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DYNAMICS OF SPECIES IN A NONAUTONOMOUS LOTKA-VOLTERRA SYSTEM

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Abstract. In this paper, we study a Lotka-Volterra model with two predators and one prey.

The explorations involve the permance, extinction, the existence, uniqueness and global asymptotic stability of a positive solution.

1. Introduction

In this paper, we consider the Lotka-Volterra model with Beddington-DeAngelis functional response of two predators and one prey.

$$
\begin{cases}
x' = x\left[a - bx\right] - \dfrac{cxy}{\alpha + \beta x + \gamma y} - mxz \\
y' = -dy + \dfrac{fxy}{\alpha + \beta x + \gamma y} - nyz \\
z' = -gz + \dfrac{hz\left[mx + ny\right]}{\xi + \eta z}
\end{cases}
\tag{1}
$$

Here $x(t)$, $y(t)$ and $z(t)$ represent the population density of the two predator species, and of the prey at times t, respectively. $a(t)$, $b(t)$, $c(t)$, $d(t)$, $m(t)$, $f(t)$, $n(t)$, $g(t)$, $h(t)$, $\beta(t)$, $\gamma(t)$, $\eta(t)$ are continuous and bounded above and below by positive constants; $\alpha(t)$, $\xi(t)$ are continuous and nonnegative.

This paper is organized as follows. Section 2 provides some definitions and notations. In section 3 we state our main result of this paper for problem (1).

2. Definition and notation

In this section we summarize the basic definitions and facts which are used later (cf. [7]). Let $\mathbb{R}^3_+ := \{(x, y, z) \in \mathbb{R}^3 | x \geqslant 0, y \geqslant 0, z \geqslant 0\}$. For a bounded

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continuous function $g(t)$ on $\mathbb{R}$, we use the following notation:

$$g^u := \sup_{t\in\mathbb{R}} g(t), \qquad g^l := \inf_{t\in\mathbb{R}} g(t).$$

Similarly to Lemma 1 in [10], one can easily prove that

Lemma 1. *Both the nonnegative and positive cones of $\mathbb{R}^3$ are positively invariant for (1).*

In the remainder of this paper, for biological reasons, we only consider the solutions $(x(t), y(t), z(t))$ with positive initial values, i.e., $x(t_o) > 0,\ y(t_o) > 0$ and $z(t_o) > 0$.

Definition 1. System (1) is said to be permanent if there exist positive constants $\delta,\ \Delta$ with $0 < \delta < \Delta$ such that

$$\min\{\liminf_{t\to\infty} x(t), \quad \liminf_{t\to\infty} y(t), \quad \liminf_{t\to\infty} z(t)\} \geqslant \delta,$$
$$\max\{\limsup_{t\to\infty} x(t), \quad \limsup_{t\to\infty} y(t), \quad \limsup_{t\to\infty} z(t)\} \leqslant \Delta$$

for all solutions of (1) with positive initial values. System (1) is said to be nonpersistent if there is a positive solution $(x(t),\ y(t),\ z(t))$ of (1) satisfying

$$\min\{\liminf_{t\to\infty} x(t),\ \liminf_{t\to\infty} y(t),\ \liminf_{t\to\infty} z(t)\} = 0.$$

Definition 2. A set A is called to be an ultimately bounded region of system (1) if for any solution $(x(t), y(t), z(t))$ of (1) with positive initial values, there exists $T_1 > 0$ such that $(x(t), y(t), z(t)) \in A$ for all $t \geqslant t_0 + T_1$.

Definition 3. A bounded nonnegative solution $(\hat{x}(t),\ \hat{y}(t),\ \hat{z}(t))$ of (1) is said to be globally asymptotically stable (or globally attractive) if any other solution $(x(t),\ y(t),\ z(t))$ of (1) with positive initial values satisfies

$$\lim_{t\to\infty} (|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)| + |z(t) - \hat{z}(t)|) = 0.$$

