

SEMI-SYMMETRY TYPE LP-SASAKIAN MANIFOLDS

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ABSTRACT. Recently the present authors have introduced the notion of *generalized quasi-conformal curvature tensor* $\mathcal{W}$, which bridges Conformal curvature tensor, Concircular curvature tensor, Projective curvature tensor and Conharmonic curvature tensor. The present paper attempts to investigate the curvature conditions (like flatness, semi-symmetry type and Ricci semi-symmetry type) of *LP-Sasakian manifold*. Further, the paper seeks to table the nature of the Ricci tensor for respective semi-symmetry type curvature conditions.

1. INTRODUCTION

In tune with Yano and Sawaki [30], recently the present authors [1] have defined and studied *generalized quasi-conformal curvature tensor* $\mathcal{W}$, in the context of $N(k, \mu)$ -contact metric manifold. The beauty of *generalized quasi-conformal curvature tensor* $\mathcal{W}$ lies in the fact that it has the flavour of Riemann curvature tensor R , conformal curvature tensor C [8], conharmonic curvature tensor $\hat{C}$ [9], concircular curvature tensor E [29, p. 84] projective curvature tensor P [29, p. 84] and m -projective curvature tensor H [17], as special cases.

In 1989 K. Matsumoto [11] introduced the notion of Lorentzian para-Sasakian (LP-Sasakian for short) manifold. In 1992, Mihai and Rosca [12] defined the same notion independently. This type of manifold is also discussed in [22], [23], [24] and the references therein.

An LP-Sasakian manifold is said to be semi-symmetry type (respectively Ricci semi-symmetry type) if the *generalized quasi-conformal curvature tensor* $\mathcal{W}$ (respectively Ricci tensor S) admits the condition

$$(1.1) \quad \omega(X, Y) \cdot \mathcal{W} = 0, \text{ (respectively } \mathcal{W}(X, Y) \cdot S = 0), \text{ for any } X, Y \text{ on } M,$$

where the dot means that $\omega(X, Y)$ acts on $\mathcal{W}$ (respectively on S) as derivation. Here ω and $\mathcal{W}$ stand for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) respectively. In particular,

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manifold satisfying the condition $R(X, Y) \cdot R = 0$ (obtained from 1.1 by setting $\bar{a} = \bar{b} = \bar{c} = 0 = a = b = c$) is said to be semi-symmetric in the sense of Cartan ([2, p. 265], and named by N. S. Sinjukov [25]). A full classification of such space is given by Z. I. Szabó ([27], [26], [28]). This type of the manifolds have been studied by several authors such as Sekigawa and Tanno [21], Sekigawa and Takagi [20], Papantoniou [15], Perrone [16], Kowalski [10], Sekigawa [19] and the references therein.

Our work is structured as follows. The section 2 is a very brief account of LP-Sasakian manifolds. Definition and some basic results of the *generalized quasi-conformal curvature tensor* $\mathcal{W}$ is discussed in section 3. LP-Sasakian manifold with vanishing *generalized quasi-conformal curvature tensor* is studied in section 4 and it is found that such a manifold M is either an Einstein space or an η -Einstein space or isometric to the Lorentz sphere $S^{2n+1}(1)$. Furthermore, the nature of the Ricci tensors for the flatness of different curvature tensors are characterized. In section 5, we investigate LP-Sasakian manifold satisfying the condition $\omega(\xi, X) \cdot \mathcal{W} = 0$. Based on this conditions and by taking into account the permutation of different curvature tensors, we obtained and tabled the expression of Ricci tensors for different semi-symmetry type conditions. The last section is devoted to study LP-Sasakian manifold admitting the condition $\mathcal{W} \cdot S = 0$. Among others an equivalent conditions that (a) M is an Einstein space, (b) M is Ricci symmetric i.e., $\nabla S = 0$, (c) $P(\xi, X) \cdot S = 0$ (or $E(\xi, X) \cdot S = 0$) for all $X \in \chi(M)$ is brought out.

2. LP-SASAKIAN MANIFOLDS

An $(2n + 1)$ -dimensional differentiable manifold M is said to be an LP-Sasakian manifold ([4], [11]) if it admits a $(1, 1)$ tensor field ϕ , a unit timelike contravariant vector field ξ , a 1-form η and a Lorentzian metric g which satisfy

$$(2.1) \quad \eta(\xi) = -1, \quad g(X, \xi) = \eta(X), \quad \phi^2 X = X + \eta(X)\xi,$$

$$(2.2) \quad g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad \nabla_X \xi = \phi X,$$

$$(2.3) \quad (\nabla_X \phi)(Y) = g(X, Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi,$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g . It can be easily seen that in an LP-Sasakian manifold, the following relations hold :

$$(2.4) \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad \text{Rank } \phi = 2n.$$

Again, if we put

$$\Omega(X, Y) = g(X, \phi Y)$$

for any vector fields X, Y then the tensor field $\Omega(X, Y)$ is a symmetric $(0, 2)$ tensor field [12]. Also, since the vector field η is closed in an LP-Sasakian manifold, we have ([11], [12])

$$(2.5) \quad (\nabla_X \eta)(Y) = \Omega(X, Y), \quad \Omega(X, \xi) = 0,$$

for any vector fields X and Y .

