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A NOTE ON SPACES WITH LOCALLY COUNTABLE WEAK-BASES

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Abstract. In this paper, we show that a regular space with a locally countable weak-base is g-metrizable. Secondly, we establish the relationships between spaces with a locally countable weak-base (resp. spaces with a locally countable weak-base consisting of $\aleph_0$-subspaces) and metric spaces (resp. locally separable metric spaces) by means of compact-covering maps, 1-sequence-covering maps, compact maps, π-maps and ss-maps, and show that all these characterizations are mutually equivalent. Thirdly, we show that 1-sequence-covering, quotient, ss-maps preserve spaces with a locally countable weak base.

1. Introduction

Weak-bases were introduced by A.V. Arhangel'skii [1]. Spaces with a locally countable weak-base were discussed in [8, 14, 18], and some results were given. For example:

Theorem A ([14]). *A regular space has a locally countable weak-base if and only if it is a quotient, π(or compact), ss-image of a metric space.*

Theorem B ([8]). *A regular space has a locally countable weak base if and only if it is a 1-sequence-covering, quotient, ss-image of a metric space.*

A space is a locally separable metric space if and only if it is a regular space with a locally countable base [2]. Thus, one may investigate the further properties of locally separable metric spaces by means of the discussion of properties of spaces with a locally countable weak-base. From the classical Nagata-Smirnov metrization theorem we know that a regular space with a locally countable base has a σ-locally finite base. So, the following question can be raised:

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Question 1. *Is a regular space with a locally countable weak-base a space with a σ-locally finite weak-base?*

Since a space with a locally countable weak-base is a generalization of a locally separable metric space, and since our purpose is to bring out properties of locally separable metric spaces by means of that of the space with a locally countable weak-base, according to Alexandroff's hypothesis, the following question can be raised:

Question 2. *By means of what map can we establish the relationship between spaces with a locally countable weak space and locally separable metric spaces?*

In this paper, we show that a regular space with a locally countable weak-base has a σ-locally finite weak base. Secondly, we further discuss spaces with a locally countable weak-base by means of compact-covering maps, 1-sequence-covering maps, π-maps, compact-map and ss-maps. Thirdly, we show that 1-sequence-covering, quotient, ss-maps preserve spaces with a locally countable weak-base.

In the following, all spaces are regular, all maps are continuous and surjective. N denotes the set of all natural numbers. ω denotes $N \cup \{0\}$. For a family $\mathcal{P}$ of subsets of a space X and a map $f\colon X \to Y$, denote $f(\mathcal{P}) = \{f(P) : P \in \mathcal{P}\}$. Readers can refer to [23, 13] for unstated definitions.

Definition 1.1. Let $f\colon X \to Y$ be a map.

(1) f is a compact-covering map ([20]) if each compact subset of Y is the image of some compact subset of X.
(2) f is a 1-sequence-covering map ([12]) if for each $y \in Y$, there exists $x \in f^{-1}(y)$ satisfying the following condition: whenever $\{y_n\}$ is a sequence of Y converging to a point y in Y, then there exists a sequence $\{x_n\}$ of X converging to a point x in X such that each $x_n \in f^{-1}(y_n)$.
(3) f is a strong sequence-covering map ([11]) if each convergent sequence (including its limit point) of Y is the image of some convergent sequence(including its limit point) of X.
(4) f is a sequence-covering map [5] if each convergent sequence(including its limit point) of Y is the image of some compact subset of X.
(5) f is a π-map if (X, d) is a metric space and for each $y \in Y$ and its open neighborhood V in $Y, d(f^{-1}(y), X \backslash f^{-1}(V)) > 0$ ([22]).
(6) f is an ss-map ([14]) if for each $y \in Y$, there exists a open neighborhood V of y in Y such that $f^{-1}(V)$ is separable in X.

It is clear that

$$\begin{array}{ccc} \text{1-sequence-covering maps} & \Rightarrow & \text{strong sequence-covering maps} \\ & & \Downarrow \\ \text{compact-covering maps} & \Rightarrow & \text{sequence-covering maps.} \end{array}$$

Every compact map on a metric space is a π-map.

Definition 1.2. Let $\mathcal{P}$ be a cover of a space X.

