UNCERTAINTY ANALYSIS OF REPAIR WORK ESTIMATION BY MONTE-CARLO SIMULATION

JAVÍTÁSI MUNKAIGÉNY BECSLÉSÉNEK MONTE-CARLO SZIMULÁCIÓS BIZONYTALANSÁGI ELEMZÉSE

Pokorádi, László[*]

ABSTRACT

Maintenance is one of the most important territories of practical engineering. From a mathematical point of view, the operation of production equipment is a discrete state space stochastic process, without after-effects, i.e., a Markov-chain. The aim of this paper is to discuss the possibilities of using the Monte-Carlo Simulation of repair processes, to determine necessary maintenance capacity. The proposed method can be implemented for the assessment of the required maintenance capacity of a welding cell, depending on an allowable estimate of uncertainty.

KIVONAT

A karbantartás a gyakorlati mérnöki munka egyik legfontosabb területe. Matematikai szempontból a termelőberendezések üzemeltetése egy diszkrét állapottérben zajló, utóhatások nélküli sztochasztikus folyamat, más szóval Markov-lánc. A dolgozat célja, hogy megvitassa a javítási folyamatok Monte-Carlo szimulációja felhasználási lehetőségeinek bemutatása a szükséges karbantartási kapacitás meghatározására. A javasolt módszer alkalmazhatóságát egy hegesztőcella szükséges karbantartási kapacitásának meghatározásán keresztül mutatjuk be a megengedhető becslési bizonytalanság függvényében.

1. INTRODUCTION

Nowadays, there are numerous papers and books discussing new methods from different aspects to help maintenance management in decision making. For example, Dodu's article analyzed the causes which led to the lack of availability of helicopters, while the rate of cannibalization and the number of unavailable spare parts increased [3].

One of the most important maintenance management questions is estimating the optimal maintenance capacity, which depends on the planned work performance (for example volume of production) [7] [11].

Balogh and Hanka discussed the applicability of Bayesian methods to probabilistic risk assessment and engineering design problems [2]. Their proposed methodology is useful for engineering managers for rare event risk analysis in other applications and other disciplines.

The Monte-Carlo Simulation (MCS) is one of the classical simulation techniques. Metropolis and Ulam named this method Monte-Carlo [12] in 1949. But an early example of the same calculation, of the motion and collision of the molecules in gas, was described by Lord Kelvin in 1901 [6]. Kelvins' calculations were aimed at demonstrating the truth of the equipartition theorem for the internal energy of a classical system. The exponential growth in computer power is a well-known story, as is its impact: the increase in computational resources led to the rise of MCS techniques in the subject of engineering simulations. Hanka demonstrated that the Queuing theory, especially the finite queue model and the Monte-Carlo simulation method are suitable and efficient tools for describing the characteristics of electric vehicle charging stations [4]. His results help investigate the operation of existing charging stations and help to plan and construct new stations.

This paper proposes a Monte-Carlo Simulation-based method to determine one the one hand, the required maintenance capacity depending on allowable estimating uncertainty and on the other hand, to determine what maintenance parameter has the greatest effect on the Required Repair Capacity.

The outline of the paper is as follows: Section 2 presents the estimation methodology of repair capacity. Section 3 details the Monte-Carlo Simulation, followed by Section 4, describing the proposed method through a case study. Section 5 summarizes the paper, outlines the prospective scientific work of the Author.

2. ESTIMATION OF NEEDED REPAIR CAPACITY

The necessary maintenance capacity can be determined by:

- production performance T;

* *full professor, Óbuda University, Institute of Mechatronics and Vehicle Engineering*

- failure rate λ;
- repair work of one failure *m*.

The first parameter can be planned directly, while the other two are stochastic ones.

The failure rate $\lambda(t)$ can be thought of as the probability that one failure will occur in a given interval, assuming no failure before time *t*. The failure rate can be determined by the following equation:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{n}{N \Delta t} \quad , \tag{1}$$

where:

n – number of failures during Δt interval;
N – number of equipment;
Δt – investigational performance interval (for example number of productions).

The failure rate λ of a given technical system changes stochastically, thus it has uncertainty. The failure rate can therefore be characterized as a random variable, which has an expected value, standard deviation, density $f(\lambda)$ and distribution function $F(\lambda)$.

To demonstrate it, Figure 1 shows a histogram of failure rates during operations of welding cells.

