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# Relation Between Static and Dynamic Modulus of Elasticity of Wood

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**Abstract –** Static and dynamic modulus of elasticity (MOE) of spruce lumber were determined under different conditions like cross head speed, bending and longitudinal vibration, and mode numbers. The characteristic time of MOE determination is introduced. Characteristic time is defined as the typical MOE determination time. Shorter characteristic times are shown to result in higher MOE values. An order of magnitude change in characteristic time resulted in a 1.7% change in MOE. We found clear evidence that creep exists on a short time scale.

**modulus of elasticity / static and dynamic determination / creep wood**

**Kivonat – Kapcsolat a faanyag statikus és dinamikus rugalmassági modulusza között.** Lucfenyő fűrészárú rugalmassági moduluszának meghatározását a mérési körülmények befolyásolják, úgy mint a az anyagvizsgáló gép sebessége, dinamikus mérés esetén a hajlító és longitudinális rezgések, illetve az alkalmazott módusok száma. A rugalmassági modulusz meghatározásának jellemzésére a karakterisztikus időt vezettük be, mely a mérésre fordított időt jelenti. A rövidebb karakterisztikus idő magasabb rugalmassági moduluszt eredményez. Egy nagyságrend változás a karakterisztikus időben, 1,7% változást jelent a rugalmassági modulusz meghatározásában. Ez egyértelmű bizonyítéka a kúszás jelenségének rövid időtartományokon.

**rugalmassági modulusz / statikus és dinamikus meghatározás / kúszás / faanyag**

## 1 INTRODUCTION

Since 1960, much attention has been paid to the non-destructive evaluation of wood, especially with regard to mechanical grading. A large number of papers deal with the determination of the modulus of elasticity in bending (MOE) and its correlation with modulus of rupture in bending (MOR). The result of this research demonstrates that the most important strength predictor parameter is MOE. This predictor has been determined by static and dynamic methods. Because the different methods of determination give a dynamic value that is about 10% higher than the static value, the two values of MOE have been handled as different parameters. Static and dynamic MOE determination were compared by several authors (Perstorper 1994; Tanaka *et al*. 1991; Kliger *et al*. 1992, Jugo and Ozarska 1996) who found

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good correlation ($r^2$: 0.90 – 0.96) between the two MOE values. In this study we demonstrate that the difference between dynamic and static MOE values can be explained by the effect of creep.

The deflection of a beam under load is composed of the sum of elastic deflection and deflection caused by creep. In practice, the effect of creep is not often taken into consideration. In the standard methods for the determination of MOE, there are regulations for controlling strain rate in the tests. In the case of dynamic MOE determination, the effect of creep is usually not taken into consideration.

## 2 METHOD OF MOE DETERMINATION

Evaluating the effect of creep on MOE determination requires the measurement of MOE as precisely as possible. We need to take into account the effects of shear and other influencing factors and to eliminate the effects of temperature and moisture changes. The air temperature and humidity of the laboratory was controlled (20C$^o$ and 70% respectively). Testing started after 4 months of conditioning, so specimen temperature was 20 Co and moisture content was 13.3+/-0.4%. To eliminate the effect of defects we used clear spruce specimens, that is, specimens without knots, slope of grain and other imperfections. All specimens were taken from one large spruce beam. The typical specimen size was 5.5 by 11.0 by 130 cm. In the study we utilised the following methods for determining MOE: stress waves, dynamic bending and static bending. These 3 methods cover a wide range in characteristic time (to be defined later) from 1 ms to 600 s.

When a stress wave of velocity $V$ is induced in a bar of density $\rho$ and length $L$ then

$$MOE_{str} = \rho V^2 = \rho\left(2Lf_{long}\right)^2 \tag{1}$$

$MOE_{str}$ is the modulus of elasticity determined by this method. Velocity of stress wave is often determined by the frequency ($f_{long}$) of longitudinal vibration. In the stress wave MOE calculation, a correction

$$f = f_0\left(1 + \frac{n^2\pi^2\mu^2\left(a^2+b^2\right)}{24L^2}\right) \tag{2}$$

was used (Rayleigh 1945) where $f$ is the limiting frequency for a very long beam, $f_o$ is the observed frequency, $\mu$ is the Poisson's ratio, $a$ and $b$ are the dimensions of the bar, $n$ is the mode number.