Lemma 2. [2] *Let h be a real number and f be a nonnegative function defined on $[h, +\infty)$ such that f is integrable on $[h, +\infty)$ and is uniformly continuous on $[h, +\infty)$, then*

$$\lim_{t\to\infty} f(t) = 0.$$

3. Main Results

Theorem 1. *If $m_1^\varepsilon > 0,\ m_2^\varepsilon > 0$ and $m_3^\varepsilon > 0$, then set Γ_ε defined by*

$$\Gamma_\varepsilon = \{(x, y, z) \in \mathbb{R}^3 |\ m_1^\varepsilon \leqslant x \leqslant M_1^\varepsilon,\ m_2^\varepsilon \leqslant y \leqslant M_2^\varepsilon,\ m_3^\varepsilon \leqslant z \leqslant M_3^\varepsilon\} \tag{2}$$

is positively invariant with respect to system (1), where

$$M_1^\varepsilon := \frac{a^u}{b^l} + \varepsilon, \qquad m_1^\varepsilon := \frac{a^l\gamma^l - c^u}{b^u\gamma^l} - \frac{m^u M_3^\varepsilon}{b^u},$$

$$M_2^\varepsilon := \frac{(f^u - d^l\beta^l)M_1^\varepsilon}{d^l\gamma^l}, \qquad m_2^\varepsilon := \frac{f^l m_1^\varepsilon - (d^u + n^u M_3^\varepsilon)(\alpha^u + \beta^u m_1^\varepsilon)}{\gamma^u(d^u + n^u M_3^\varepsilon)},$$

$$M_3^\varepsilon := \frac{h^u(m^u M_1^\varepsilon + n^u M_2^\varepsilon) - g^l \xi^l}{g^l \eta^l}, \quad m_3^\varepsilon := \frac{h^l(m^u m_1^\varepsilon + n^u m_2^\varepsilon) - g^u \xi^u}{g^u \eta^u}$$

and $\varepsilon \geqslant 0$ *is constant.*

Proof. We know that the logistic equation

$$X'(t) = A(t)X(t)[B - X(t)] \qquad (B \neq 0)$$

has a unique solution

$$X(t) = \frac{BX_0 \; e^{\int_{t_0}^t A(s)Bds}}{X_0[e^{\int_{t_0}^t A(s)Bds} - 1] + B}$$

where $X_0 := X(t_0)$. By Lemma 1, we have $x(t) > 0,\ y(t) > 0$ and $z(t) > 0$ for all $t \geq t_0$. Because $M_1^\varepsilon > x_0 > 0$ and (1), we have

$$x'(t) \leqslant x(t)[a(t) - b(t)x(t)] \leqslant x(t)[a^l - b^u x(t)] = b^u x(t)(M_1^0 - x).$$

Thus, by using a standard comparison argument, we obtain that

$$x(t) \leqslant \frac{x_0 M_1^0 e^{a^u(t-t_0)}}{x_0[e^{a^u(t-t_0)} - 1] + M_1^0} \leqslant \frac{x_0 M_1^\varepsilon e^{a^u(t-t_0)}}{x_0[e^{a^u(t-t_0)} - 1] + M_1^\varepsilon} < M_1^\varepsilon, \quad t \geqslant t_0. \tag{3}$$

Similarly, because

$$\begin{aligned} y' &\leqslant -d^l y + \frac{f^u xy}{\alpha^l + \beta^l x + \gamma^l y} \leqslant -d^l y + \frac{f^u M_1^\varepsilon y}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y} \leqslant \\ &\leqslant \frac{[(f^u - d^l\beta^l)M_1^\varepsilon - d^l\gamma^l y]y}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y} = \frac{d^l\gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y} y[M_2^\varepsilon - y], \quad t \geqslant t_0, \end{aligned} \tag{4}$$

and $0 < y_0 < M_2^\varepsilon$, we have

$$y(t) \leqslant M_2^\varepsilon \; \frac{y_0 \; e^{\int_{t_0}^t \frac{M_2^\varepsilon d^l \gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y(s)} ds}}{y_0[e^{\int_{t_0}^t \frac{M_2^\varepsilon d^l \gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y(s)} ds} - 1] + M_2^\varepsilon} < M_2^\varepsilon, \quad t \geqslant t_0. \tag{5}$$