Let M be an $(2n + 1)$ -dimensional LP-Sasakian manifold with structure (ϕ, ξ, η, g) . Then the following relations hold ([11], [12]):

$$(2.6) \quad g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z) = g(Y, Z)\eta(X) - g(X, Z)\eta(Y),$$

$$(2.7) \quad R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X,$$

$$(2.8) \quad R(X, Y)\xi = \eta(Y)X - \eta(X)Y,$$

$$(2.9) \quad S(X, \xi) = 2n\eta(X),$$

for any vector fields X, Y, Z where R is the Riemannian curvature tensor of the manifold.

3. THE GENERALIZED QUASI CONFORMAL CURVATURE TENSOR

The *generalized quasi-conformal curvature tensor* is defined as

$$(3.1) \quad \begin{aligned} \mathcal{W}(X, Y)Z = & \frac{2n-1}{2n+1} [(1+2na-b) - \{1+2n(a+b)\}c] C(X, Y)Z \\ & + [1-b+2na] E(X, Y)Z + 2n(b-a)P(X, Y)Z \\ & + \frac{2n-1}{2n+1} (c-1)\{1+2n(a+b)\}\hat{C}(X, Y)Z \end{aligned}$$

for all X, Y and $Z \in \chi(M)$, the set of all vector field of the manifold M , where a, b and c are real constants. And C, E, P and $\hat{C}$ stand for Conformal, Conccircular, Projective and Conharmonic curvature tensor respectively. These curvature tensor are defined as follows

$$(3.2) \quad \begin{aligned} C(X, Y)Z = & R(X, Y)Z \\ & - \frac{1}{2n-1} [S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY] \\ & + \frac{r}{2n(2n-1)} [g(Y, Z)X - g(X, Z)Y], \end{aligned}$$

$$(3.3) \quad E(X, Y)Z = R(X, Y)Z - \frac{r}{2n(2n+1)} [g(Y, Z)X - g(X, Z)Y],$$

$$(3.4) \quad P(X, Y)Z = R(X, Y)Z - \frac{1}{2n} [S(Y, Z)X - S(X, Z)Y],$$

$$(3.5) \quad \begin{aligned} \hat{C}(X, Y)Z = & R(X, Y)Z - \frac{1}{2n-1} [S(Y, Z)X - S(X, Z)Y \\ & + g(Y, Z)QX - g(X, Z)QY], \end{aligned}$$

for all X, Y & $Z \in \chi(M)$, where R, S, Q and r being Christoffel Riemannian curvature tensor, Ricci tensor, Ricci operator and scalar curvature respectively.

In particular, the *generalized quasi-conformal curvature tensor* $\mathcal{W}$ reduced to

- (1) Riemann curvature tensor R , if $a = b = c = 0$,
- (2) conformal curvature tensor C , if $a = b = -\frac{1}{2n-1}$, $c = 1$,
- (3) conharmonic curvature tensor $\hat{C}$, if $a = b = -\frac{1}{2n-1}$, $c = 0$,
- (4) concircular curvature tensor E , if $a = b = 0$ and $c = 1$,
- (5) projective curvature tensor P , if $a = -\frac{1}{2n}$, $b = 0$, $c = 0$ and
- (6) m -projective curvature tensor H , if $a = b = -\frac{1}{4n}$, $c = 0$.

The m -projective curvature tensor is introduced by G. P. Pokhariyal and R. S. Mishra [17]. Which is defined as follows

$$(3.6) \quad H(X, Y)Z = R(X, Y)Z - \frac{1}{4n} \left[S(Y, Z)X - S(X, Z)Y + g(Y, Z)QX - g(X, Z)QY \right].$$

Using (3.2), (3.3), (3.4) and (3.5) in (3.1), the *generalized quasi-conformal curvature tensor* $\mathcal{W}$ becomes

$$(3.7) \quad \begin{aligned} \mathcal{W}(X, Y)Z &= R(X, Y)Z + a \left[S(Y, Z)X - S(X, Z)Y \right] \\ &\quad + b \left[g(Y, Z)QX - g(X, Z)QY \right] \\ &\quad - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \left[g(Y, Z)X - g(X, Z)Y \right]. \end{aligned}$$

Note that our *generalized quasi-conformal curvature tensor* $\mathcal{W}$ is not a generalized curvature tensor (see, [13], [6], [5], [7]), as it does not satisfy the condition $\mathcal{W}(X_1, X_2, X_3, X_4) = \mathcal{W}(X_3, X_4, X_1, X_2)$, where

$$\mathcal{W}(X_1, X_2, X_3, X_4) = g(\mathcal{W}(X_1, X_2)X_3, X_4),$$

for all X_1, X_2, X_3, X_4 . Moreover our $\mathcal{W}$ is not a proper generalized curvature tensor [13], as it does not satisfy the second Bianchi identity

$$(3.8) \quad (\nabla_{X_1}\mathcal{W})(X_2, X_3)X_4 + (\nabla_{X_2}\mathcal{W})(X_3, X_1)X_4 + (\nabla_{X_3}\mathcal{W})(X_1, X_2)X_4 = 0.$$

4. LP-SASAKIAN MANIFOLDS WITH FLAT GENERALIZED QUASI-CONFORMAL CURVATURE TENSOR

Definition 4.1. An LP-Sasakian manifold $M^{2n+1}(\phi, \xi, \eta, g)$ is said to be η -Einstein, if its Ricci tensor S of the metric g satisfies

$$S(X, Y) = \alpha g(X, Y) + \beta \eta(X)\eta(Y),$$

for some real constant α and β .

