

ON THE CONVERGENCE OF CESÀRO MEANS OF NEGATIVE ORDER OF WALSH-FOURIER SERIES

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ABSTRACT. In this paper we investigate the convergence of Cesàro means of negative order of Walsh-Fourier series of functions of generalized bounded oscillation.

Let $r_0(x)$ be a function defined on $R := (-\infty, \infty)$ by

$$r_0(x) = \begin{cases} 1, & \text{if } x \in [0, \frac{1}{2}) \\ -1, & \text{if } x \in [\frac{1}{2}, 1) \end{cases}, \quad r_0(x+1) = r_0(x).$$

The Rademacher system is defined by

$$r_n(x) = r_0(2^n x), \quad n \geq 1 \text{ and } x \in [0, 1).$$

Let $w_0, w_1, \dots$ represent the Walsh functions, i.e., $w_0(x) = 1$ and if $k = 2^{n_1} + \dots + 2^{n_s}$ is a positive integer with $n_1 > n_2 > \dots > n_s$ then $w_k(x) = r_{n_1}(x) \times \dots \times r_{n_s}(x)$.

The idea of using products of Rademacher's functions to define the Walsh system originated from Paley [16].

The Walsh-Dirichlet kernel is defined by

$$D_n(x) = \sum_{k=0}^{n-1} w_k(x).$$

Recall that

$$D_{2^n}(x) = \begin{cases} 2^n, & \text{if } x \in [0, \frac{1}{2^n}) \\ 0, & \text{if } x \in [\frac{1}{2^n}, 1). \end{cases}$$

Suppose that f is a Lebesgue integrable function on $[0, 1]$ and 1-periodic. Then its Walsh-Fourier series is defined by

$$\sum_{k=0}^{\infty} \widehat{f}(k) w_k(x),$$

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where

$$\widehat{f}(k) = \int_0^1 f(t) w_k(t) dt$$

is called the k -th Walsh-Fourier coefficient of the function f . Denote by $S_n(f, x)$ the n -th partial sum of the Walsh-Fourier series of the function f , namely

$$S_n(f, x) = \sum_{k=0}^{n-1} \widehat{f}(k) w_k(x).$$

The Cesàro (C, α) -means of the Walsh-Fourier series are defined as

$$\sigma_n^\alpha(f, x) = \frac{1}{A_n^\alpha} \sum_{k=0}^n A_{n-k}^\alpha \widehat{f}(k) w_k(x),$$

where

$$A_0^\alpha = 1, \\ A_n^\alpha = \frac{(\alpha + 1) \cdots (\alpha + n)}{n!}, \quad \alpha \neq -1, -2, \dots$$

Let $C([0, 1])$ denote the space of continuous functions f with period 1. If $f \in C([0, 1])$, then the function

$$\omega(\delta, f) = \sup \{ |f(x') - f(x'')| : |x' - x''| \leq \delta, x', x'' \in [0, 1] \}$$

is called the modulus of continuity of the function f . The modulus of continuity of an arbitrary function $f \in C([0, 1])$ has the following properties:

- 1) $\omega(0) = 0$,
- 2) $\omega(\delta)$ is nondecreasing,
- 3) $\omega(\delta)$ is continuous on $[0, 1]$,
- 4) $\omega(\delta_1 + \delta_2) \leq \omega(\delta_1) + \omega(\delta_2)$ for $0 \leq \delta_1 \leq \delta_2 \leq \delta_1 + \delta_2 \leq 1$.

An arbitrary function $\omega(\delta)$ which is defined on $[0, 1]$ and has properties 1) – 4) is called a modulus of continuity. If the modulus of continuity $\omega(\delta)$ is given, then H^ω denotes the class of functions $f \in C([0, 1])$ for which

$$\omega(\delta, f) = O(\omega(\delta)) \quad \text{as} \quad \delta \rightarrow 0.$$

$C_w([0, 1])$ is the collection of functions $f: [0, 1) \rightarrow R$ that are uniformly continuous from the dyadic topology of $[0, 1)$ to the usual topology of R , or for short: uniformly W -continuous.