(1) $\mathcal{P}$ is a network X if for whenever $x \in V$ with V open in X, then $x \in P \subset V$ for some $P \in \mathcal{P}$.
(2) $\mathcal{P}$ is a k-network for X if for each compact subset K of X and its open neighborhood V, there exists a finite subfamily $\mathcal{P}'$ of $\mathcal{P}$ such that $K \subset \mathcal{P}' \subset V$ ([21]).
(3) $\mathcal{P}$ is a cs-network for X if for each $x \in X$, its open neighborhood and a sequence $\{x_n\}$ converging to x, there exist $P \in \mathcal{P}$ such that $\{x_n : n \geq m\} \cup \{x\} \subset P \subset V$ for some $m \in N$ ([6]).

A space is a cosmic space if it has a countable network ([20]).

A space is an $\aleph_0$-space if it has a countable k-network, and it is equivalent to a space with a countable cs-network ([20]).

A space X is an $\aleph$-space if X has a σ-locally finite k-network ([21]).

Definition 1.3 ([4]). For a space X and $x \in P \subset X$, P is a sequential neighborhood of x in X if whenever $x_n \to x$, then $\{x_n : x \geq m\} \cup \{x\} \subset P$ for some $m \in N$. P is a sequential open set of X if for each $x \in P$, P is a sequential neighborhood of x in X.

A space X is a sequential space if each sequential open set of X is open in X.

Definition 1.4. Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a family of subsets of a space X satisfying that for each $x \in X$,

(1) $\mathcal{P}_x$ is a network of x in X.
(2) If $U, V \in \mathcal{P}_x$, then $W \subset U \cap V$ for some $W \in \mathcal{P}_x$.

$\mathcal{P}$ is a weak-base for X if $G \subset X$ such that for each $x \in G$, there exists $P \in \mathcal{P}_x$ satisfying $P \subset G$, then G is open in X. $\mathcal{P}$ is an sn-network ([12]) (i.e., an sequential neighborhood network) for X if each element of $\mathcal{P}_x$ is a sequential neighborhood of x in X, here $\mathcal{P}_x$ is an sn-network of x in X.

A space X is a g-first countable space (resp. a sn-first countable space [13]) if X has a weak-base (resp. a sn-network) $\mathcal{P}$ such that each $\mathcal{P}_x$ is countable ([1]).

A space X is a g-second countable space if X has a countable weak-base ([1]).

A space X is a g-metrizable space if X has a σ-locally finite weak-base ([23]).

For a space, weak-base $\Rightarrow$ sn-network $\Rightarrow$ cs-network. An sn-network for a sequential space is a weak-base (see [12]).

We have the following implications for a space X [23, 24, 13, 3].

metrizable $\Rightarrow$ g-metrizable $\iff$ symmetrizable $+\aleph$-space $\iff$ g-first countable$+\aleph$-space $\Rightarrow$ symmetrizable $\Rightarrow$ k-space $\Leftarrow$ sequential space $\Leftarrow$ g-first countable $\Rightarrow$ sn-first countable $\Rightarrow$ α_4-space.

2. Result

Lemma 2.1 ([14]). *The following are equivalent for a space X:*

(1) *X has a locally countable weak-base.*
(2) *X is a g-first countable space with a locally countable k-network.*
(3) *X is a topological sum of g-second countable spaces.*

Theorem 2.2. *A space has a locally countable weak-base if and only if it is a locally Lindelöf, g-metrizable space.*

Proof. The 'if' part is obvious, because every σ-locally finite cover in any locally Lindelöf space is locally countable.

The 'only if' part: Suppose a space X has a locally countable weak-base. Then X is a g-first countable space with a locally countable k-network by Lemma 2.1, and so X is a k-space with a locally countable k-network. By Theorem 1 in [9], X is an $\aleph$ -space. Thus X is g-metrizable by Theorem 2.4 in [3]. By Lemma 2.1, X is a topological sum of g-second countable spaces. Since g-second countable spaces is Lindelöf, then X is locally Lindelöf. □

From Theorem 2.2 and Theorem 1.13 in [23], the following holds.

Corollary 2.3. *Let X be a space with a locally countable weak-base. If (1) or (2) below holds, then X is metrizable.*

(1) *X is a Fréchet space.*
(2) *X is a q-space.*

Lemma 2.4 ([24]). *Suppose (X, d) is a metric space and $f\colon X \to Y$ is a quotient map. Then Y is a symmetric space if and only if f is a π-map.*

Theorem 2.5. *The following are equivalent for a space X:*

(1) *X has a locally countable weak-base.*
(2) *X is a compact-covering, 1-sequence-covering, quotient, π, ss-image of a metric space.*
(3) *X is a quotient, π, ss-image of a metric space.*
(4) *X is a 1-sequence-covering, quotient, ss-image of a metric space.*