Such notion was first introduced and studied by Okumura [14] and named by Sasaki [18] in his lecture notes 1965. In particular, if $\beta = 0$, we say that the manifold is Einstein.

Let the manifold be generalized quasi-conformally flat. Therefore, using (2.7) and (2.9) in (3.7) we can easily bring out after a straightforward calculation that

$$(4.1) \quad S(X, Z) = \kappa g(X, Z) + (\kappa - 2n)\eta(X)\eta(Z),$$

where

$$(4.2) \quad \kappa = -\frac{1}{a} \left\{ 1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\}.$$

Remark 4.2. The Ricci tensor S in the equation (4.1) can also be written as

$$(4.3) \quad S(X, Z) = \kappa g(X, Z) + (\kappa - 2n)\eta(X)\eta(Z),$$

where

$$(4.4) \quad \kappa = -\frac{1}{b} \left\{ 1 + 2na - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\}.$$

Thus, we see that LP-Sasakian manifold with $\mathcal{W}(X, Y)Z = 0$ is an η -Einstein space provided $a \neq 0$ (or $b \neq 0$).

Again, an LP-Sasakian manifold with $R = 0$ or $E = 0$ (i.e., for the case $a = 0$ and $b = 0$), one can easily determine that such manifold is an Einstein. This leads to the followings:

Theorem 4.3. *Let $M^{2n+1}(\phi, \xi, \eta, g)$, $n > 1$; be an LP-Sasakian manifold with vanishing generalized quasi-conformal curvature tensor. Then M is either an Einstein space or an η -Einstein space or isometric to the Lorentz sphere $S^{2n+1}(1)$.*

Again, from (4.1), one can easily determine the following theorem-

Theorem 4.4. *Every LP-Sasakian manifold (M^{2n+1}, g) , $n > 1$ with vanishing generalized quasi-conformal curvature tensor is necessarily η -parallel.*

Further, if we choose $a = b$, then from (4.1) we can state the following.

Theorem 4.5. *A generalized quasi-conformally flat LP-Sasakian manifold (M^{2n+1}, g) , $(n > 1)$ is a manifold of quasi-constant curvature with associated scalars*

$$A = -\left(1 + \frac{b}{a}\right) \kappa a + \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right), \quad B = a(\kappa + 2n),$$

provided that $a \neq 0$.

The notion of a *manifold of quasi-constant curvature* was first introduced by Chen and Yano [3] for a Riemannian geometry.

5. LP-SASAKIAN MANIFOLDS WITH SEMI-SYMMETRY TYPE CURVATURE CONDITION

Definition 5.1. A $(2n + 1)$ -dimensional ($n > 1$) LP-Sasakian manifold is said to be semi-symmetry type if the condition $\omega(X, Y) \cdot \mathcal{W} = 0$ holds, for any vector fields X, Y on the manifold and $\omega(X, Y)$ acts on $\mathcal{W}$ as derivation, where ω and $\mathcal{W}$ stand for *generalized quasi-conformal curvature tensor* with the associated scalar triples $(\bar{a}, \bar{b}, \bar{c})$ and (a, b, c) respectively.

Now, let us consider a $(2n + 1)$ -dimensional LP-Sasakian manifold M , satisfying the condition

$$(5.1) \quad (\omega(\xi, X) \cdot \mathcal{W})(Y, Z)U = 0,$$

which is equivalent to

$$(5.2) \quad g(\omega(\xi, X)\mathcal{W}(Y, Z)U, \xi) - g(\mathcal{W}(\omega(\xi, X)Y, Z)U, \xi) \\ - g(\mathcal{W}(Y, \omega(\xi, X)Z)U, \xi) - g(\mathcal{W}(Y, Z)\omega(\xi, X)U, \xi) = 0.$$

Putting $X = Y = \{e_i\}$ in (5.2) where $\{e_1, e_2, e_3, \dots, e_{2n}, e_{2n+1} = \xi\}$ is an orthonormal basis of the tangent space at each point of the manifold M and then taking the summation over i , $1 \leq i \leq 2n + 1$, we get

$$(5.3) \quad \sum_{i=1}^{2n+1} \left[g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi) - g(\mathcal{W}(\omega(\xi, e_i)e_i, Z)U, \xi) \right. \\ \left. - g(\mathcal{W}(e_i, \omega(\xi, e_i)Z)U, \xi) - g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi) \right] = 0.$$

From the equation (3.7), we can easily estimate the followings

$$(5.4) \quad \eta(\mathcal{W}(\xi, U)Z) = \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2na - 2nb - 1 \right] \eta(Z)\eta(U) \\ + \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2nb - 1 \right] g(Z, U) - aS(Z, U),$$

$$(5.5) \quad \sum_{i=1}^{2n+1} \tilde{\mathcal{W}}(e_i, Z, U, e_i) = (1 - b + 2na)S(Z, U) \\ + \left\{ br - \frac{2ncr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} g(Z, U),$$

$$(5.6) \quad \sum_{i=1}^{2n+1} \eta(\mathcal{W}(e_i, Z)e_i) = -2n(1 - a + 2nb)\eta(Z) \\ - \left\{ ar - \frac{2ncr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \eta(Z),$$

$$(5.7) \quad \sum_{i=1}^{2n+1} S(\mathcal{W}(e_i, Z)U, e_i) = \left\{ ar + \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} S(Z, U) \\ - (a + b - 1)S^2(Z, U) - \frac{cr^2}{2n+1} \left(\frac{1}{2n} + a + b \right) g(Z, U) + bS(Qe_i, e_i)g(Z, U),$$

$$(5.8) \quad \sum_{i=1}^{2n+1} \eta(e_i)\eta(\mathcal{W}(Qe_i, Z)U) \\ = -2n \left[1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] g(Z, U) - 2naS(Z, U) \\ - 2n \left[1 + 2n(a + b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] \eta(Z)\eta(U)$$

and

$$(5.9) \quad \sum_{i=1}^{2n+1} S(e_i, Z)\eta(\mathcal{W}(e_i, \xi)U) \\ = 2n \left[1 + 2n(a + b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] \eta(Z)\eta(U) \\ + \left\{ 1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} S(Z, U) + aS^2(Z, U).$$

