covers the profit requirement.

The NPV rule shows not only loss-making projects are to be rejected but also those profitable ones where the IRR is smaller than the required rate of return. The positive NPV is the same as the discounted sum of yearly surplus profits generated over the required rate of return.

In the case of non-conventional cash flows, the NPV calculated with zero interest rate also quantifies the nominal profit sum generated during the whole life of the project. (This can be easily seen if the NPV formula is written with the zero interest rate.) In the case of the analyzed project type, there is a loss at the zero interest rate. Consequently, there is no generated source to cover all of the expenditures during the lifetime of the project. In addition to this, a positive sign of NPV can emerge. As a starting point, there is a need to explore how this positive NPV is created.

The methodological handling of surplus profit is the key to the problem

The yield is a positive difference in annual sales revenue and annual expenditure. This amount is a surplus compared to the next year's financial needs of the project. Therefore, the amount of the yield leaves the project in that year when it is generated. Usually, the circumstances of the use of the amounts leaving the project will not affect the evaluation of the examined project. Unlike the actual financial processes, the automatism of the NPV method handles the content ingredients of the yield differently. This method focuses on the present value of the resulting surplus profit, so the surplus profit remains within the calculation. In the calculations, this amount will continue to be included as a part of project indicators, although in reality this yield part also goes out from the project. The consequences of this are radically different in the case of conventional and non-conventional investment projects.

Formula (1) describes the net future value (NFV) of a conventional project where the yearly yields are in chronological order. (In the case of the multi-year investment process, the cash flow line starts with more than one negative amount.)

$$NFV = -E_0(1+i)^n + H_1(1+i)^{n-1} + H_2(1+i)^{n-2} + \dots + H_{n-1}(1+i) + H_n \quad (1)$$

where:

E_0 = initial investment;

H_t = yield, which is the difference in revenues and expenditures at the end of the year number t . In

the case of a project with conventional cash flows, the magnitude relation is always $H_t > 0$;

t = the serial number of the years;

n = lifetime of the project;

i = required rate of return.

Calculated year by year, at first the yields reimburse the initial investment and the profit requirement continuously. After the return requirement has been met, the generated yields are the surplus profits with a positive sign. The surplus profits are no longer need to cover any requirement. They remain within the calculation, as in the NPV case. The method omits from the calculation only those amounts that are needed to cover the return requirement (Illés 2016).

According to this NFV method, the surplus profits are increased by the required rate of return up to the end of the period. The resulting interest incomes are not real, as the surplus profits have left the project. These false interest sums are technical items. They are needed only for the summation of the surplus profits generated at different times. The false interest amounts automatically disappear when the NFV is discounted to the present value. The discounting takes effect from the year of surplus profit creation. So, in this case, the different

handling of content ingredients of the yield does not lead to an error. For the project with conventional cash flows, the IRR method does not create surplus profit. The rate of return includes all the generated profits.

The above context is not valid for the analyzed non-conventional investment projects. For these special projects, in the first year, all of the initial capital and its profit requirement are returned; furthermore, also a very high temporary surplus profit is generated. The high expenditure surplus in the second year will sweep away the temporary surplus profit. The difference between the expenditure surplus and temporary surplus profit will be the loss. The temporary surplus profit goes out of the calculation. Therefore, discounting can no longer eliminate false interest income.

It follows from the above that according to the calculation of the NFV, the temporary surplus profit generated at the end of the first year will be increased by the interest for one year. The method does not charge interest on the full first year's yield, but only on the temporary surplus profit. This interest income is false, as the temporary surplus profit also leaves the project as a part of the yield at the end of the first year. The temporary surplus profit can yield real interest only after being invested in another project. The false interest income, if it is large enough, will cover the loss, the profit requirement and the remainder will be the false NFV. The discounted amount of the latter is a false NPV.

For the analyzed project type, the calculation mechanism of IRR also applies the utilization of non-existing interest income. Here, the amount of the false interest income exactly equals the sum of the loss and the profit according to the IRR.