Let f be defined on $[0, 1)$. We shall represent the dyadic modulus of continuity by

$$\dot{\omega}(\delta, f) = \sup_{0 \leq h \leq \delta} \sup_x |f(x \oplus h) - f(x)|,$$

where $\oplus$ denotes dyadic addition (see [12] or [18]).

The problems of summability of Cesàro means of the Walsh-Fourier series were studied in [4], [7], [10], [9], [8], [16], [18], [17].

Tevzadze [19] has studied the uniform convergence of Cesàro means of negative order of the Walsh-Fourier series. In particular, in terms of modulus of

continuity and variation of function $f \in C_w([0, 1])$ he has proved the criterion for the uniform summability by the Cesàro method of negative order of Fourier series with respect to the Walsh system.

In [9] Goginava investigated the problem of estimating the deviation of $f \in L_p$ from its Cesàro means of negative order of Walsh-Fourier series in the L_p -metric, $p \in [1, \infty)$. Analogous results for Walsh-Kaczmarz system were proved by Nagy [15] and Gát, Nagy [6].

In his monograph [23, part 1, chapter 4] Zhizhiashvili investigated the behaviour of Cesàro means of negative order of trigonometric Fourier series in detail.

The notion of a function-bounded variation was introduced by Jordan [13]. Generalizing this notion Wiener [21] considered the class of function V_p . Young [22] introduced the notion of the function of bounded Φ -variation. Waterman [20] studied the class of function of bounded Λ -variation, and Chanturia [3] defined the notion of the modulus of variation of a function. In 1990, Kita and Yoneda [14] introduced the notion of the generalized Wiener's class $BV(p(n) \uparrow p)$. Generalizing the class $BV(p(n) \uparrow p)$, Akhobadze [1, 2] considered the classes of function $BV(p(n) \uparrow p, \varphi)$ and $B\Lambda(p(n) \uparrow p, \varphi)$.

Definition 1. [11] Let $1 \leq p(n) \uparrow p$ as $n \rightarrow \infty$ where $1 \leq p \leq \infty$. We say that a function belongs to the $BO(p(n) \uparrow p)$ class if

$$O(f; p(n) \uparrow p) := \sup_n \left\{ \sum_{l=0}^{2^n-1} \sup_{t, u \in [l2^{-n}, (l+1)2^{-n}]} |f(t) - f(u)|^{p(n)} \right\}^{\frac{1}{p(n)}} < \infty.$$

When $p(n) = p$ for all n , $BO(p(n) \uparrow p)$ coincides with the class of p -bounded fluctuation BF_p [18].

Estimates of the Fourier coefficients of functions of bounded fluctuation with respect to the Vilenkin system were studied by Gát and Toledo [5].

In [11] Goginava proved that the following statements are true.

Theorem 1. *Let f be a function in the class $BO(p(n) \uparrow \infty)$ and*

$$\dot{\omega}\left(\frac{1}{2^n}, f\right) = o\left(\frac{1}{p(n+1) \log_2 p(n+1)}\right) \text{ as } n \rightarrow \infty.$$

Then the Walsh-Fourier series of the function f converges uniformly in $[0, 1]$.

Theorem 2. *Let $p(2n) \leq cp(n)$, $n \in P$ and $p(n) \log_2 p(n) = o(n)$ as $n \rightarrow \infty$. If ω satisfies the condition*

$$\lim_{n \rightarrow \infty} \sup \omega\left(\frac{1}{n}\right) p([\log_2 n]) \log_2 p([\log_2 n]) = c_0 > 0,$$

then there exists a function in the class $H^\omega \cap BO(p(n) \uparrow \infty)$ for which the Walsh-Fourier series diverges at some point.