Now,

$$(5.10) \quad \sum_{i=1}^{2n+1} g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi) \\ = \left[\frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right] \{ \bar{\mathcal{W}}(e_i, Z, U, e_i) + \eta(\mathcal{W}(e_i, Z)U)\eta(e_i) \} \\ - \bar{a} [S(\mathcal{W}(e_i, Z)U, e_i) + 2n\eta(\mathcal{W}(e_i, Z)U)\eta(e_i)] \\ = \left[\frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right] \bar{\mathcal{W}}(e_i, Z, U, e_i) - \bar{a}S(\mathcal{W}(e_i, Z)U, e_i) \\ + \left[\frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 - 2n\bar{a} \right] \eta(\mathcal{W}(\xi, U)Z).$$

In consequence of (5.5) and (5.7), the equation (5.10) becomes

$$(5.11) \quad \sum_{i=1}^{2n+1} g(\omega(\xi, e_i)\mathcal{W}(e_i, Z)U, \xi) = \\ \left[\left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} (1 + 2na - b) \right]$$

$$\begin{aligned}
& -\bar{a} \left\{ ar + \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \Big] S(Z, U) + \bar{a}(a+b-1)S^2(Z, U) \\
& + \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n(\bar{a} + \bar{b}) - 1 \right\} \eta(\mathcal{W}(\xi, U)Z) \\
& + \left[\left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} \left\{ br - \frac{2n\bar{c}r}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right. \\
& \left. - \bar{a} \left\{ b \parallel Q \parallel^2 - \frac{cr^2}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right] g(Z, U).
\end{aligned}$$

Again,

$$\begin{aligned}
(5.12) \quad & \sum_{i=1}^{2n+1} g(\mathcal{W}(\omega(\xi, e_i)e_i, Z)U, \xi) \\
& = \left[2n \left\{ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) \right\} + \bar{a}r + 2n\bar{b} - 2n\bar{a} \right] \eta(\mathcal{W}(\xi, U)Z) \\
& + 2n\bar{b} \left[1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] g(Z, U) + 2n\bar{a}\bar{b}S(Z, U) \\
& + 2n\bar{b} \left[1 + 2n(a+b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] \eta(Z)\eta(U).
\end{aligned}$$

By virtue of (5.9), we have

$$\begin{aligned}
(5.13) \quad & \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, \omega(\xi, e_i)Z)U, \xi) \\
& = \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} \eta(\mathcal{W}(\xi, U)Z) \\
& + 2n\bar{a} \left[1 + 2n(a+b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] \eta(Z)\eta(U) \\
& + \bar{a} \left\{ 1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} S(Z, U) + a\bar{a}S^2(Z, U).
\end{aligned}$$

Finally,

$$\begin{aligned}
(5.14) \quad & \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi) \\
& = \left\{ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) \right\} g(e_i, U)\eta(\mathcal{W}(e_i, Z)\xi) \\
& + \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \eta(\mathcal{W}(e_i, Z)e_i)\eta(U) \\
& + \bar{a}S(e_i, U)\eta(\mathcal{W}(e_i, Z)\xi)
\end{aligned}$$

$$= \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \eta(\mathcal{W}(e_i, Z)e_i)\eta(U).$$

In view of (5.6), the equation (5.14) turns into

$$(5.15) \quad \sum_{i=1}^{2n+1} g(\mathcal{W}(e_i, Z)\omega(\xi, e_i)U, \xi) \\ = - \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \left[2n(1-a+2nb) \right. \\ \left. + \left\{ ar - \frac{2ncr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right] \eta(U)\eta(Z).$$

By virtue of (5.4), (5.11), (5.12), (5.13) and (5.15), we obtain from (5.3) that

$$(5.16) \quad \left[\left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} (1+2na-b) \right. \\ - \bar{a} \left\{ ar + \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} + a\bar{a}r \\ + 2na \left\{ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) \right\} \\ \left. - \bar{a} \left\{ 1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right] S(Z, U) \\ + \left[\left\{ 2n(1-a+2nb) + ar - \frac{2ncr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \times \right. \\ \times \left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{a} - 1 \right\} \\ + \left\{ \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2n(a+b) - 1 \right\} \times \\ \times \left\{ 2n \left\{ -\bar{b} + \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 1 - 2n\bar{b} \right\} - \bar{a}r \right\} \\ \left. - 2n(\bar{a} + \bar{b}) \left\{ 1 + 2n(a+b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right] \eta(U)\eta(Z) \\ + \left[\left\{ \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) - 2n\bar{b} - 1 \right\} \left\{ br - \frac{2ncr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right. \\ + \left\{ \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2nb - 1 \right\} \times \\ \times \left\{ -2n \left\{ 1 + 2n\bar{b} - \frac{\bar{c}r}{2n+1} \left(\frac{1}{2n} + \bar{a} + \bar{b} \right) \right\} - \bar{a}r - 2n\bar{b} \right\} \\ \left. - \bar{a} \left\{ b \parallel Q \parallel^2 - \frac{cr^2}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \right]$$

$$-2n\bar{b} \left\{ 1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right\} \Big] g(Z, U) \\ + \bar{a}(b-1)S^2(Z, U) = 0.$$

From (5.16), one can easily bring out the following.