Proof. (1) ⇒ (2). Suppose $\mathcal{P}$ is a locally countable weak-base for X, then $\mathcal{P}$ is a sn-network for X. Denote $\mathcal{P} = \{P_\alpha : \alpha \in A\}$. For each $i \in N$, let A_i be a copy of A, and it is endowed with discrete topology. Put

$$M=\left\{\alpha = (\alpha_n) \in \prod_{n\in N} A_n : \{P_{\alpha_n} : n \in N\} \text{ is a network of some point } x_\alpha \text{ in } X\right\}$$

and give M the subspace topology induced from the product topology of the product space $\prod_{n\in N} A_n$. The point x_α is unique in M because X is T_2. We define $f\colon M \to X$ by $f(\alpha) = x_\alpha$. Obviously, M is a metric space.

(i) f is an ss-map.

Let $\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}$ be a locally countable sn-network for X, and $\mathcal{P}_x = \{P_{\alpha_n} : n \in N\}$, $\alpha = (\alpha_n)$, then $\alpha \in M$ and $f(\alpha) = x$. Thus f is surjective. For each $\alpha = (\alpha_n) \in M$, we have $f(\alpha) = x_\alpha$. If U is an open neighborhood of x_α in X, then there exists $n \in N$ with $x_\alpha \in P_{\alpha_n} \subset U$ because $\{P_{\alpha_n} : n \in N\}$ is a network of x_α in X. Put $W = \{\beta \in M : \text{the } n\text{-th coordinate of } \beta \text{ is } \alpha_n\}$, then W is an open neighborhood V of x in X such that $\{\alpha \in A : V \cap P_\alpha \neq\}$ is countable. Put $L = \left(\prod_{n\in N} \{\alpha \in A_n : V \cap P_\alpha \neq\}\right) \cap M$, then L is a second countable subspace of M, and so L is a hereditarily separable subspace of M. Since $f^{-1}(V) \subset L$, thus $f^{-1}(V)$ is a separable subspace of M. Hence f is an ss-map.

(ii) f is a 1-sequence-covering map.

Put $\beta = (\alpha_i)$, then $\beta \in f^{-1}(x)$. Denote $B_n = \{(\gamma_i) \in M : \text{if } i \leq n, \text{ then } \gamma_i = \alpha_i\}$. Then $\{B_n : n \in N\}$ is a monotonic decreasing neighborhood base of β in M. For each $n \in N$, it is easy to check that $f(B_n) = \bigcap_{i\leq n} P_{\alpha_i}$. For a convergent sequence $\{x_j\}$ of X with $x_j \to x$, since $f(B_n)$ is a sequential neighborhood of x in X, there exists $i(n) \in N$ such that if $i \geq i(n)$, then $x_i \in f(B_n)$. Thus $f^{-1}(x_i) \cap B_n \neq$. We may assume $1 < i(n) < i(n+1)$. For each $j \in N$, let

$$\beta_j \in \begin{cases} f^{-1}(x_j), & \text{if } j < i(1), \\ f^{-1}(x_j) \cap B_n, & \text{if } i(n) \leq j < i(n+1), n \in N. \end{cases}$$

Then it is easy to show that the sequence $\{\beta_j\}$ converges to β in M. Hence f is 1-sequence-covering.

(iii) f is a compact-covering map.

For each compact subset K of X. Since X has a locally countable k-network $\mathcal{F}$ by Lemma 2.1, then $\{F \cap K : F \in \mathcal{F}\}$ is a countable k-network for subspace K. Thus K is metrizable because a compact spaces with a countable k-network is metrizable. Similar to the proof of Theorem 2 in [11], we can prove that f is compact-covering.

(iv) f is a quotient map.

By (ii) and Proposition 2.1.16(2) in [10], f is a quotient map.

(v) f is a π-map.

By (iv), Theorem 2.2 and Lemma 2.4, f is a π-map.

(2) ⇒ (3) and (2) ⇒ (4) are obvious.

(3) ⇒ (1). Suppose X is a quotient, π, ss-image of a metric space. By Lemma 2.4, X is a symmetric space, so X is a g-first countable space. By Corollary 2.8.9 in [10], X has a locally countable k-network. Hence X has a locally countable weak-base by Lemma 2.1.