MATHEMATICAL PROOF

Proving the false outcome by a mathematical model

According to the above, the model of the analyzed investment project type is loss-making, with two IRRs, and the lifetime is two years. The amount of the initial investment is relatively low, a high revenue surplus (yield) is generated at the end of the first year as compared to the initial investment, and then the second year also finishes with a similarly high expenditure surplus. Formula (2) describes this model mathematically.

$$-E_0 + H_1 - U_2 = M_s < 0 \quad \|E_0| < H_1; \quad |E_0| < |U_2|; \quad r_1 \neq r_2 \quad (2)$$

where

M_s = the loss generated during the lifetime of the project;

H_1 = yield, which is the difference in revenues and expenditures at the end of the first year;

U_2 = expenditure surplus, which is the difference in revenues and expenditures at the end of the

second year;
 r_1 and r_2 = the two IRRs.

(Further magnitude order conditions are necessary between E_0 and H_1 and U_2 for the existence of the two IRRs. From the aspect of the purpose of the paper, the presentation of these is unnecessary.)

Based on formula (2):

$$H_1 < E_0 + U_2 \quad (3)$$

That is, the first year's yield does not cover even at face value the sum of the initial investment and the second year's expenditure surplus. According to (3):

$$H_1 - E_0 < U_2 \quad (4)$$

As stated above, the first year's yield completely leaves the project. However, a part of the yield, namely the temporary surplus profit, remains within the calculation mechanism. The temporary surplus profit (ΔH_s), appearing at the end of the first year, is the difference of the first year's yield and the sum of the initial investment and its one-year profit requirement. Mathematically:

$$\Delta H_s = H_1 - E_0 - E_0 i \quad (5)$$

According to (5), the structure of the first year's yield is as follows:

$$H_1 = \Delta H_s + E_0 + E_0 i \quad (6)$$

The substitution of the detailed inscription of H_1 [according to (6)] into Formula (4):

$$\Delta H_s + E_0 + E_0 i - E_0 < U_2$$

That is:

$$\Delta H_s + E_0 i < U_2 \quad (7)$$

Consequently, (8) also is true:

$$\Delta H_s < U_2 \quad (8)$$

The NFV at the end of the second year:

$$NFV = -E_0(1+i)^2 + H_1(1+i) - U_2 \quad (9)$$

The discounted NFV is equal to the NPV:

$$NFV \frac{1}{(1+i)^2} = NPV = -E_0 + H_1 \frac{1}{1+i} - U_2 \frac{1}{(1+i)^2} \quad (10)$$

The substitution of the detailed inscription of H_1 [according to (6)] into Formula (9):

$$NFV = -E_0(1+i)^2 + (\Delta H_s + E_0 + E_0 i)(1+i) - U_2 \quad (11)$$

Performance of the assigned operations at (11):

$$NfV = -E_0 - E_0 i^2 - 2E_0 i + \Delta H_s + E_0 + E_0 i + \Delta H_s i + E_0 i + E_0 i^2 - U_2 \quad (12)$$

By rearranging (12), step by step, a simple formula of the NFV is available.

$$NfV = \Delta H_s + \Delta H_s i - U_2 \quad (13)$$

The false interest income is in (13) as $\Delta H_s i$. In the case of $NPV > 0$, NfV also > 0 , and Formula (13) can be rearranged to Formula (14).

$$\Delta H_s + \Delta H_s i > U_2 \quad (14)$$

However, according to (8) the temporary surplus profit is lower than the second year's surplus expenditure. The simultaneous fulfillment of (8) and (14) can only be true if the false interest income is used in the financing of the project. Accordingly, it is unequivocal that a positive NPV of the loss-making projects can only happen if the false interest income is used in financing. In this way, the NFV and the NPV calculated from this is also false.

The content structure of the false interest income

A clear exploration of the content structure of the false interest income can be solved using the IRR method. For IRR, NPV is equal to 0. The zero value of NPV can only occur if NFV is also equals to 0.