Theorem 5.2. *Let (M^{2n+1}, g) , $n > 1$ be an LP-Sasakian manifold. Then the nature of Ricci tensor for respective semi-symmetry type curvature conditions are given in Table 1.*

6. LP-SASAKIAN MANIFOLD SATISFYING THE CONDITION $\mathcal{W} \cdot S = 0$

Let $M^{2n+1}(\phi, \xi, \eta, g)$ ($n > 1$), be an LP-Sasakian manifold, satisfying the condition

$$(6.1) \quad \mathcal{W}(\xi, Y) \cdot S = 0,$$

i.e.

$$\mathcal{W}(\xi, Y)S(Z, U) - S(\mathcal{W}(\xi, Y)Z, U) - S(Z, \mathcal{W}(\xi, Y)U) = 0,$$

i.e.

$$(6.2) \quad S(\mathcal{W}(\xi, Y)Z, U) + S(Z, \mathcal{W}(\xi, Y)U) = 0.$$

Taking $U = \xi$ in (6.2) and using (2.9), we get

$$(6.3) \quad 2n\eta(\mathcal{W}(\xi, Y)Z) + S(Z, \mathcal{W}(\xi, Y)\xi) = 0.$$

In view of (2.7), (2.9) and (3.7), we have

$$(6.4) \quad \eta(\mathcal{W}(\xi, Y)Z) = \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2na - 2nb - 1 \right] \eta(Z)\eta(Y) \\ + \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 2nb - 1 \right] g(Z, Y) - aS(Z, Y).$$

$$(6.5) \quad S(Z, \mathcal{W}(\xi, Y)\xi) = \left[1 + 2na - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] S(Y, Z) \\ + 2n \left[1 + 2n(a+b) - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] \eta(Y)\eta(Z) + bS^2(Y, Z).$$

By virtue of (6.4) and (6.5), the equation (6.3) yields

$$(6.6) \quad S^2(Y, Z) = \frac{2n}{b} \left[1 + 2nb - \frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) \right] g(Y, Z) \\ + \frac{1}{b} \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a + b \right) - 1 \right] S(Y, Z)$$

for all Y, Z where $b \neq 0$, provided $S^2(X, Y) = S(QX, Y)$.

This leads to Theorem 6.1.

<i>Curvature condition</i>	<i>Expression for Ricci tensor</i>
$R(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0$ & $a = b = c = 0$)	$S = 2ng,$ an Einstein space.
$R(\xi, X) \cdot C = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0,$ $a = b = -\frac{1}{2n-1}$ & $c = 1$)	$S = (\frac{r}{2n} - 1)g + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an η - Einstein space.
$R(\xi, X) \cdot \hat{C} = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0,$ $a = b = -\frac{1}{2n-1}$ & $c = 0$)	$S = (\frac{r}{2n} - 1)g + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an η - Einstein space.
$R(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0,$ $a = b = 0$ & $c = 1$)	$S = 2ng,$ an Einstein space.
$R(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0,$ $a = -\frac{1}{2n}$ & $b = c = 0$)	$S = 2ng + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an η - Einstein space.
$R(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = \bar{b} = \bar{c} = 0,$ $a = b = -\frac{1}{4n}$ & $c = 0$)	$S = (\frac{r+4n^2}{4n+1})g + \{\frac{r-2n(2n+1)}{4n+1}\}\eta \otimes \eta,$ an η - Einstein space.
$E(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = \bar{b} = 0, \bar{c} = 1,$ & $a = b = c = 0$)	$S = 2ng,$ an Einstein space.
$E(\xi, X) \cdot C = 0$ (Obtain by $\bar{a} = 0 = \bar{b}, \bar{c} = 1$ $a = b = -\frac{1}{2n-1}, c = 1$)	$S = (\frac{r}{2n} - 1)g + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an η - Einstein space.
$E(\xi, X) \cdot \hat{C} = 0$ (Obtain by $\bar{a} = 0 = \bar{b}, \bar{c} = 1$ $a = b = -\frac{1}{2n-1}$ & $c = 1$)	$S = (\frac{r}{2n} - 1)g + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an Einstein space
$E(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = 0 = \bar{b}, \bar{c} = 1$ $a = b = 0$ & $c = 1$)	$S = 2ng,$ an Einstein space.
$E(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = 0 = \bar{b}, \bar{c} = 1$ $a = -\frac{1}{2n}, b = 0$ & $c = 1$)	$S = 2ng + (\frac{r}{2n} - 2n - 1)\eta \otimes \eta,$ an η - Einstein space.