Bending vibration provides a rather quick and precise method for determining MOE. In this method the beam is supported at the nodal points by soft material. In *Figure 1*, two vibration modes and associated nodal points are shown. Timoshenko's beam theory (Timoshenko and Young 1954) and Hearmon's correction (Hearmon 1966) were used in the evaluation of MOE. Since MOE is included in Hearmon's formula, an iterative process is necessary to determine the correction factor.

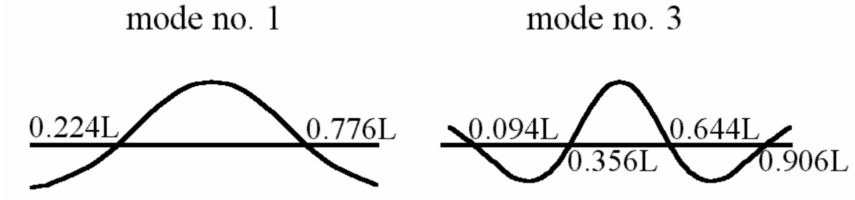


*Figure 1. Locations of nodal points in free bending vibration of bar*

*for the first and third vibration mode*

A testing machine was used to measure the static modulus of elasticity ($MOE_{stat}$). In the case of 3 point loading, the following formula accounts for the effect of shear:

$$MOE_{stat} = \frac{Fl^3}{48I\Delta S - 3Flb^2 / 2G} \quad (3)$$

where: *F*: the applied force

*ΔS*: deflection (within the elastic range)

*l*: span

*b*: depth of specimen

*G*: shear modulus

*I*: $ab^3/12$ (*a* is the width of the specimen)

The shear modulus (*G*) of the specimen was determined by torsional vibration, based on the method described by Hearmon (Hearmon 1966).

It is not easy to compare the three modulus of elasticity determinations. The stress distributions in the beam are different for the different methods and, due to the heterogeneity of wood, the determined MOE values are also different. However, with the use of defect free specimens, it is possible to keep the effect of different stress distributions in the different methods to a minimum. Another main difference is the characteristic time of determination. With this approach, the effect of time causing creep can be evaluated. We strongly believe that creep exists not only over a long time scale, but also over a short duration.

We need to determine the characteristic time of the measurement. The characteristic time of stress wave MOE determination is the time of one period of longitudinal vibration: *T=1/f*. This is true for beams shorter than 2.5 m. However, for longer beams, the longitudinal stress waves separate (see *Figure 2*). In this case, the characteristic time cannot be determined by *1/f* because it becomes constant at about 1 ms.

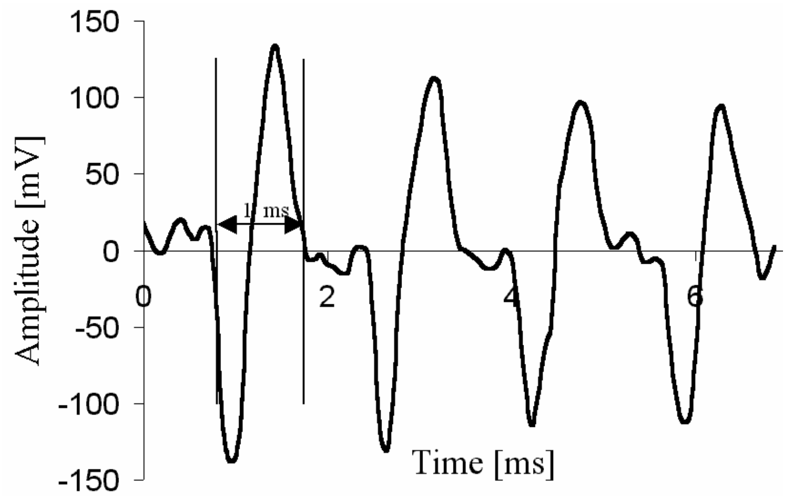


*Figure 2. Separation of longitudinal stress waves in a 427 cm long 2 by 4 specimen.*

The characteristic time for dynamic bending MOE determination is simply the time of one period of longitudinal vibration regardless of the specimen dimensions.