In the case of false IRR, the false interest income is equal to the product arithmetical of the temporary surplus profit and IRR. According to the logical approach, the generated false interest income is equal to the sum of the loss and the profit according to the interest rate. Mathematically:

$$\Delta H_s i = E_0 i + M_s \quad | \quad i = r; NfV = 0 \quad (15)$$

The way to prove the correctness of Formula (15) is to describe its elements in detail and then rearrange them into the well-known IRR formula. A detailed description of (15) is as follows:

$$(H_1 - E_0 - E_0 i)i = E_0 i + E_0 - H_1 + U_2 \quad | \quad i = r; NfV = 0 \quad (16)$$

Rearranged:

$$\begin{aligned} H_1 i - E_0 i - E_0 i^2 &= E_0 i + E_0 - H_1 + U_2; \\ H_1 i + H_1 - U_2 &= E_0 + 2E_0 i + E_0 i^2; \\ H_1 i + H_1 - U_2 &= E_0 (1+i)^2, \\ -E_0 (1+i)^2 + H_1 (1+i) - U_2 &= 0 \quad | \quad i = r; NfV = 0 \end{aligned} \quad (17)$$

If $NfV = 0$, then (17) equals (9). After discounting (17):

$$-E_0 + H_1 \frac{1}{1+i} - U_2 \frac{1}{(1+i)^2} = 0 \quad | \quad i = r; NfV = 0 \quad (18)$$

(18) is the well known IRR formula. This proves that the false interest income structure according to (15) is correct. In this case, the components of false interest amount indeed are the profit by IRR and the loss.

Based on the evidence of (15), it is clear that the positive NPV of the examined model can only occur if the sum of the false interest income is greater than the sum of the profit requirement and the loss. So when evaluating such projects, it must be the starting point that the loss is a loss.

The model analyzed is relatively simple, so the argumentation is relatively straightforward and the relationships are transparent. Even a relatively small modification in the model leads to more complicated possibilities of analysis.

Based on the above, it is obvious (but mathematically is not proven) that the NPV is always false when the NPV curve has a section where the increase in the interest rate increases the NPV. It has been known for more than half a century that the IRR is not suitable for assessing investment projects with non-conventional cash flow. Now it has been proven that the NPV is not appropriate, either. Project-specific methods need to be developed to assess the potential benefits of such projects.

EVALUATION OPTIONS

In the case of conventional cash flow lines, the loss-making project cannot have a positive NPV. Therefore, realization of this is automatically not recommended by the NPV rule. Examined in itself, the business efficiency of a loss-making project is inadequate in the case of non-conventional cash flows neither. However, in some cases, further examinations may be recommended. For the analyzed project type, it can be examined whether the relatively high yield in the first year could be a considerable advantage for the company, despite the later loss. Although the methodology for the general solution is not known, there are several possibilities for examinations based on management logic. Below are two variants of the combined method for this.

The critical value of the reinvestment rate

According to the above, the total amount of the first year's yield will leave the project. One of the main questions is as follows: what reinvestment rate is needed for this yield to ensure the return requirement for the original project at the end of the second year? In this

case, a critical profitability rate can be determined that would ensure that the reinvestment project compensates for the loss and fulfills the originally required rate of return.

In this case, the following two projects are analyzed together. One is the original project, and the other one is the one-year investment of the first year's yield.

The cash flow rows are as follows.

- Original project:
 $-E_0; +H_1; -U_2$
- Reinvestment project:
 $0; -H_1; +H_1(1+r)$
- The two projects together:
 $-E_0; 0; -U_2 + H_1(1+r)$

where r = the profitability rate (that is the IRR) of the reinvested first year's yield.

The NFV of the two projects (the return requirement of the original project contains the original required rate of return):

$$NFV = -E_0(1+i)^2 - U_2 + H_1(1+r) \quad (19)$$

The critical profitability rate is where $NFV = 0$. After rearranging Formula (19):

$$E_0(1+i)^2 + U_2 = H_1(1+r_k) \quad | \quad NFV = 0 \quad (20)$$

where r_k = critical profitability rate of the first year's yield (the return requirement of the original project cannot be met if the realizable profit rate is lower than this).

According to (20), the one-year investment and its yield must reimburse the nominal value of the initial investment and its profit requirement for two years and the surplus expenditure at the end of the second year.

The first question for the evaluation: how realistic is the feasibility of critical profitability rate? The second question is whether it is appropriate to use a substantial part of the new investment's profit for loss financing. The answer can be given only by knowing the specific circumstances, numbers and options. (The calculation may also be made according to the after-tax variant of the temporary surplus profit.)