TABLE 1

<i>Curvature condition</i>	<i>Expression for Ricci tensor</i>
$E(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = 0 = \bar{b}, \bar{c} = 1$ $a = b = -\frac{1}{4n}$ & $c = 1$)	$S = \left(\frac{r+4n^2}{4n+1}\right)g + \left\{\frac{r-2n(2n+1)}{4n+1}\right\}\eta \otimes \eta,$ an η -Einstein space.
$\hat{C}(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0$ & $a = b = c = 0$)	$2S = (r + 2n)g + (r - 2n)\eta \otimes \eta - S^2.$
$\hat{C}(\xi, X) \cdot \hat{C} = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0,$ $a = b = -\frac{1}{2n-1}$ & $c = 0$)	$(2n - 1)S = (\ Q\ ^2 - 2n)g - 2nr\eta \otimes \eta + 2nS^2.$
$\hat{C}(\xi, X) \cdot C = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0,$ $a = b = -\frac{1}{2n-1}$ & $c = 1$)	$(2n - 1)S = (\ Q\ ^2 - 2n)g + \left\{\frac{r^2}{2n} - (2n + 1)r\right\}\eta \otimes \eta - 2nS^2$
$\hat{C}(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0$ & $a = b = c = 0$)	$2S = (r + 2n)g + S^2 + (2n - r)\left\{\frac{r}{2n(2n+1)} - 1\right\}\eta \otimes \eta.$
$\hat{C}(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0$ $a = -\frac{1}{2n}, b = 0$ & $c = 0$)	$2S = (r + 2n)g + \left(\frac{r}{2n} - 2n - 1\right)\eta \otimes \eta + S^2.$
$\hat{C}(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 0$ $a = b = -\frac{1}{4n}$ & $c = 0$)	$\frac{6n+1}{4n}S = \left(\frac{2n+1}{4n}r + n + \frac{1}{4n}\ Q\ ^2\right)g + \left(\frac{r}{4n} - \frac{2n+1}{2}\right)\eta \otimes \eta - \left(1 + \frac{1}{4n}\right)S^2.$
$P(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ & $a = b = c = 0$)	$\left(\frac{1}{2n} - 1\right)S = \left(\frac{r}{2n} - 2n\right)g + \left(\frac{r}{2n} - 2n - 1\right)\eta \otimes \eta + \frac{1}{2n}S^2.$
$P(\xi, X) \cdot \hat{C} = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ $a = b = -\frac{1}{2n-1}$ & $c = 0$)	$\frac{4n^2+1}{2n}S = \left\{\frac{2n+1}{2n}r - 2n - \frac{\ Q\ ^2}{2n}\right\}g + \left\{\frac{2n+1}{2n}r - (2n + 1)^2\right\}\eta \otimes \eta + S^2.$
$P(\xi, X) \cdot C = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ $a = b = -\frac{1}{2n-1}$ & $c = 1$)	$-\frac{4n^2+1}{2n}S = \left\{\frac{r}{2n} - (2n + 1)\right\}^2\eta \otimes \eta - S^2 + \left(\frac{r}{2n} - 2n\right)\left(\frac{r}{2n} - 1\right)g - \frac{1}{2n}\left(\frac{r^2}{2n} - \ Q\ ^2\right)g.$
$P(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ $a = b = 0$ & $c = 1$)	$\frac{1-2n}{2n}S = \left(\frac{r}{2n} - 2n\right)g - \frac{1}{2n+1}\left\{\frac{r}{2n} - (2n + 1)\right\}^2\eta \otimes \eta - \frac{1}{2n}S^2.$
$P(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ $a = -\frac{1}{2n}, b = 0$ & $c = 0$)	$(1 - 2n)S = (r - 4n^2)g - S^2.$
$P(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = -\frac{1}{2n}, \bar{b} = 0, \bar{c} = 0$ $a = b = -\frac{1}{4n}$ & $c = 0$)	$S = \left(n - \frac{1}{8n^2}\ Q\ ^2\right)g + \frac{1}{2n}\left(1 + \frac{1}{4n}\right)S^2.$