(4) ⇒ (1). Suppose $f\colon M \to X$ is a 1-sequence-covering, quotient, ss-map, where M is a metric space. Let $\mathcal{B}$ be a σ-locally finite base for M. For each

$x \in X$, there exists $\beta_x \in f^{-1}(x)$ satisfying Definition 1.1(2). Put

$$\mathcal{P}_x = \{f(B) : \beta_x \in B \in \mathcal{B}\},$$

$$\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}.$$

Then, it is easy to check that $\mathcal{P}$ is a locally countable sn-network for X. Since X is a sequential space, thus $\mathcal{P}$ is a locally countable weak-base. □

Theorem 2.6. *The following are equivalent for a space X:*

(1) *X has a locally countable weak-base consisting of cosmic subspaces.*
(2) *X has a locally countable weak-base consisting of $\aleph_0$-subspaces.*
(3) *X is a compact-covering, 1-sequence-covering, quotient, π, ss-image of a locally separable metric space.*
(4) *X is a 1-sequence-covering, quotient, ss-image of a locally separable metric space.*

Proof. (1) ⇒ (2) follows from Theorem 7(2) in [19].

(2) ⇒ (3). Let $\mathcal{P}$ be a locally countable weak base for X consisting of $\aleph_0$-subspaces. Denote $\mathcal{P} = \{P_\alpha : \alpha \in \bigwedge\}$. For each $\alpha \in \bigwedge$, P_α is an $\aleph_0$-subspace, then P_α has a countable cs-network. For each $x \in P_\alpha$, $\{P_\beta \cap P_\alpha : x \in P_\beta \text{ and } \beta \in \bigwedge\}$ is a countable sn-network of x in subspace P_α, then P_α is a sn-first countable space, and so P_α is an α_4-space (see [13]). By Theorem 3.18 in [13], P_α has a countable sn-network. Let $\mathcal{P}_\alpha$ be a countable sn-network for subspace P_α. Denote $\mathcal{P}_\alpha = \{B_a : a \in A_\alpha\}$, here A_α is countable. Endow A_α with discrete topology. Put

$$M_\alpha = \{\beta = (a_i) \in A_\alpha^\omega : \{B_{a_i} : i \in N\} \text{ forms a network at some point } x(\beta) \text{ in } P_\alpha\}$$

and endow M_α with the subspace topology induced from the product topology of the usual product space A_α^ω, then M_α is a separable metric space. Define $f_\alpha \colon M_\alpha \to P_\alpha$ by $f_\alpha(\beta) = x(\beta)$ for each $\beta \in M_\alpha$. As in the proof of Theorem 2.5, we can prove that f_α is a compact-covering, 1-sequence-covering map. Put

$$M = \bigoplus_{\alpha \in \bigwedge} M_\alpha,\ Z = \bigoplus_{\alpha \in \bigwedge} P_\alpha \text{ and } f = \bigoplus_{\alpha \in \bigwedge} f_\alpha : M \to Z.$$

Then, M is a locally separable metric space and f is a compact -covering, 1-sequence-covering map. Define $g \colon Z \to X$ a natural map, and let $h = g \circ f \colon M \to X$. Then g is a compact-covering, 1-sequence-covering map, and so h is a compact-covering, 1-sequence-covering map (see [7, Theorem 2.3, Corollary 2.4]). Because X is a sequential space, then h is a quotient map. Thus, h is a π-map by Lemma 2.4.

For each $x \in X$, since $\mathcal{P}$ is locally countable, there exists an open neighborhood U of x in X such that $\{\alpha \in \bigwedge : P_\alpha \cap U \neq \Phi\}$ is countable. Because $h^{-1}(U) \subset \bigoplus\{M_\alpha : \alpha \in \bigwedge \text{ and } P_\alpha \cap U \neq \Phi\}$, then $f^{-1}(U)$ is separable in M. Hence h is an ss-map.

(3) ⇒ (4) is clear.

(4) $\Rightarrow$ (1). Let $f\colon M \to X$ be a 1-sequence-covering, quotient, ss-map, where M is a locally separable metric space. Suppose $\mathcal{B}$ is a σ-locally finite base for M consisting of separable subspace, then $f(\mathcal{B})$ consists of cosmic subspaces. For each $x \in X$, there exists $\beta_x \in f^{-1}(x)$ satisfying Definition 1.1(2). Put

$$\mathcal{P}_x = \{f(B) : \beta_x \in B \in \mathcal{B}\},$$

$$\mathcal{P} = \cup\{\mathcal{P}_x : x \in X\}.$$

Obviously, $\mathcal{P} \subset f(\mathcal{B})$. Thus, $\mathcal{P}$ is a locally countable weak-base of cosmic subspaces. □