The theorem of Tevzadze [19] implies that if $p < \frac{1}{\alpha}$ and $f \in BF_p \cap C_\omega$, then the Cesàro mean $\sigma_n^{-\alpha}(f)$ of Walsh-Fourier series uniformly converges to the

function f . On the other hand, for $p = \frac{1}{\alpha}$ there exists a continuous function f for which $\sigma_n^{-\alpha}(f, 0)$ diverges. On the basis of the above facts the following problems arise naturally:

Let $f \in BO\left(p(n) \uparrow \frac{1}{\alpha}\right)$, $0 < \alpha < 1$. Under what condition on the sequence $\{p(n) : n \geq 1\}$ the uniform convergence of Cesàro $(C, -\alpha)$ means of Walsh-Fourier series of the function f holds?

The following theorem is true.

Theorem 3. *Let $f \in C_w([0, 1]) \cap BO\left(p(n) \uparrow \frac{1}{\alpha}\right)$, $0 < \alpha < 1$, $2^k \leq n \leq 2^{k+1}$. Then*

$$\left\| \sigma_n^{-\alpha}(f) - f \right\|_c \leq c(\alpha) \left\{ \sum_{r=0}^k 2^{r-k} \dot{\omega}\left(\frac{1}{2^r}, f\right)_c + \frac{\left(\dot{\omega}\left(\frac{1}{2^k}, f\right)\right)^{1-\alpha p(k)}}{1 - \alpha p(k)} \right\}.$$

Corollary 1. *Let $f \in C_w([0, 1]) \cap BO\left(p(n) \uparrow \frac{1}{\alpha}\right)$, $0 < \alpha < 1$ and*

$$\frac{\left(\dot{\omega}\left(\frac{1}{2^k}, f\right)\right)^{1-\alpha p(k)}}{1 - \alpha p(k)} \rightarrow 0 \quad \text{as } k \rightarrow \infty.$$

Then

$$\left\| \sigma_n^{-\alpha}(f) - f \right\|_c \rightarrow 0.$$

In order to prove Theorem 3 we need the following lemmas proved by Goginava in [9, 8].

Lemma 1 (Goginava [9]). *Let $f \in C_w([0, 1])$. Then for every $\alpha \in (0, 1)$ the following estimation holds*

$$\frac{1}{A_n^{-\alpha}} \left\| \int_0^1 \sum_{\nu=0}^{2^{k-1}-1} A_{n-\nu}^{-\alpha} w_\nu(u) [f(\cdot \oplus u) - f(\cdot)] du \right\|_c \leq c(p, \alpha) \sum_{r=0}^{k-1} 2^{r-k} \dot{\omega}(1/2^r, f)_p,$$

where $2^k \leq n < 2^{k+1}$.

Lemma 2 (Goginava [8]). *Let $f \in C_w([0, 1])$ and $2^k \leq n < 2^{k+1}$. Then for every $\alpha \in (0, 1)$ the following estimations hold*

$$\begin{aligned} & \frac{1}{A_n^{-\alpha}} \left| \int_0^1 \sum_{\nu=2^{k-1}}^{2^k-1} A_{n-\nu}^{-\alpha} w_\nu(u) [f(\cdot \oplus u) - f(\cdot)] du \right| \\ & \leq c(\alpha) \left(\sum_{j=1}^{2^{k-1}-1} \frac{1}{j^{1-\alpha}} \left| f\left(x \oplus \frac{2j}{2^k}\right) - f\left(x \oplus \frac{2j+1}{2^k}\right) \right| \right), \\ & \frac{1}{A_n^{-\alpha}} \left| \int_0^1 \sum_{\nu=2^k}^n A_{n-\nu}^{-\alpha} w_\nu(u) [f(\cdot \oplus u) - f(\cdot)] du \right| \\ & \leq c(\alpha) \left(\sum_{j=1}^{2^k} \frac{1}{j^{1-\alpha}} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right| \right). \end{aligned}$$