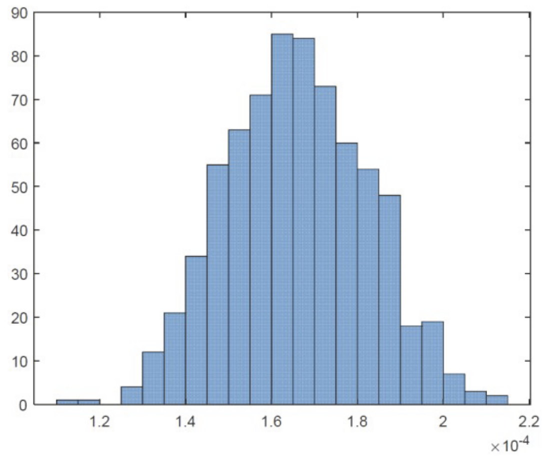

Figure 1. Histogram of Failure Rates of Welding Cells [1/Working Hour]

The repair work of one failure *m* is a stochastic one too. Therefore, it can be described to be its expectation, deviation, density $f(m)$ and distribution function $F(m)$.

Figure 2 shows a histogram of repair times of welding cells.

Based on the calculation indicated above, the required repair capacity per unit of working performance can be determined by the following equation

$$W_{unit} = m\lambda \quad , \tag{2}$$

and – knowing planned production performance – the total required repair capacity is

$$W_{total} = TW_{unit} \quad . \tag{3}$$

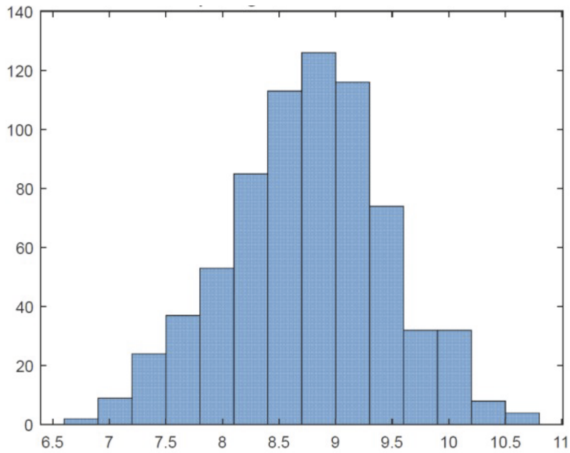

Figure 2. Histogram of Repair Works of Welding Cells [Working Hour]

3. THE MONTE-CARLO SIMULATION

At the core of MCS there is a computational procedure in which the performance measure is estimated using samples drawn randomly from a data set with appropriate statistical properties (see Figure 3.)

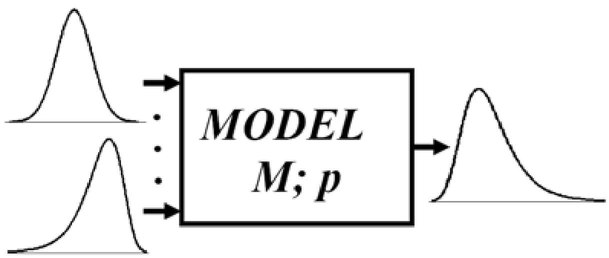

Figure 3. The Monte-Carlo Simulation (source: [8])

There are a number of books and papers on the theory of the MCS and its applications. Rubinstein offered a detailed treatment of the theoretical backgrounds and the statistical aspects of these methods in his book [12]. Dagpunar provided an introduction to the theory and practice of MC and Simulation methods [2]

The most common features of MCSs are as follows:

{1} A known probability density function *f(x)* over the set of system inputs.
{2} Random sampling of inputs based on the distribution specified in feature {1}, and simulation of the system under the selected inputs.
{3} Numerical aggregation of experimental data collected from multiple simulations conducted according to feature {2} [9].

Numerical experiments of MCS had led the modelers to run the simulation on many sampled inputs before they were able to infer the values of the system performance measures of interest. At its substance is a computational procedure in which a performance measure is estimated using samples drawn randomly from a population with appropriate statistical properties. The selection of samples, in turn, requires an appropriate random number generator.

Basically, three generation methods are used [10]:

- Inverse Transform Method (ITM);
- Composition Method (CM);
- Acceptance–Rejection Method (ARM).

The third is associated with John von Neumann and consists of sampling a random variant from an appropriate distribution and subjecting it to a test to determine whether or not it will be acceptable for use [6].

4. THE CASE STUDY

In this section, MCS of welding cells' operational process will be carried out to determine the required repair work depending on estimation uncertainty. Based on the simulation results, theoretical and practical conclusions can be deduced for a maintenance management decision.

