New problems – competition A (see page 419): **A. 783.** A *polyomino* is a figure which consists of unit squares joined together by their sides. (A polyomino may contain holes.) Let $n \geqslant 3$ be a positive integer. Consider a grid of unit square cells which extends to infinity in all directions. Find, in terms of n, the greatest positive integer C which satisfies the following condition: For every colouring of the cells of the grid in n colours, there is some polyomino within the grid which contains at most $n-1$ colours and whose area is at least C. (Submitted by *Nikolai Beluhov,* Stara Zagora, Bulgaria and *Stefan Gerdjikov,* Sofia, Bulgaria) **A. 784.** Let n, s, t be positive integers and $0 < \lambda < 1$. A simple graph on n vertices with at least λn^2 edges is given. We say that $(x_1, \ldots, x_s, y_1, \ldots y_t)$ is a*good insertion,* if letters x_i and y_j denote not necessarily distinct vertices and every $x_i y_j$ is an edge of the graph ($1 \leqslant i \leqslant s$, $1 \leqslant j \leqslant t$). Prove that the number of good insertions is at least $\lambda^{st} n^{s+t}$. (Submitted by *Kada Williams,* Cambridge) **A. 785.** Let $k \geqslant t \geqslant 2$ positive integers. For integers $n \geqslant k$ let p_n be the probability that if we choose k from the first n positive integers randomly, any t of the k chosen integers have greatest common divisor 1. Let q_n be the probability that if we choose $k-t+1$ from the first n positive integers the product is not divisible by a perfect t-th power that is greater then 1. Prove that sequences p_n and q_n converge to the same value. (Submitted by *Dávid Matolcsi,* Budapest)

Problems in Physics

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M. 398. Measure the rolling resistance between a cylinder and the ground. Use two different cylinders of the same radius and carry out the measurement for two different surfaces. (The two different cylinders can be for example a paper cylinder of a roll of plastic wrap, and an aluminium foil roll, whilst the two different surfaces can be the floor of the room with and without a soft carpet.) Investigate how much the deceleration of the cylinder can be considered constant.

G. 717. A bat flies parallel to the wall of a cave at a speed of 45.0 m/s. It emits a short ultrasound signal, the echo of which is heard after 0.120 s. How far does the bat fly from the wall? The speed of ultrasound in the cave is 333 m/s. **G. 718.** Suppose the material of the Sun consists of carbon and oxygen. (In the old days, this idea came up seriously.) At most how much would the total lifespan of the Sun be if the coal burns perfectly and the energy radiated in a unit time is the same as it is now? (In the calculations, let us use the actual mass of the Sun.) **G. 719.** A closed beverage can of size 330 ml is floating in water. The can is made of aluminium, and the mass of the empty can is 13 g. How many millilitres of gas is in the closed can, if it contains exactly 330 ml of soft drink of density approximately the same as that of water? **G. 720.** In the Tour de France cycling race, the riders go uniformly at a speed of 50 km/h on a horizontal road. The distance between the peloton and the breakaway riders is 1 km. When the riders reach an approximately 5 km long climb their speed soon decreases to 40 km/h, and when they move downwards also along a distance of 5 km their speed soon increases to 60 km/h. Sketch the distance between the peloton and the breakaway group as a function of time from the moment when the breakaway reaches the climb, until the moment it reaches the end of the downhill slope.