The time for static MOE measurement varies according to the maximum deflection attained during the test. The standard methods for small clear specimens specify the rate of strain or the rate of crosshead movement. For example ASTM D143-83 specifies 2.5 mm/min. for crosshead speed. In case of construction size timber the crosshead speed depends on the dimensions of the specimen. For the timber specimens we used, the time it took to reach the 4 MPa stress level was taken as the characteristic time. This method of determination of characteristic time is arbitrary. The 4 MPa stress level was chosen because with this characteristic time we got the best fit between static and dynamic measurements. This characteristic time represents the time needed for 1 mm deflection at 1 m span in 3 point loading.

## 3 RESULTS AND DISCUSSION

We have defined a creep parameter (η'), in order to analyse the effect of creep on MOE determination as:

$$\eta' = \frac{MOE_t - MOE_0}{MOE_0} 100 \tag{4}$$

where: $MOE_0$: reference modulus of elasticity,
$MOE_t$: the MOE at *t* characteristic time.

*Figure 3a* shows the measured MOE for 21 clear spruce specimens as a function of characteristic time. Static MOE at 0.1, 1, 10 and 100 mm/min crosshead speed (600, 60, 6 and 0.6 sec characteristic time respectively), dynamic bending MOE at a mode number 1 and 3, and stress wave MOE were determined. The time scale is logarithmic. The tendency is that a higher MOE is obtained with shorter characteristic time. Using these data we calculated the creep parameter *η'* using formula (4) where the MOE measured at 10 mm/min. crosshead speed was chosen as the reference: $MOE_0$. *Figure 3b* shows the creep parameter (*η'*) as a function of characteristic time.

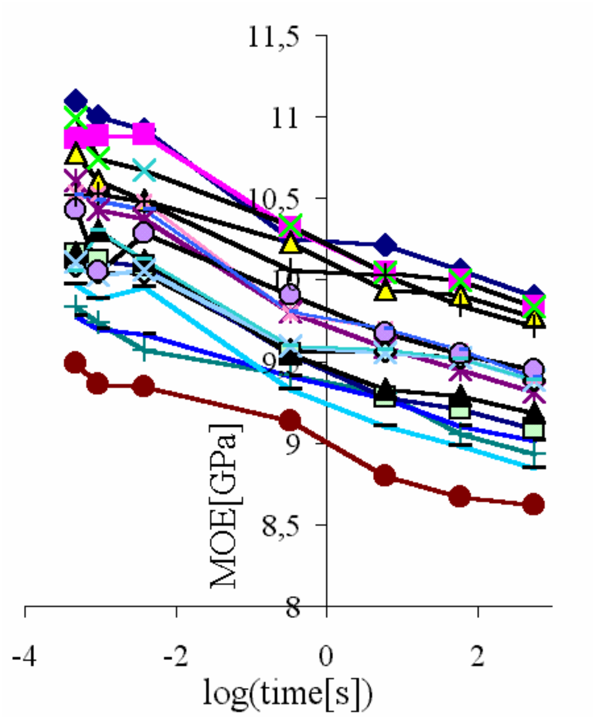


*Figure 3a.*
*Measured stress wave, dynamic bending and static MOE as a function of characteristic time*