TABLE 2

<i>Curvature condition</i>	<i>Expression for Ricci tensor</i>
$H(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{4n}, \bar{c} = 0$ & $a = b = c = 0$)	$\frac{1-2n}{4n} S = \left(\frac{r}{4n} - n\right) g$ $+ \left(\frac{r}{4n} - \frac{1}{2}\right) \eta \otimes \eta - \frac{1}{4n} S^2.$
$H(\xi, X) \circ \hat{C} = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{4n}, \bar{c} = 0$ $a = b = -\frac{1}{2n-1}$ & $c = 0$)	$\frac{4n^2+1}{4n} S = \left\{ \frac{2n+1}{4n} r - n - \frac{\ Q\ ^2}{4n} \right\} g$ $+ \left\{ \frac{4n+1}{4n} r - \frac{(2n+1)^2}{2} \right\} \eta \otimes \eta + \frac{1}{2n} S^2.$
$H(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{4n}, \bar{c} = 0$ $a = b = 0$ & $c = 1$)	$\frac{1-2n}{4n} S = \left(\frac{r}{4n} - n\right) g - \frac{1}{4n} S^2$ $+ \left(\frac{r}{2n} - 1\right) \left(\frac{1}{2} - \frac{r}{4n(2n+1)}\right) \eta \otimes \eta.$
$H(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{4n}, \bar{c} = 0,$ $a = -\frac{1}{2n}, b = 0$ & $c = 0$)	$\frac{1-2n}{4n} S = \left(\frac{r}{4n} - n\right) g$ $+ \frac{1}{2} \left\{ 2n + 1 - \frac{r}{2n} \right\} \eta \otimes \eta - \frac{1}{4n} S^2.$
$H(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{4n}, \bar{c} = 0$ $a = b = -\frac{1}{4n}$ & $c = 0$)	$S = \frac{1}{2n} \left(1 + \frac{1}{4n}\right) S^2$ $+ \left(n - \frac{1}{8n^2} \ Q\ ^2\right) g$ $+ \left(\frac{r}{4n} - \frac{2n+1}{2}\right) \eta \otimes \eta.$
$C(\xi, X) \cdot R = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = b = c = 0$)	$\left(2 - \frac{r}{2n}\right) S = 2ng - S^2$ $+ (r - 2n) \eta \otimes \eta.$
$C(\xi, X) \cdot \hat{C} = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = b = -\frac{1}{2n-1}, c = 0$)	$(2n - 1 - r) S =$ $\left\{ \left(r - \frac{r^2}{2n}\right) - 2n + \ Q\ ^2 \right\} g$ $+ \left(r - \frac{r^2}{2n}\right) \eta \otimes \eta - 2n S^2.$
$C(\xi, X) \cdot C = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = b = -\frac{1}{2n-1}, c = 0$)	$(2n - 1 - r) S =$ $\left\{ \left(r - \frac{r^2}{2n}\right) - 2n + \ Q\ ^2 \right\} g - 2n S^2.$
$C(\xi, X) \cdot P = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = -\frac{1}{2n}, b = c = 0$)	$\left(2 - \frac{r}{2n}\right) S = 2ng - S^2$ $+ \left(\frac{r}{2n} - 1\right) \left(2n + 1 - \frac{r}{2n}\right) \eta \otimes \eta.$
$C(\xi, X) \cdot E = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = b = 0, c = 1$)	$\left(2 - \frac{r}{2n}\right) S = 2ng - S^2$ $+ \left(\frac{r}{2n} - 1\right) \left\{ 2n(2n+1) - \frac{r}{2n+1} \right\} \eta \otimes \eta.$
$C(\xi, X) \cdot H = 0$ (Obtain by $\bar{a} = \bar{b} = -\frac{1}{2n-1}, \bar{c} = 1$ $a = b = -\frac{1}{4n}, c = 0$)	$\frac{1}{n} \left\{ \left(3n + \frac{1}{2}\right) - \left(r + \frac{r^2}{4n}\right) \right\} S$ $= -\left(2 + \frac{1}{2n}\right) S^2$ $+ \left\{ \frac{r}{2n} \left(1 - \frac{r}{2n}\right) + 2n + \frac{\ Q\ ^2}{2n} \right\} g$ $+ \left(\frac{r}{2n} - 2n - 1\right) \left(1 - \frac{r}{2n}\right) \eta \otimes \eta.$

TABLE 3

Theorem 6.1. *Let $M^{2n+1}(\phi, \xi, \eta, g)$ be an LP-Sasakian manifold with $\mathcal{W} \cdot S = 0$. Then the Ricci tensor S admit the relation (6.6) provided $b \neq 0$.*

Now, for $b = 0$, the equation (6.6) reduces to

$$(6.7) \quad \left[\frac{cr}{2n+1} \left(\frac{1}{2n} + a \right) - 1 \right] \{S(Y, Z) - 2ng(Y, Z)\} = 0.$$

i.e.,

$$(6.8) \quad S(Y, Z) = 2ng(Y, Z) \quad \text{or} \quad \frac{cr}{2n+1} \left(\frac{1}{2n} + a \right) - 1 \neq 0$$

Theorem 6.2. *Let $M^{2n+1}(\phi, \xi, \eta, g)$ ($n > 1$), be a LP-Sasakian manifold. Then the following conditions are equivalent:*

- (a) M is an Einstein space.
- (b) M is Ricci symmetric i.e., $\nabla S = 0$.
- (c) $P(\xi, X) \cdot S = 0$ (or $E(\xi, X) \cdot S = 0$) for all $X \in \chi(M)$.

Example 6.3. (see [24, p. 286-287]) Let $M^3(\phi, \xi, \eta, g)$ be an LP-Sasakian manifold (M^3, g) with a ϕ -basis

$$e = e^z \frac{\partial}{\partial x}, \quad \phi e = e^{z-\alpha x} \frac{\partial}{\partial x}, \quad \xi = \frac{\partial}{\partial x}, \quad \text{where } \alpha \text{ is non-zero constant.}$$

Then from Koszul's formula for Lorentzian metric g , we can obtain the Levi-Civita connection as follows

$$\begin{aligned} \nabla_e \xi &= \phi e, & \nabla_e \phi e &= 0, & \nabla_e e &= -\xi, \\ \nabla_{\phi e} \xi &= e, & \nabla_{\phi e} \phi e &= \alpha e^z e, & \nabla_{\phi e} e &= \alpha e^z \phi e, \\ \nabla_\xi \xi &= 0, & \nabla_\xi \phi e &= 0, & \nabla_\xi e &= 0. \end{aligned}$$