4.1. The Simulation

Firstly, the momentary failure rates and repair time data were analyzed statistically.

Due to the relatively small number of available data, the goodness-of-fit tests have been left out. According to general engineering practice it is assumed that the measured parameters have a normal (Gauss) probability distribution. The correlation between failure rates and repair times is less the 1%, thus it can be declared that they are independent of 1. Table 1 shows the statistical analysis of data demonstrated by Figures 1 and 2.

Table 1. Statistical Data

	Failure Rate	Repair Work
Number of Samples	715	
Mean Value	$1.659\ 10^{-4}$	8.773
Standard Deviation	$1.654\ 10^{-5}$	0.739
Correlation	0.0097	

As the next step – using the determined input data and equation (2) above – the required repair capacity per unit of production performance should be defined.

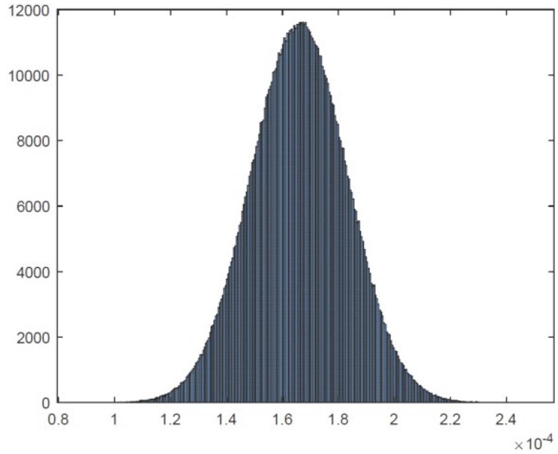

Figure 4. Histogram of Failure Rates [1/Working Hour]

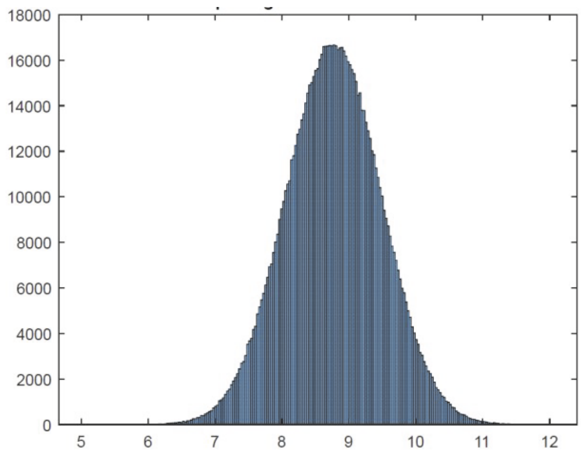

Figure 5. Histogram of Repair Work of one Failure [Working Hour]

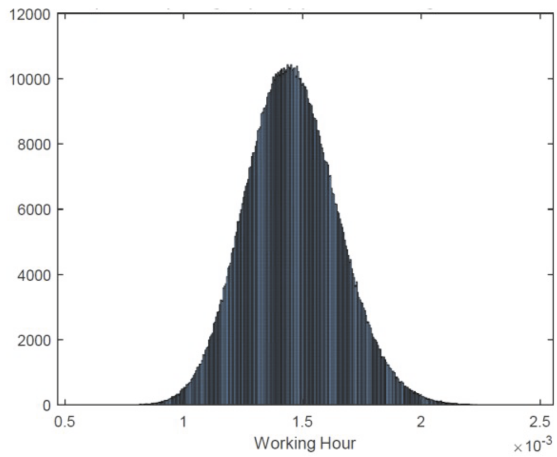

Figure 6. Histogram of Required Repair Capacity per Unit Production Performance [Working Hour]

The number of excitations was increased until the relative difference of results of the last two simulations fell below one thousandth. This excitation number is 1000000, which can provide sufficient statistical data, such that correct conclusions can be drawn from the results of the simulations.

Figures 4 – 7 show the histograms of the simulations.