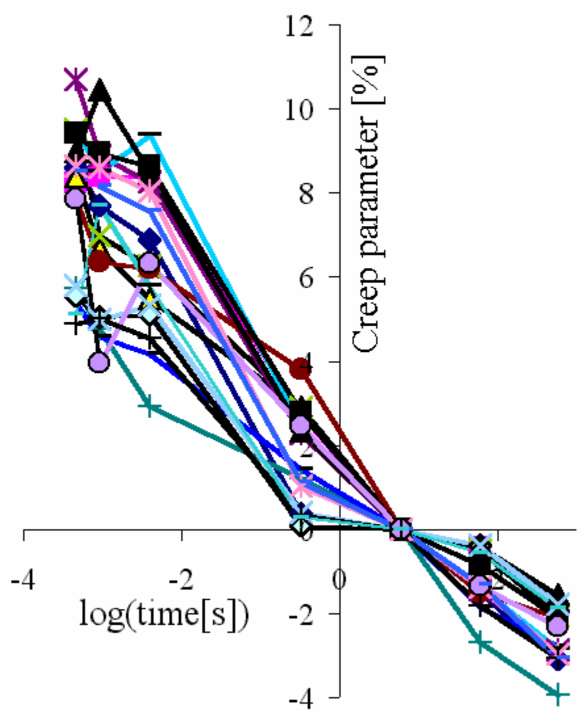


*Figure 3b.*
*Creep parameter, η', as a function of characteristic time*

To define the most likely relationship we used the average MOE of 21 specimens. This averaging reduced the scatter caused by the heterogeneity of wood and measurement error. The average MOE values are plotted in *Figure 4*.

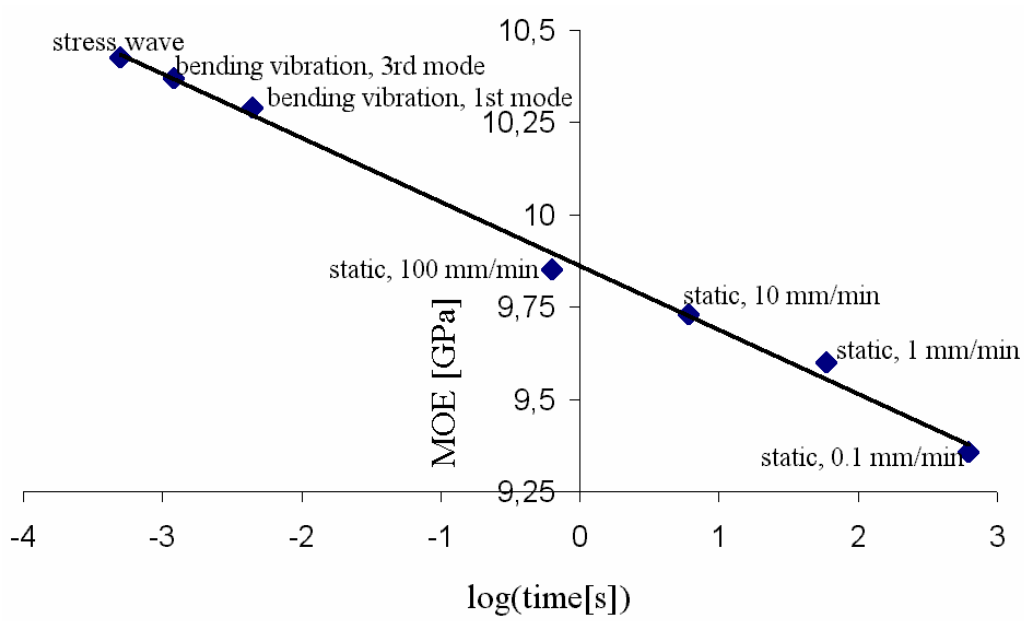


*Figure 4. Average MOE as a function of characteristic time.*

The averaged points were fitted with a straight line. Practical use of this result is a formula that calculates the effect of the characteristic time:

$$MOE_{t1} = MOE_{t2}\left(1 + 0.017\log\left(t_2 / t_1\right)\right) \quad (5)$$

where: $t_1$: the characteristic time of $MOE_{t1}$ determination
$t_2$: the characteristic time of $MOE_{t2}$ determination

For example, using the formula (5), it is possible to predict the static MOE using dynamic MOE data. In this case, $t_1$ equals 25 seconds, which is the characteristic time of the standard static MOE determination for a small clear specimen.

The effect of creep appears in a paper by Nakao (Nakao *et al*. 1995) where MOE of wood is plotted as a function of resonance order. At higher resonance order (shorter characteristic time) the author observed higher MOE. The line shown in *Figure 4* also demonstrates the effect of creep in short time scales. One order of magnitude change in characteristic time results 1.7% change in MOE. The effect of creep on dynamic modulus of elasticity determination is important, not only for wood, but also for other materials where creep is not negligible, such as plastics.

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