And because $0 < z_0 < M_3^\varepsilon$ and

$$\begin{aligned} z'(t) &\leqslant -g^l z + \frac{h^u z(m^u x + n^u y)}{\xi^l + \eta^l z} \\ &\leqslant -g^l z + \frac{h^u z(m^u M_1^\varepsilon + n^u M_2^\varepsilon)}{\xi^l + \eta^l z} \\ &= \frac{g^l \eta^l}{\xi^l + \eta^l z} z(M_3^\varepsilon - z), \end{aligned}$$

we also get that

$$z(t) < M_3^\varepsilon, \quad t \geqslant t_0. \tag{6}$$

Now, from (1), (3), (5) and (6), we have

$$\begin{aligned} x'(t) &\geqslant x\left(a^l - b^u x - \frac{c^u y}{\alpha^l + \gamma^l y} - m^u z\right) \\ &\geqslant x\left(a^l - b^u x - \frac{c^u M_2^\varepsilon}{\alpha^l + \gamma^l M_2^\varepsilon} - m^u M_3^\varepsilon\right) \\ &= b^u x(m_1^\varepsilon - x) \end{aligned}$$

and $x_0 > m_1^\varepsilon$. Thus,

$$x(t) \geqslant \frac{m_1^\varepsilon x_0 e^{b^u m_1^\varepsilon (t-t_0)}}{x_0[e^{b^u m_1^\varepsilon (t-t_0)} - 1] + m_1^\varepsilon} > m_1^\varepsilon, \quad \text{for all} \quad t \geqslant t_0.$$

Similarly, we have

$$\begin{aligned} y'(t) &\geqslant -d^u y + \frac{f^l m_1^\varepsilon}{\alpha^u + \beta^u m_1^\varepsilon + \gamma^u y}\, y - n^u M_3^\varepsilon y \\ &= \frac{\gamma^u (d^u + n^u M_3^\varepsilon)}{\alpha^u + \beta^u m_1^\varepsilon + \gamma^u y}\, y[m_2^\varepsilon - y], \end{aligned}$$

for which follows that $y(t) > m_2^\varepsilon$, for all $t \geqslant t_0$, and

$$\begin{aligned} z'(t) &\geqslant -g^u z + \frac{h^l z(m^u m_1^\varepsilon + n^u m_2^\varepsilon)}{\xi^u + \eta^u z} \\ &= \frac{g^u \eta^u}{\xi^u + \eta^u z}\, z(m_3^\varepsilon - z), \end{aligned}$$

for which follows that $z(t) > m_3^\varepsilon$, for all $t \geqslant t_0$. So the proof is complete. $\square$

Corollary 1. *If* $m_1^\varepsilon > 0,\ m_2^\varepsilon > 0$ *and* $m_3^\varepsilon > 0$, *then we have*

$$\begin{aligned} &\liminf_{t\to\infty} x(t) \geqslant m_1^\varepsilon, \qquad \limsup_{t\to\infty} x(t) \leqslant M_1^\varepsilon, \\ &\liminf_{t\to\infty} y(t) \geqslant m_2^\varepsilon, \qquad \limsup_{t\to\infty} y(t) \leqslant M_2^\varepsilon, \\ &\liminf_{t\to\infty} z(t) \geqslant m_3^\varepsilon, \qquad \limsup_{t\to\infty} z(t) \leqslant M_3^\varepsilon. \end{aligned}$$

Proof. From (3) we have $\limsup_{t\to\infty} x(t) < M_1^\varepsilon$. Thus there exists $t_1 \geqslant t_0$ such that

$$x(t) \leqslant M_1^\varepsilon,\ t \geqslant t_1.$$

Similarly to (4) and (5) we obtain that

$$y(t) \leqslant M_2^\varepsilon\ \frac{y_1\ e^{\int_{t_1}^t \frac{M_2^\varepsilon d^l \gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y(s)} ds}}{y_1[e^{\int_{t_1}^t \frac{M_2^\varepsilon d^l \gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y(s)} ds} - 1] + M_2^\varepsilon}. \tag{7}$$

Thus, $0 < y(t) \leqslant \max\{M_2^\varepsilon, y_1\}$ for all $t \geqslant t_1$, where $y_1 := y(t_1)$. Therefore, from (7) we get that

$$\limsup_{t\to\infty} y(t) \leqslant M_2^\varepsilon.$$

As a consequence, there exists $t_2 \geqslant t_1$ such that $y(t) \leqslant M_3^\varepsilon$ for all $t \geqslant t_2$.