Proof of Theorem 3. We can write

$$\begin{aligned}
 \sigma_n^{-\alpha}(f, x) - f(x) &= \frac{1}{A_n^{-\alpha}} \int_0^1 \sum_{\nu=0}^n A_{n-\nu}^{-\alpha} w_\nu(x) [f(x \oplus u) - f(x)] du \\
 &= \frac{1}{A_n^{-\alpha}} \int_0^1 \sum_{\nu=0}^{2^{k-1}-1} A_{n-\nu}^{-\alpha} w_\nu(x) [f(x \oplus u) - f(x)] du \\
 (1) \quad &+ \frac{1}{A_n^{-\alpha}} \int_0^1 \sum_{\nu=2^{k-1}}^{2^k-1} A_{n-\nu}^{-\alpha} w_\nu(x) [f(x \oplus u) - f(x)] du \\
 &+ \frac{1}{A_n^{-\alpha}} \int_0^1 \sum_{\nu=2^k}^n A_{n-\nu}^{-\alpha} w_\nu(x) [f(x \oplus u) - f(x)] du \\
 &= I + II + III.
 \end{aligned}$$

From Lemmas 1 and 2 we have

$$(2) \quad \|I\|_c \leq c(\alpha) \sum_{\nu=0}^{k-1} 2^{r-k} \omega\left(\frac{1}{2^r}, f\right)_c,$$

$$|II| \leq c(\alpha) \left(\sum_{j=1}^{2^{k-1}-1} \frac{1}{j^{1-\alpha}} \left| f\left(x \oplus \frac{2j}{2^k}\right) - f\left(x \oplus \frac{2j+1}{2^k}\right) \right| \right)$$

and

$$|III| \leq c(\alpha) \left(\sum_{j=1}^{2^k-1} \frac{1}{j^{1-\alpha}} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right| \right).$$

Using Abel's transformation, we get

$$\begin{aligned}
 |III| &\leq c(\alpha) \left(\sum_{j=1}^{2^k-2} \left(\frac{1}{j^{1-\alpha}} - \frac{1}{(j+1)^{1-\alpha}} \right) \right. \\
 (3) \quad &\times \sum_{l=1}^j \left| f\left(x \oplus \frac{2l}{2^{k+1}}\right) - f\left(x \oplus \frac{2l+1}{2^{k+1}}\right) \right| \\
 &+ \frac{1}{(2^k-1)^{1-\alpha}} \sum_{j=1}^{2^k-1} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right| \Bigg) \\
 &= III_1 + III_2.
 \end{aligned}$$

Let $\varepsilon_k := \alpha p_k < 1$, $s_k := \frac{p(k)}{\varepsilon_k}$, $\frac{1}{s_k} + \frac{1}{t_k} = 1$. Then using Hölder's inequality for III_2 we can write

$$\begin{aligned}
III_2 &= \frac{1}{(2^k - 1)^{1-\alpha}} \sum_{j=1}^{2^k-1} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right|^{\varepsilon_k} \\
&\quad \times \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right|^{1-\varepsilon_k} \\
&\leq \frac{c(\alpha)}{2^{k(1-\alpha)}} \left(\sum_{j=1}^{2^k-1} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right|^{p(k)} \right)^{\frac{\varepsilon_k}{p(k)}} \\
&\quad \times \left(\sum_{j=1}^{2^k-1} \left| f\left(x \oplus \frac{2j}{2^{k+1}}\right) - f\left(x \oplus \frac{2j+1}{2^{k+1}}\right) \right|^{(1-\varepsilon_k)t_k} \right)^{\frac{1}{t_k}} \\
&\leq \frac{c(\alpha)}{2^{k(1-\alpha)}} \left(BO\left(f, p(k) \uparrow \frac{1}{\alpha}\right) \right)^{\varepsilon_k} \left(\dot{\omega}\left(f, \frac{1}{2^k}\right) \right)^{1-\varepsilon_k} 2^{\frac{k}{t_k}} \\
&\leq c(\alpha) \left(BO\left(f, p(k) \uparrow \frac{1}{\alpha}\right) \right)^{\varepsilon_k} \left(\dot{\omega}\left(f, \frac{1}{2^k}\right) \right)^{1-\varepsilon_k} 2^{k\left(\alpha - \frac{1}{s_k}\right)} \\
&= c(\alpha) \left(BO\left(f, p(k) \uparrow \frac{1}{\alpha}\right) \right)^{\varepsilon_k} \left(\dot{\omega}\left(f, \frac{1}{2^k}\right) \right)^{1-\varepsilon_k} 2^{k\left(\alpha - \frac{\varepsilon_k}{p(k)}\right)} \\
&= c(\alpha) \left(BO\left(f, p(k) \uparrow \frac{1}{\alpha}\right) \right)^{\varepsilon_k} \left(\dot{\omega}\left(f, \frac{1}{2^k}\right) \right)^{1-\alpha p(k)} \longrightarrow 0
\end{aligned}$$