P. 5250. A car travels at a constant speed along a long, straight road. Consider a point on the rim of the wheel of the car. Investigate the whether *a*) the average speed of this point is greater, smaller or equal to the speed of the car; *b*) the magnitude of the average velocity of this point is greater, smaller or equal to the speed of the car. **P. 5251.** A small body of mass m is released from rest at point A of a fixed prism shown in the *figure.* The body slides frictionlessly along the straight slope on the left side and along the

circular path of radius R. The straight slope on the right is not frictionless, the coefficient of friction is μ. *a*) What force is exerted by the body on the prism at the lowest point of the path? *b*) What is the speed of the body at point C? *c*) To what height h will the body go up? *Data:* $m = 0.6$ kg, $R = 30$ cm, $\alpha = 60^\circ$, $\mu = \frac{1}{2}\tan\alpha$. **P. 5252.** A thin-walled tube of mass M is rolled by pulling a yarn wound around the tube as shown in the *figure*. The tube rolls at a constant speed along the horizontal floor without slipping. Inside the tube there is a small object, which slides on the wall of the tube and remains at a constant angular position, the coefficient of kinetic friction is μ. What is the tension in the yarn when the speed of the tube is constant? **P. 5253.** The average depth of the lake Pécsi-tó, which is next to the village Orfű, is 3.3 m. How much should the temperature of the water change from 25 °C in order that the water level decrease by 0.5 cm? **P. 5254.** One mole of air initially at standard conditions is compressed isothermally to half of its initial volume and then it is allowed to expand adiabatically to its original volume. *a*) What is the total work done on the gas during the process? *b*) How much heat is released by the gas? *c*) What is the change in the internal energy of the gas? *d*) What is the final temperature of the gas? **P. 5255.** A point-like object of charge $Q = 3 \cdot 10^{-7}$ C is fixed above a very long piece of insulating thread of mass $m = 10$ g, $d = 5$ cm above the midpoint of the thread. The insulating thread is also fixed and then charged uniformly, to a linear charge density of $\sigma = -2 \cdot 10^{-6}$ C/m. At what acceleration does the thread begin to move if it is released without any initial speed? **P. 5256.** How does the capacitance of a parallel plate capacitor change if the space between its plates is filled with two types of uniform, insulating material of two different dielectric constants, and the surface which separates them is *a*) perpendicular to the plates; *b*) parallel to the plates of the condenser? **P. 5257.** *Roland Eötvös* demonstrated the law named after his teacher at Koenigsberg – *Franz Ernst Neumann* (1798–1895) – as follows: he stretched two long pieces of metal wires in a room horizontally and parallel to each other at a high position and connected their ends on one side through a sensitive galvanometer. To the other ends a piece of moveable metal rod was connected perpendicularly to the wires. Then he slid the rod along the wires such that it remained perpendicular to the wires, whose distance was 2 m. According to the measurements at that time the angle between direction of the magnetic field of the Earth and the horizontal was 62°. The horizontal component of the magnetic field was measured to be 0.2 oersted, in the CGS system of units used at that time. At what speed did Eötvös pull the rod if the reading on the galvanometer was 80 μV? **P. 5258.** We would like to create a sharp image of the filament of an incandescent lamp with a converging lens exactly below the lamp on a white sheet of paper lying on the tabletop. At least how many dioptres is the power of the lens if the paper is 40 cm below the lamp? **P. 5259.** In a particle accelerator a beam of deuteron of energy 200 keV hits a target, the current is 0.3 mA. The deuterons stop in the target. *a*) How much thermal energy should be taken away from the target in each second if the target does not warm up? *b*) Will the result change if instead of deuterons, the same energy electrons or α particles, which give the same current, hit the target? **P. 5260.** A piece of thread runs around a fixed cylinder having a horizontal axis. If an object of mass m is attached to the left end of the rope and another object of mass $3m$ is attached to the right end of the rope, the objects, which were released from rest, move with an acceleration of 2 m/s^2. *a*) What is the acceleration of the objects if the mass of the bodies at both sides is first doubled, and then tripled? *b*) What is the acceleration of the objects if on the right side the object of $3m$ remains, but to the left end of the rope an object of mass $8m$ is attached? *c*) How should the mass of the object at the left be changed if on the right the object of mass $3m$ is not changed and the system stays at rest after releasing the objects? The rope is very light, and the coefficients of static and kinetic friction between the rope and the cylinder are the same.