Similarly, we claim that

$$\limsup_{t\to\infty} z(t) \leqslant M_3^\varepsilon, \qquad \liminf_{t\to\infty} x(t) \geqslant m_1^\varepsilon,$$

$$\liminf_{t\to\infty} y(t) \geqslant m_2^\varepsilon, \qquad \liminf_{t\to\infty} z(t) \geqslant m_3^\varepsilon.$$

The proof is complete. □

Corollary 2. *If $m_1^\varepsilon > 0$, $m_2^\varepsilon > 0$ and $m_3^\varepsilon > 0$, then system (1) is permanent and set Γ_ε with $\varepsilon \geqslant 0$ defined by (2) is an ultimately bounded region of system (1).*

Using Corollary 1, it is easy to verify the statement of the above lemma.

Theorem 2. *If $M_2^0 < 0$ or $M_3^0 < 0$, then $\lim\limits_{t\to\infty} y(t) = 0$ or $\lim\limits_{t\to\infty} z(t) = 0$ respectively.*

Proof. We see that if $M_2^o < 0$ or $M_3^o < 0$ then $M_2^\varepsilon < 0$ or $M_3^\varepsilon < 0$, respectively, with ε is sufficiently small. Therefore, similarly to the proof of Theorem 1 we get that

$$y'(t) \leqslant \frac{d^l \gamma^l}{\alpha^l + \beta^l M_1^\varepsilon + \gamma^l y}\, y[M_2^\varepsilon - y] \tag{8}$$

and

$$z'(t) \leqslant \frac{g^l \eta^l}{\xi^l + \eta^l z}\, z[M_3^\varepsilon - z].$$

If $M_2^\varepsilon < 0$ then $y'(t) < 0$. Thus, $0 < y(t) \leqslant y(t_0)$, for all $t \geqslant t_0$ and then there exists $C_1 \geqslant 0$ such that

$$\lim_{t\to\infty} y(t) = C_1.$$

If $C_1 > 0$ then from system (1), (8) and $0 < C_1 \leqslant y(t) \leqslant y(t_0)$, $t \geqslant t_0$, we have that there exists $\mu > 0$ such that $y'(t) < -\mu$ for all $t \geqslant t_0$. Thus $y(t) < -\mu(t - t_0) + y_0$. Therefore, $\lim\limits_{t\to\infty} y(t) = -\infty$. So we have a contraction to the fact that $y(t) > 0$ for all $t \geqslant t_0$. Hence,

$$\lim_{t\to\infty} y(t) = 0.$$

Similarly, if $M_3^\varepsilon < 0$ then $\lim\limits_{t\to\infty} z(t) = 0$. The proof is complete. □

Theorem 3. *Let $(x^*(t),\ y^*(t),\ z^*(t))$ be bounded positive solutions of system (1). If $m_1^\varepsilon > 0$, $m_2^\varepsilon > 0$ and $m_3^\varepsilon > 0$ and the following conditions hold:*

$$\inf_{t\geqslant t_0} \Big\{ b(t) - \frac{\alpha(t) f(t)}{u(t, m_1^\varepsilon, m_2^\varepsilon)} - \frac{[\beta(t)c(t) + f(t)\gamma(t)]M_2^\varepsilon}{u(t, m_1^\varepsilon, m_2^\varepsilon)} \\ - \frac{m(t)h(t)\xi(t)}{v(t, m_3^\varepsilon)} - \frac{m(t)h(t)\eta(t)M_3^\varepsilon}{v(t, M_3^\varepsilon)} \Big\} > 0,$$

$$\inf_{t\geqslant t_0}\left\{\frac{f(t)\gamma(t)m_1^\varepsilon}{u(t,m_1^\varepsilon,M_2^\varepsilon)}-\frac{\alpha(t)c(t)}{u(t,m_1^\varepsilon,m_2^\varepsilon)}-\frac{h(t)\xi(t)n(t)}{v(t,m_3^\varepsilon)}\right.$$
$$\left.-\frac{\beta(t)c(t)M_1^\varepsilon}{u(t,M_1^\varepsilon,m_2^\varepsilon)}-\frac{h(t)n(t)\eta(t)M_3^\varepsilon}{v(t,M_3^\varepsilon)}\right\}>0,$$

$$\inf_{t\geqslant t_0}\{\frac{m(t)h(t)\eta(t)m_1^\varepsilon+n(t)h(t)\eta(t)m_2^\varepsilon}{v(t,M_3^\varepsilon)}-m(t)-n(t)\}>0, \tag{9}$$

where

$u(t,x,y):=[\alpha(t)+\beta(t)x^*+\gamma(t)y^*(t)][\alpha(t)+\beta(t)x(t)+\gamma(t)y(t)],$