Merging with an independent project

If a company has a project with conventional cash flow, and it can be combined with the examined non-conventional project so that the two projects together as a project combination also will be conventional, then this possibility might be appropriate to examine. It is a very important rule that in this case only the IRR method can be used for analysis. NPV and its traditionally

derived indicators are not suitable for comparing the new project and the project combination. (Transforming the NPV to a comparable index number leads to the surplus profitability rate, that is, to the difference between IRR and the required rate of return; see Illés 2012). In addition, information on aggregate capital needs is also required.

The aggregate capital need is a capital sum that quantifies the sum of capital needed throughout the whole lifetime of the project. For this, the capital tied-up must be determined for each year and then added up. This shows such content as if this whole amount would be invested for one year. The measurement unit is one unit of tied-up capital for one year. (This is a new conception and a new business economics category. For modelling and an example of its calculation see Illés 2019b.)

The course of the calculation: It is necessary to quantify the IRR and aggregate capital needs of the project with originally conventional cash flow, and then the IRR and aggregate capital needs of the merged project.

As decision-making information, the IRR and the aggregate capital needs of the new independent project and the merged project should be compared. The economic impact of the original non-conventional project is favorable if the indicators of the merged project are more favorable than the indicators of the new independent project. That is, the merged project is advantageous if the larger amount of capital results in the higher IRR. Because of the opposite effects (aggregate capital need decreases, the IRR increases), further analysis may be necessary.

NUMERICAL DEMONSTRATIONS OF THE REVEALED RELATIONSHIPS

The analyzed non-conventional project example is Project A. That has the following main features.

- Cash flow line: -20; +125; -125
- Loss: -20
- The two IRRs are 25% and 400%.

The false interest income as a financial source

The data in Table 2 show how the temporary surplus profits are generated and how the NPV and IRR methods use false interest incomes for financing.

Table 2
Generating the temporary surplus profits and false interest incomes for Project A, at variable interest rates

Measurement unit: unit

PER-CENT	STRUCTURE OF 125 UNITS YIELD AT END OF 1ST YEAR			RESOURCE CALCULATED AT END OF 2ND YEAR		NFV	NPV
	For capital returns	For interest	Temporary surplus profit	Carried over	False interest income		
	$-E_0$	$-E_0i$	$\Delta H_s = H_1 - E_0 - E_0 i$	ΔH_s	$\Delta H_s i$	$\Delta H_s + \Delta H_s i - U_2$	$NFV \frac{1}{(1+i)^2}$
15%	20	3	102	102	15.3	$117.3 - 125 = -7.7$	-5.8
30%	20	6	99	99	29.7	$128.7 - 125 = 3.7$	2.2
25%	20	5	100	100	25.0	$125 - 125 = 0$	0
400%	20	80	25	25	100.0	$125 - 125 = 0$	0

In the row of 15%, the 15.3 units of false interest income are not enough to cover the 20-unit loss and the 3-unit profit requirements. The lack is 7.7 units. The present value is a lack of 5.8 units. The conventional method results in the same NPV:

$$NPV_{15\%} = -20 + 125 \times 0.86957 - 125 \times 0.75614 = -20 + 108.7 - 94.5 = -5.8$$

In the row of 30%, the 29.7 units of false interest income cover the 20-unit loss and the 6-unit profit requirements, and 3.7 units of surplus are also generated. In this case, the NPV is 2.2 units. According to the conventional method, the NPV calculation is as follows:

$$NPV_{30\%} = -20 + 125 \times 0.76923 - 125 \times 0.59172 = -20 + 96.2 - 74.0 = 2.2$$

At the next two rates of IRRs (25% and 400%), the false interest incomes cover the 20-unit loss and the 5 units and 80 units, respectively, for interest.

Finding the critical reinvestment rate

Following from section 4.1, this method also uses a complementary project. This project is the reinvestment of the yield that exits from Project A at the end of the first year. The cash flow rows and the profit requirements are as follows:

- Original project (Project A): $-20; +125; -125$;
- Reinvestment project: $0; -125; +125(1+r_k)$;
- The two projects together: $-20; -125; +125(1+r_k)$;
- The required rate of return for Project A is 15%.

The critical return rate calculation for the re-invested amount:

$$20 \times 1.3225 + 125 = 125 \times (1 + r_k);$$

$$\frac{151.5}{125} = 1 + r_k = 1.212; \quad r_k = 0.212$$

The result of the calculation shows that if an investment opportunity of at least 21.2% profitability for a year can be found, the two projects together will meet the entire return requirement. However, for the sake of clarity, it is advisable to review the forming of the profits of each project separately.