as $k \rightarrow \infty$.

Fix $m_0(k)$ and define it later

$$\begin{aligned}
III_1 &\leq c(\alpha) \sum_{j=1}^{m_0(k)} \frac{1}{j^{2-\alpha}} \sum_{l=1}^j \left| f\left(x \oplus \frac{2l}{2^{k+1}}\right) - f\left(x \oplus \frac{2l+1}{2^{k+1}}\right) \right| \\
&\quad + \sum_{j=m_0(k)+1}^{2^k-1} \frac{1}{j^{2-\alpha}} \sum_{l=1}^j \left| f\left(x \oplus \frac{2l}{2^{k+1}}\right) - f\left(x \oplus \frac{2l+1}{2^{k+1}}\right) \right| \\
&\leq c(\alpha) \left\{ \sum_{j=1}^{m_0(k)} \frac{1}{j^{2-\alpha}} j \dot{\omega}\left(\frac{1}{2^k}, f\right) \right. \\
&\quad \left. + \sum_{j=m_0(k)+1}^{2^k-1} \frac{1}{j^{1+1/p(k)-\alpha}} \left(\sum_{l=1}^j \left| f\left(x \oplus \frac{2l}{2^{k+1}}\right) - f\left(x \oplus \frac{2l+1}{2^{k+1}}\right) \right|^{p(k)} \right)^{\frac{1}{p(k)}} \right\} \\
&\leq c(\alpha) \left\{ (m_0(k))^\alpha \dot{\omega}\left(\frac{1}{2^k}, f\right) + \frac{m_0(k)^{\alpha - \frac{1}{p(k)}}}{\frac{1}{p(k)} - \alpha} BO\left(f, p(k) \uparrow \frac{1}{\alpha}\right) \right\}.
\end{aligned}$$

Set

$$m_0(k) = \left(\frac{1}{\dot{\omega}\left(\frac{1}{2^k}, f\right)} \right)^{p(k)}.$$

Then we have

(4)

$$III_1 \leq c(\alpha) \left\{ \dot{\omega} \left(\frac{1}{2^k}, f \right)^{1-\alpha p(k)} + \frac{\dot{\omega} \left(\frac{1}{2^k}, f \right)^{1-\alpha p(k)}}{\frac{1}{p(k)} - \alpha} \right\} \leq c(\alpha) \frac{\dot{\omega} \left(\frac{1}{2^k}, f \right)^{1-\alpha p(k)}}{1 - \alpha p(k)}.$$

Combining (3) – (4) we have

$$(5) \quad |III| \leq c(\alpha) \frac{\dot{\omega} \left(\frac{1}{2^k}, f \right)^{1-\alpha p(k)}}{1 - \alpha p(k)}.$$

Analogously we can prove that

$$(6) \quad |II| \leq c(\alpha) \frac{\dot{\omega} \left(\frac{1}{2^k}, f \right)^{1-\alpha p(k)}}{1 - \alpha p(k)}.$$

Combining (1), (2), (5) and (6) we complete the proof of Theorem 3. $\square$

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