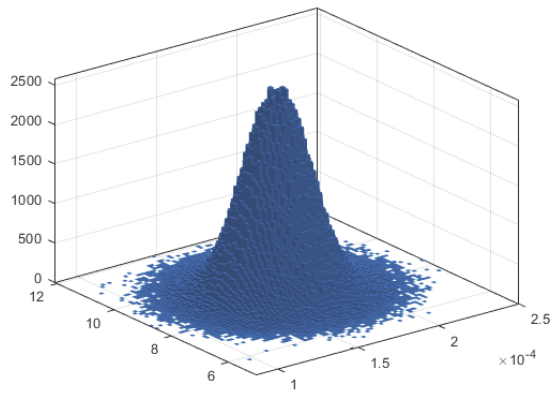

Figure 7. Three-dimensional Histogram of Required Repair Capacity per Unit Production Performance

Table 2 shows results of the simulation,

Table 2. The First Results of Simulation

		W_{unit}
Expected Value	m_{WU}	0.0015
Standard Deviation	s_{WU}	$1.9249 \cdot 10^{-4}$

4.2. Determination of Requested Repair Capacity

The results of model simulation can mainly be useful from the point of view of maintenance management. The most important question is the determination of Required Repair Capacity (RRC) depending on the required estimating uncertainty. This uncertainty indicates the probability that the planned repair capacity will not be sufficient for the occurred failures.

By applying the expected value and the standard deviation of the Required Repair Capacity per Unit Performance, – based on the standard normal distribution – Required Repair Capacity can be determined. For example, if the estimated uncertainty is 10%:

$$RRC = T\left(m_{WU} + 1.29 s_{WU}\right) \quad . \tag{4}$$

Table 3. Required Repair Capacity

Estimating Uncertainty	RRC [Working Hour]
0.1	174.83121
0.05	181.76085
0.02	189.65294
0.01	195.04266
0.005	199.66242

In the case of 100000 productions' welding the RRCs are shown in Table 3.

4.3. Correlation Analysis

The results of the simulation can be used for correlation analysis of parameters. It can determine that the independent variable (in this case: failure rate and repair work of one failure) has the greatest effect on the output parameter (RRC).

The correlation coefficient characterizes the strength of stochastic interdependencies of the random variables [8].

The correlation coefficient $r_{\eta\mu}$ can be determined empirically by the equation:

$$r_{\eta\mu} = \frac{\sum_{i=1}^{n}\left(x_i - \sum_{j=1}^{n} x_j\right)\left(y_i - \sum_{j=1}^{n} y_j\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \sum_{j=1}^{n} x_j\right)^2 \left(y_i - \sum_{j=1}^{n} y_j\right)^2}} \tag{5}$$

using the samples $x_1; x_2; \ldots x_n$ and $y_1; y_2; \ldots y_n$ which belong to the variables η and μ [8].

Table 4 shows the coefficients of Required Repair Capacity per Unit Performance.

Table 4. Correlation Coefficients

Correlation between W_{unit} and λ	0.7842
Correlation between W_{unit} and m	0.6159

Knowing the correlation coefficients, it can be said that the best way to improve the maintenance of the welding cell under test is by reducing the failure rate. This task should be solved by an engineering solution and not by "statistical corrections."

5. CONCLUSIONS

This paper discussed a Monte-Carlo Simulation-based method of repair process analysis. Its potential implementation was demonstrated by a case study. The following conclusions can be deduced from the results of modeling and analysis:

1. The proposed method can be used:
   - for analyzing of maintenance processes;
   - for supporting decision making in maintenance management;
   - for estimating the Requested Repair Capacity depending on the required estimating uncertainty;
   - to determine the input parameter that has the most effect on RRC.

2. The drawback of MCS is that its elapsed time increases significantly if the number of excitations rises (see Figure 8).

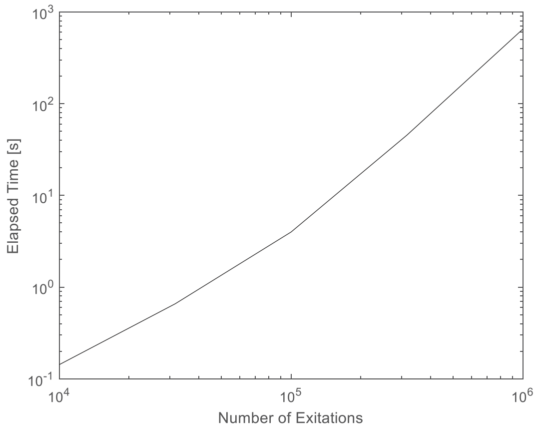

Figure 8. Elapsed Time of Monte-Carlo Simulation depends on Number of Excitations

The Author's planned prospective scientific research related to this field of applied mathematics and maintenance management decision making; includes the study of methodologies of mathematical tools for analysis of maintenance systems and processes, for example stochastic model and simulation-based sensitivity analysis of maintenance systems and processes.

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