Theorem 2.7. *The following are equivalent for a space X:*

1. *X has locally countable weak-base.*
2. *X is a compact-covering, quotient, compact, ss-image of a locally separable metric space.*
3. *X is a quotient, compact, ss-image of a locally separable metric space.*
4. *X is a quotient, π, ss-image of a locally separable metric space.*
5. *X is a 1-sequence-covering, quotient, ss-image of a locally separable metric space.*

Proof. (1) $\Rightarrow$ (2). Suppose X has a locally countable weak-base. By Lemma 2.1, X is a topological sum of g-second countable spaces. Let $X = \bigoplus_{\alpha\in\wedge} X_\alpha$, where each X_α is a g-second countable space. By Corollary 4.7 in [16], there are a separable metric space M_α and a compact-covering, quotient, compact map f_α from M_α onto X_α. Put

$$M = \bigoplus_{\alpha\in\wedge} M_\alpha \text{ and } f = \bigoplus_{\alpha\in\wedge} f_\alpha : M \to X.$$

Then, M is a locally separable metric space and f is a quotient, compact, ss-map. It will suffice to show that f is a compact-covering map.

For each compact subset K of X, $K \subset \bigcup_{i=1}^{n} X_{\alpha_i}$ for some finitely many $\alpha_i \in \wedge$. Since every X_{α_i} is both open and closed in X, $K \cap X_{\alpha_i}$ is compact in X_{α_i}, and so $f_{\alpha_i}(L_i) = K \cap X_{\alpha_i}$ for some compact subset L_i of M_{α_i} for each $i \le n$. Let $L = \bigoplus_{i=1}^{n} L_i$. Then L is compact in M with $f(L) = K$. Hence f is compact-covering.

(2) $\Rightarrow$ (3) $\Rightarrow$ (4) are clear.

(4) $\Rightarrow$ (1) is similar to the proof of Theorem 2.5 (3)$\Rightarrow$(1).

(1) $\Rightarrow$ (5). Suppose X has a locally countable weak-base. By Lemma 2.1, X is a topological sum of g-second countable spaces. Let $X = \bigoplus_{\alpha\in\wedge} X_\alpha$, where each X_α is g-second countable. As in the proof of Theorem 2.6 (2) $\Rightarrow$ (3), there are a separable metric space M_α and a 1-sequence-covering map f_α from

M_α onto X_α. Put

$$M = \bigoplus_{\alpha \in \wedge} M_\alpha \text{ and } f = \bigoplus_{\alpha \in \wedge} f_\alpha : M \to X.$$

Then, M is a locally separable metric space and f is a 1-sequence-covering, quotient, ss-map from M onto X. Thus X is a 1-sequence-covering, quotient, ss-image of a locally separable metric space.

(5) $\Rightarrow$ (1) is similar to the proof of Theorem 2.5 (4) $\Rightarrow$ (1). □

Remark 2.8. A compact-covering, quotient, compact image of a locally compact metric space $\not\Rightarrow$ a space with a point-countable cs-network; see Example 9.8 in [5] or Example 2.9.27 in [10]. Thus, the condition "ss-" in Theorem 2.7 (1) $\sim$ (4) cannot be omitted.

By Theorem 2.5-2.7, we have

Corollary 2.9. *The following conditions (a) $\sim$ (c) are mutually equivalent for a space X:*

(a) *Theorem 2.5 (1) $\sim$ (4).*
(b) *Theorem 2.6 (1) $\sim$ (4).*
(c) *Theorem 2.7 (2) $\sim$ (4).*

Lemma 2.10 ([14]). *Suppose Y is a quotient ss-image of a sequential space X with a locally countable k-network, then Y has a locally countable k-network.*

Theorem 2.11. *Let $f\colon X \to Y$ be a 1-sequence-covering, quotient, ss-map such that X has a locally countable weak-base, then Y has a locally countable weak-base.*

Proof. Let $f\colon X \to Y$ be a 1-sequence-covering, quotient, ss-map, where X has a locally countable weak-base. By Lemma 2.1, X is a sequential space with a locally countable k-network. Thus, Y has a locally countable k-network by Lemma 2.10. Since 1-sequence-covering quotient maps preserve g-first countable spaces([17, Corollary 3]), then Y is g-first countable. By Lemma 2.1, Y has a locally countable weak-base. □

Remark 2.12. The space of Example 2.14(1) in [24] has a countable weak-base, but its image under a perfect map is not g-first countable. Thus, spaces with a locally countable weak-base are not necessarily preserved under perfect maps.

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