- The loss of Project A:
 $-20 + 125 - 125 = -20.0$ units
- Profit of the reinvestment project if the criterion is met: $0 - 125 + 151.5 = +26.5$ units
- Total nominal profits of the two projects together: $+6.5$ units.

The results of the additional calculation clarify the very essence of the problem. On the one hand, it is unlikely that an investment opportunity with 21.2% profitability for one year can be found, while the required rate of return of Project A is 15%. On the other hand, if such an option were available, probably it would not be appropriate for a large part of the profit to be absorbed with the loss, meaning that instead of a profit of 26.5 units a profit of 6.5 units would be realized. Probably without Project A, even in the case of borrowing the 125 units, more profits would remain in the company than from the two projects together.

Analysis of a merged project

The merged project method also uses an independent conventional project. This is Project B. The cash flow rows are as follows.

- Project A: $-20; +125; -125$;
- Project B: $-100; +40; -125$;
- Merged Project: $-120; +165; 0$

The main calculated data of Project B and Merged Project are in Table 3.

Table 3
The IRR and the aggregate capital needs

Measurement unit: unit

PROJECT	TOTAL PROFIT (FACE VALUE)	AGGREGATE CAPITAL NEEDS	IRR %
Project B	65	$100 + (100 - 6.4) = 193,6$	33.6
Merged Project	45	120	37.5

(The calculation of the second year tied-up capital for Project B is as follows: At the end of the first year, there is a yield of 40 units. The content distribution of this: 33.6 units are the profit requirement [100×0.336] and the difference [$40 - 33.6 = 6.4$ units] is the returned part of the capital. Thus, the tied-up capital for the second year is $100 - 6.4 = 93.6$ units.)

According to the data in Table 3, the profitability of Merged Project is 3.9 percentage points higher than the profitability of Project B. This is a favorable effect. However, the total profit is lowered by the loss of Project A, and the higher profitability applies to a considerably lower principal amount. The first-year yield of Project A is very high, so this results in a significant reduction in capital needs in Merged Project.

In this case, decision-makers have to decide which is more favorable for the company: 33.6% profitability for the capital of 193.6 units or 37.5% profitability for the capital of 120 units. The 33.6 % profitability of project B is excellent. However, the question is whether the critical profitability of the difference in the aggregate capital needs of 73.6 units can be reached. The critical profitability rate calculation:

$$120 \times 0.375 + 73.6 r_k = 65;$$

$$\text{From this : } r_k = 0.272$$

The critical profitability rate of the difference between the two aggregate capital needs is 27.2 %. If this can be achieved, then the average profitability of the capital sum corresponding to 193.6 units of aggregate capital needs originally required for Project B will not fall below 33.6%. If the profitability for the difference in aggregate capital needs is predictably higher than 27.2%, then the project combination may be advantageous.

In general, the critical profitability of the aggregate capital needs difference depends on the differences in

the parameters of the original projects in the merged project.

CONCLUSIONS

In the case of non-conventional cash flows the NPV of loss-making projects may be positive. This is a false indication of business efficiency. Obviously, a loss-making project cannot be profitable. The economic content of the positive NPV calculated for such cases is false. The two IRRs are false as well. The paper has proven that in the case of a special variant of non-conventional projects a false interest income covers the loss, the required profit, and furthermore may show some surplus profit. The discounted amount of surplus of the false interest income gives positive NPV. This alone demonstrates that in the case of non-conventional cash flows, the NPV is not suitable for evaluating projects. This finding differs from today's general academic opinion.

Even the NPV of profitable (not at a loss) variants of non-conventional cash flow may contain false interest income. In the literature special NPV curves can often be seen, which have a section where the increase in the interest rate increases the NPV. When the surplus profit is temporary, then its false interest incomes cannot disappear during discounting. So these are incorporated as the yields of the project, and the shown returns are not real. In these cases, the NPV will also be false. The mathematical proof of the general context of that problem is not resolved yet.

An additional research objective may be to explore whether there is a project type that is profitable and non-conventional where the NPV components cannot include false yield elements.

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