$v(t,z)\ :=[\xi(t)+\eta(t)z^*][\xi(t)+\eta(t)z]$

then $(x^*(t),y^*(t),z^*(t))$ *is globally asymptotically stable.*

Proof. Let $(x(t),\ y(t),\ z(t))$ be any solution of (1) with positive initial value. Since Γ_ε is an ultimately bounded region of (1), there exists $T_1>0$ such that $(x(t),\ y(t),\ z(t))\in\Gamma_\varepsilon$ and

$$(x^*(t),\ y^*(t),\ z^*(t))\in\Gamma_\varepsilon \text{ for all } t\geqslant t_0+T_1.$$

Considering a Liapunov function defined by

$$V(t)=|\ln(x(t))-\ln(x^*(t))|+|\ln(y(t))-\ln(y^*(t))|+|\ln(z(t))-\ln(z^*(t))|,t\geqslant t_0$$

a direct calculation of the right derivative $D^+V(t)$ of $V(t)$ along the solution of (1) produces

(10)

$$D^+V(t) = \operatorname{sgn}(x-x^*)\left(\frac{x'}{x} - \frac{x^{*\prime}}{x^*}\right) + \operatorname{sgn}(y-y^*)\left(\frac{y'}{y} - \frac{y^{*\prime}}{y^*}\right)$$

$$+ \operatorname{sgn}(z-z^*)\left(\frac{z'}{z} - \frac{z^{*\prime}}{z^*}\right)$$

$$= \operatorname{sgn}(x-x^*)\left[a - bx - \frac{cy}{\alpha+\beta x+\gamma y} - mz - a + bx^* + \frac{cy^*}{\alpha+\beta x^*+\gamma y^*} + mz^*\right]$$

$$+ \operatorname{sgn}(y-y^*)\left[-d + \frac{fx}{\alpha+\beta x+\gamma y} - nz + d - \frac{fx^*}{\alpha+\beta x^*+\gamma y^*} + nz^*\right]$$

$$+ \operatorname{sgn}(z-z^*)\left[\frac{h(mx+ny)}{\xi+\eta z} - \frac{h(mx^*+ny^*)}{\xi+\eta z^*}\right]$$

$$= \operatorname{sgn}(x-x^*)\left[-b(x-x^*) - m(z-z^*) - c\left(\frac{y}{\alpha+\beta x+\gamma y} - \frac{y^*}{\alpha+\beta x^*+\gamma y^*}\right)\right]$$

$$- n \ \operatorname{sgn}(y-y^*)(z-z^*) + f \ \operatorname{sgn}(y-y^*)\frac{\alpha(x-x^*)+\gamma(xy^*-x^*y)}{u(t,x,y)}$$

$$+ h \ \operatorname{sgn}(z-z^*) \ \frac{(mx+ny)(\xi+\eta z^*) - (mx^*+ny^*)(\xi+\eta z)}{v(t,z)}$$

$$\leqslant -b|x-x^*| + m|z-z^*| + \frac{\alpha c|y-y^*|}{u(t,x,y)} + \operatorname{sgn}(x-x^*) \ \frac{\beta c(xy^*-x^*y)}{u(t,x,y)}$$

$$+ n|z-z^*| + \frac{\alpha f|x-x^*|}{u(t,x,y)} + \operatorname{sgn}(y-y^*) \ \frac{f\gamma(xy^*-x^*y)}{u(t,x,y)}$$

$$+ h \ \operatorname{sgn}(z-z^*) \ \frac{(mx+ny)(\xi+\eta z^*) - (mx^*+ny^*)(\xi+\eta z)}{v(t,z)}.$$

We have

$$(mx+ny)(\xi+\eta z^*) - (mx^*+ny^*)(\xi+\eta z) =$$
$$= m\xi(x-x^*) + n\xi(y-y^*) + m\eta(xz^*-x^*z) + n\eta(yz^*-y^*z)$$

and

$$\begin{aligned} xy^* - x^*y &= x(y^* - y) + y(x - x^*), \\ xz^* - x^*z &= x(z^* - z) + z(x - x^*), \\ yz^* - y^*z &= y(z^* - z) + z(y - y^*). \end{aligned}$$