Using the above relations, one can easily calculate the non-vanishing components of the curvature tensor R , Ricci tensor S , scalar curvature r and generalized quasi-conformal curvature tensor $\mathcal{W}$ as follows

$$\begin{aligned} R(\phi e, \xi) \xi &= -\phi e, & R(e, \xi) \xi &= -e, & R(e, \phi e) \phi e &= (1 - \alpha^2 e^{2z}) e, \\ R(\phi e, \xi) \phi e &= -\xi, & R(e, \xi) e &= -\xi, & R(e, \phi e) e &= -(1 - \alpha^2 e^{2z}) \phi e, \\ S(e, e) &= -\alpha^2 e^{2z}, & S(\phi e, \phi e) &= -\alpha e^z, & S(\xi, \xi) &= -2, \quad r = 2(1 - \alpha^2 e^{2z}), \\ \mathcal{W}(e, \xi) \xi &= -\lambda_1 e, & \mathcal{W}(\phi e, \xi) \xi &= -\lambda_1 \phi e, \\ \mathcal{W}(e, \xi) e &= -\lambda_2 \xi, & \mathcal{W}(\phi e, \xi) \phi e &= -\lambda_2 \xi, \\ \mathcal{W}(e, \phi e) \phi e &= \lambda_3 e, & \mathcal{W}(e, \phi e) e &= -\lambda_3 \phi e, \\ \omega(e, \xi) \xi &= -\bar{\lambda}_1 e, & \omega(\phi e, \xi) \xi &= -\bar{\lambda}_1 \phi e, \\ \omega(e, \xi) e &= -\bar{\lambda}_2 \xi, & \omega(\phi e, \xi) \phi e &= -\bar{\lambda}_2 \xi, \end{aligned}$$

where,

$$\lambda_1 = \left[1 + 2a - b\alpha^2 e^{2z} - \frac{c}{3}(1 + 2a + 2b)(1 - \alpha^2 e^{2z}) \right],$$

$$\begin{aligned}
\lambda_2 &= \left[1 - a\alpha^2 e^{2z} + 2b - \frac{c}{3}(1 + 2a + 2b)(1 - \alpha^2 e^{2z}) \right], \\
\lambda_3 &= \left[(1 - \alpha^2 e^{2z}) \left(1 - \frac{c}{3} \right) + (a + b)\alpha^2 e^{2z} \left(\frac{2c}{3} - 1 \right) \right], \\
\bar{\lambda}_1 &= \left[1 + 2\bar{a} - \bar{b}\alpha^2 e^{2z} - \frac{\bar{c}}{3}(1 + 2\bar{a} + 2\bar{b})(1 - \alpha^2 e^{2z}) \right], \\
\bar{\lambda}_2 &= \left[1 - \bar{a}\alpha^2 e^{2z} + 2\bar{b} - \frac{\bar{c}}{3}(1 + 2\bar{a} + 2\bar{b})(1 - \alpha^2 e^{2z}) \right]
\end{aligned}$$

and the components which can be obtained from these by the symmetry properties. we calculate the covariant derivatives of the non-vanishing components of the curvature tensor as follows:

$$\begin{aligned}
(\omega(e, \xi) \cdot \mathcal{W})(\phi e, \xi) \phi e &= \bar{\lambda}_1(\lambda_2 - \lambda_3)e, & (\omega(e, \xi) \cdot \mathcal{W})(e, \phi e) \phi e &= \bar{\lambda}_2(\lambda_2 - \lambda_3)\xi, \\
(\omega(e, \xi) \cdot \mathcal{W})(e, \phi e)e &= 0, & (\omega(e, \xi) \cdot \mathcal{W})(e, \phi e)\xi &= (\lambda_1 \bar{\lambda}_2 - \bar{\lambda}_1 \lambda_3)\phi e, \\
(\omega(\phi e, \xi) \cdot \mathcal{W})(e, \phi e)\xi &= (\bar{\lambda}_1 \lambda_3 - \lambda_1 \bar{\lambda}_2)e, & (\omega(\phi e, \xi) \cdot \mathcal{W})(e, \xi)e &= \bar{\lambda}_1(\lambda_2 - \lambda_3), \\
(\omega(\phi e, \xi) \cdot \mathcal{W})(e, \phi e) \phi e &= 0, & (\omega(\phi e, \xi) \cdot \mathcal{W})(\phi e, \xi) \phi e &= (\bar{\lambda}_1 \lambda_2 - \lambda_1 \bar{\lambda}_2)\phi e, \\
(\omega(\phi e, \xi) \cdot \mathcal{W})(e, \phi e)e &= \bar{\lambda}_2(\lambda_3 - \lambda_2)\xi, & (\omega(e, \xi) \cdot \mathcal{W})(e, \xi)e &= (\bar{\lambda}_1 \lambda_2 - \lambda_1 \bar{\lambda}_2)e, \\
(\omega(e, \xi) \cdot \mathcal{W})(\phi e, \xi)e &= (\bar{\lambda}_1 \lambda_3 - \lambda_1 \bar{\lambda}_2)\xi, & (\omega(\phi e, \xi) \cdot \mathcal{W})(\phi e, \xi)e &= 0, \\
(\omega(e, \xi) \cdot \mathcal{W})(e, \xi) \phi e &= 0, & (\omega(\phi e, \xi) \cdot \mathcal{W})(e, \xi) \phi e &= (\bar{\lambda}_1 \lambda_3 - \lambda_1 \bar{\lambda}_2)e.
\end{aligned}$$

From the above it is clear that the manifold (M^3, g) under consideration is a semi-symmetry type LP-Sasakian manifold if $\bar{\lambda}_1 = \lambda_2 = \lambda_3$.

Lemma 6.4. *There exists a semi-symmetry type LP-Sasakian manifold provided $\bar{\lambda}_1 = \lambda_1$, $\bar{\lambda}_2 = \lambda_2 = \lambda_3 = \bar{\lambda}_3$.*

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