Thus,

$$D^+V(t) \leqslant \left[-b + \frac{hm\xi}{v(t,z)} + \frac{\alpha f}{u(t,x,y)} + \frac{\beta cy + f\gamma y}{u(t,x,y)} + \frac{hm\eta z}{v(t,z)}\right]|x - x^*| +$$

$$+ \left[\frac{\alpha c}{u(t,x,y)} + \frac{\beta cx - f\gamma x}{u(t,x,y)} + \frac{h\xi n + hn\eta z}{v(t,z)}\right]|y - y^*| + \tag{11}$$

$$+ \left[m + n - \frac{hm\eta x + hn\eta y}{v(t,z)}\right]|z - z^*|.$$

Because $(x(t),\ y(t),\ z(t)) \in \Gamma_\varepsilon$, for all $t \geqslant t_0 + T_1$, we get that

(12)

$$D^+V(t) \leqslant -\left[b(t) - \frac{\alpha(t)f(t)}{u(t,m_1^\varepsilon,m_2^\varepsilon)} - \frac{\beta(t)c(t) + f(t)\gamma(t)}{u(t,m_1^\varepsilon,M_2^\varepsilon)}\ M_2^\varepsilon - \right.$$

$$\left. - \frac{m(t)h(t)\xi(t)}{v(t,m_3^\varepsilon)} - \frac{m(t)h(t)\eta(t)M_3^\varepsilon}{v(t,M_3^\varepsilon)}\right]|x - x^*| -$$

$$- \left[\frac{f(t)\gamma(t)m_1^\varepsilon}{u(t,m_1^\varepsilon,M_2^\varepsilon)} - \frac{\alpha(t)c(t)}{u(t,m_1^\varepsilon,m_2^\varepsilon)} - \frac{h(t)\xi(t)n(t)}{v(t,m_3^\varepsilon)} - \right.$$

$$\left. - \frac{\beta(t)c(t)M_1^\varepsilon}{u(t,M_1^\varepsilon,m_2^\varepsilon)} - \frac{h(t)n(t)\eta(t)M_3^\varepsilon}{v(t,M_3^\varepsilon)}\right]|y - y^*| -$$

$$- \left[\frac{m(t)h(t)\eta(t)m_1^\varepsilon + h(t)n(t)\eta(t)m_2^\varepsilon}{v(t,M_3^\varepsilon)} - n(t) - m(t)\right]|z - z^*|,$$

for all $t \geqslant t_0 + T_1$. ¿From (9) follows that there exists a positive constant $\mu > 0$ such that

$$D^+V(t) \leqslant -\mu[|x(t) - x^*(t)| + |y(t) - y^*(t)| + |z(t) - z^*(t)|], \tag{13}$$

for all $t \geqslant t_0 + T_1$.

Integrating on both sides of (13) from $t_0 + T_1$ to t produces

$$V(t) + \mu \int_{t_0+T_1}^{t} [|x(s) - x^*(s)| + |y(s) - y^*(s)| + |z(s) - z^*(s)|]ds \leqslant V(t_0 + T_1) < +\infty,$$

for all $t \geqslant t_0 + T_1$.

Then

$$\int_{t_0+T_1}^{t}[|x(s)-x^*(s)|+|y(s)-y^*(s)|+|z(s)-z^*(s)|]ds \leqslant \mu^{-1}V(t_0+T_1) < +\infty,$$

for all $t \geqslant t_0 + T_1$,

Hence, $|x - x^*| + |y - y^*| + |z - z^*| \in L^1([t_0 + T_1, +\infty))$.

The boundedness of x^*, y^*, z^* and the ultimate boundedness of $x(t)$, $y(t)$, $z(t)$ imply that $x(t)$, $y(t)$, $z(t)$, $x^*(t)$, $y^*(t)$ and $z^*(t)$ all have bounded derivatives for $t \geqslant t_0 + T_1$ (from the equations satisfied by them). As a consequence

$$|x(s) - x^*(s)| + |y(s) - y^*(s)| + |z(s) - z^*(s)|$$

is uniformly continuous on $[t_0 + T_1, +\infty)$.

By Lemma 2 we have

$$\lim_{t\to\infty}(|x(t) - x^*(t)| + |y(t) - y^*(t)| + |z(t) - z^*(t)|) = 0$$

which completes the proof. □

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